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ORIGINAL ARTICLE

Optimising influence in social networks using bounded rationality models

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Abstract Influence models enable the modelling of the spread of ideas, opinions and behaviours in social networks. Bounded rationality in social networks suggests that players make non-optimum decisions due to the limitations of access to information. Based on the premise that adopting a state or an idea can be regarded as being 'rational', we propose an influence model based on the heterogeneous bounded rationality of players in a social network. We employ the quantal response equilibrium model to incorporate the bounded rationality in the context of social influence. We hypothesise that bounded rationality of following a seed or adopting the strategy of a seed is negatively proportional to the distance from that node, and it follows that closeness centrality is the appropriate measure to place influencers in a social network. We argue that this model can be used in scenarios where there are multiple types of influencers and varying pay-offs of adopting a state. We compare different seed placement mechanisms to compare and contrast the optimum method to minimise the existing social influence in a network when there are multiple and conflicting seeds. We ascertain that placing of opposing seeds according to a measure derived from a combination of the betweenness centrality values from the seeds, and the closeness centrality of the network provide the maximum negative influence. Further, we extend this model to a strategic decision-making scenario where each seed operates a strategy in a strategic game.

Keywords Social influence · Game theory · Bounded rationality

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1 Introduction

Influence modelling in social networks is a key research problem with many applications over different domains. As a motivating example, consider how the present discussion on global warming takes place in online social media and in social networks in general. With the issue of global warming, the actions of individuals, organisations and governments are deeply influenced by several key individuals who may be scientists, political figures and social figures. Thus, modelling the influence of such key players over the rest of the network is an important research problem as it affects the spread of information over the network. This information spread may be key in determining the subsequent actions that affect the resolution or the aggravation of the issue at hand.

Numerous attempts have been made to model the influence in a social context. Two classical models are linear threshold model and the independent cascade model (Kempe et al. 2003). Both these models take into account the neighbourhood effect of adopting a particular state by a node in the social network. Social influence modelling tries to address the optimisation problem of finding the optimum configuration of seeds to maximise the social influence. Under both these models, the optimisation problem of selecting the most influential nodes has been shown to be an NP-hard problem (Kempe et al. 2003, 2005). Therefore, greedy algorithm is often used to come up with an approximated solution (Kempe et al. 2005). Another approach to model social influence has been to use the PageRrank algorithm-based models, especially with respect to measuring the influence of microblogs (Huang and Xiong 2013). Game-theoretic influential models too have been suggested to model social influence, where social influence is modelled as a strategic game (Clark and Poovendran 2011; Tzoumas et al. 2012). However, these models assume the prevalence of perfect rationality in players making their decision to adopt a particular state, even though in real-world players are boundedly rational (Gigerenzer and Selten 2002; Kasthurirathna and Piraveenan 2015).

In this work, we present a social influence model that is based on the bounded rationality of players in a social network. In the proposed model, the rationality of following an influencing node or adopting a strategy is negatively proportional to the distance from the seed. The bounded rationality-based probability of following is quantified using the quantal response equilibrium model (Goeree et al. 2008), which is based on a logit function. Based on the proposed model, we propose an optimisation technique to select the topological positions of the most influential nodes in a social network. Further, we argue that the proposed optimisation technique is more computationally efficient than the standard linear threshold and independent cascade models. Also, we propose an optimisation technique to place the negative influencers, provided that a set of influencers are already existing in a network. Thereby, we show that the average of closeness centrality and the betweenness centrality from the original set of influencers results in a measure that can be used to identify the optimum positions to place the negative influencers. We apply the proposed optimisation technique to the Wiki-vote real-world network (Leskovec and Krevl 2014) to further validate the proposed social influence model and the optimisation technique. However, when we place the opposing strategies in an alternating fashion, the closeness centrality gives the best performance in inhibiting the influence of the original set of influencers. Since the alternating placement of opposing strategies gives the local optimal solution, the global optimal solution is not achieved.

Finally, we extend the proposed bounded rationality-based social influence model to strategic games, where the social influence is proportional to the tendency of playing the most optimum strategy, using the prisoner's dilemma game as an example. The above-mentioned optimisation techniques are then shown to be applicable in strategic game scenarios as well.

The rest of this paper is organised as follows. In the next section, we discuss the relevant background knowledge for our work. In particular, we discuss social networks, existing social network influence models and game theory. Then, we present our influence model based on bounded rationality and quantal response equilibrium model. In the subsequent section, we extend this model into a strategic decision-making scenario. Next, we simulate the propagation of social influence when seeds are placed at different configurations. We propose an efficient mechanism to find the optimum placement of seeds to counter the influence of existing seeds, when there are multiple types of contending seeds. Finally, we discuss our results and present our conclusions.

2 Background

2.1 Social networks

A social network can be considered as a network structure that consists of social actors (Knoke and Yang 2008). Analysis of social networks can be performed based on different disciplines, such as psychology, sociology and statistics (Knoke and Yang 2008). With the advent of complex network analysis (Barabási and Albert 1999; Albert and Barabási 2002; Ghoshal et al. 2014; Abnar et al. 2015), there has been keen interest in using network analysis to study social networks as complex systems (Albert and Barabási 2002; Newman 2003). Numerous aspects of social networks such as the evolution of coordination and network robustness have been extensively studied (Kasthurirathna et al. 2013a, b; Perc and Szolnoki 2008; Borbora et al. 2013; Piraveenan et al. 2013, 2012). In particular, many social networks are shown to display both scale-free and small-world characteristics (Albert and Barabási 2002).

2.1.1 *Scale-free model*

It has been observed that most of the real-world complex networks, including social networks, possess a power-law degree distribution. The networks with such power-law degree distributions are called scale-free networks, because their topology is independent of scale. Formally put, the degree distribution of a scale-free network fits the following equation,

$$p_k = \alpha k^{-\gamma} \tag{1}$$

where k is node degree and γ is called the power-law exponent.

Barabasi–Albert model (1999), which is based on growth and preferential attachment, can be used to generate scale-free networks. Preferential attachment suggests that nodes choose candidates to create links based on a degree-based preference (Barabási and Albert 1999; Albert and Barabási 2002).

In addition to the scale-free networks, we are using Erdos–Renyi (ER) random networks (Albert and Barabási 2002) and well-mixed networks in our study. The ER random networks are generated by randomly connecting links, while in well-mixed networks, all nodes are connected to each other. These models are used as reference models. These models help us to evaluate how the social influence model that we propose performs in different topologies.

2.2 Influence modelling in social networks

Modelling of influence in social networks has gained much interest in the recent past. This is partly due to the potential that the emergence of online social networks is present, in myriad fields from online marketing of products to political campaigns (Chen et al. 2009; Domingos and Richardson 2001). Especially due to advent of 'viral marketing' where word of mouth is used as a form of advertising through social media, the importance of social influence modelling has become even more prevalent (Brown and Reingen 1987; Domingos and Richardson 2001). One key advantage about online social networks is that it is possible to deduce the meta-information about the social network, such as the underlying topology and the weights of the links, based on the data that are captured from the social interactions (Domingos and Richardson 2001). The key challenge in social influence modelling is to identify the placement of 'seeds' or the influencing agents that are able to create a cascading effect in the network, where the maximum possible number of nodes in the network is affected. This problem becomes even more complex when there are multiple types of competing seeds in operation (Chen et al. 2011; Clark and Poovendran 2011). Two main classes of influence or diffusion models are found in the literature, namely the linear threshold model and independent cascade model. Apart from that, recent interest has emerged on network topological influence models based on the PageRank algorithm, and even based on game-theoretic models (Huang and Xiong 2013; Clark and Poovendran 2011). Following is a brief introduction to the most common social influence models found in the literature.

2.2.1 *Linear threshold model*

One of the most common models for social influence is the linear threshold model (Kempe et al. 2003). The assumption that is made in this model is that a node has a binary state of being active or inactive with respect to a particular influence. Each node has a random variable that dictates the fraction of nodes based on whose state which, it will switch or keep its current state. Formally put, each node v has a threshold $\theta_v \epsilon [0, 1]$ that is randomly selected, which denotes the fraction of neighbours of node v that has to be active in order for node v to be active and vice-versa. Each node is affected by each neighbour w according to a weight $b_{v,w}$ such that $\sum_{w \epsilon Q(v)} b_{v,w} \leq 1$ where $Q(v)$ is the set of neighbours of v. A node is activated when the total weight of its active neighbours is at least θ_v:

$$\sum\nolimits_{w \epsilon Q(v)} b_{v,w} \geq \theta_v$$

The random assignments of threshold θ_v account for the lack of knowledge of intrinsic latent tendencies of nodes to adopt neighbour strategies.

The classical linear threshold model is designed so that a node will have a single binary state. However, in a social network, there could be opposing or conflicting influences in place. To account for this possible negative influence, as extension to the linear threshold model has been suggested called the competitive linear threshold model (CLT) (He et al. 2012) that accounts for the possible negative influences that maybe present in a network. Thus, instead of the two states inactive and +active, there are three possible states in a CLT model, namely inactive, +active and −active.

Notice that both the LT and CLT model assume that nodes switch in a discrete fashion and not in continuous fashion. The stochastic nature is captured in the randomness of the threshold. Also, even under the CLT model, it is not possible to model influencing scenarios where more than one positive or negative influence is present. In other words, there may be scenarios where multiple options are available for an individual in a social network (such as an election), where the CLT model could not be applicable.

2.2.2 *Independent cascade model*

In the independent cascade model (Kempe et al. 2003), when a node v becomes active, it has a single chance of activating each currently inactive neighbour w. Each activation attempt succeeds with probability p_{vw}. Here too, the influence is defined in a binary fashion while multiple influence types are not considered. An extension for the independent cascade model has been proposed which allows the inclusion of negative opinions (Chen et al. 2011).

2.3 Game theory

In our influence model, we employ a game-theoretic approach to quantify influence. Game theory is the science of strategic decision-making (Barron 2013). Further, repeated strategic games have been extensively used to model the strategic decision-making in populations of players (Perc et al. 2013). Different games such as the prisoner's dilemma game and the coordination game have been proposed to model different strategic decision-making scenarios (Barron 2013). One of the pivotal concepts of game theory is Nash equilibrium (1950). Nash equilibrium suggests that there exists one of more equilibria in strategic decision-making scenarios, from which no player benefits by deviating. The key assumption in Nash equilibrium is that the players are fully rational (Goeree et al. 2008). In other words, it assumes that the players have full knowledge of the strategies and the pay-offs of the opponents and do not have any cognitive or temporal limitation in calculating the optimum strategy (Goeree et al. 2008).

The assumption of full rationality does not hold for real-world players as they tend to have 'bounded rationality'. Bounded rationality is the non-optimal rationality of players due to the limitations of cognitive capacity, information availability or the computational time available for a player (Gigerenzer and Selten 2002). For this reason, the behaviour of real-world players may deviate substantially from Nash equilibrium (Haile et al. 2008). In this study, in order to model the behaviour of boundedly rational followers, we employ the quantal response equilibrium (QRE) model (Goeree et al. 2008).

2.3.1 Quantal response equilibrium

Quantal response equilibrium is a generalisation of Nash equilibrium, which accounts for the boundedly rational or 'noisy' agents. Probabilistic choice models are often used to incorporate stochastic elements in to the analysis of individual decisions. Quantal response equilibrium (QRE) (Goeree et al. 2008) presents an analogous way to model games with noisy players. Probabilistic choice models such as logit and probit models are based on quantal response functions. They have the inherent feature where the deviations of optimal decisions are negatively correlated with the associated costs. Thus, in the QRE model, players are likely to make better choices than worse choices, although there is no guarantee that they will always select the best possible choice. The logit function given in Eq. (2) is often used to derive the equilibrium probabilities at QRE (Goeree et al. 2008; Zhang 2013).

$$P_j^i = \frac{e^{\lambda_i \mathrm{Eu}^i(s_j^i, P_j)}}{\sum_k e^{\lambda_i \mathrm{Eu}^i(s_k^i, P_k)}} \tag{2}$$

Here, P_j^i is the probability of player i selecting the strategy j. $\mathrm{Eu}^i(s_j^i, P_j)$ is the expected utility to player i in choosing strategy j, given that other players play according to the probability distribution P_j. The total number of strategies that player i can choose from is given by k.

QRE produces a mixed strategy equilibrium, where the choice probabilities give the equilibrium probabilities of a player with a particular value of bounded rationality. In the logit QRE function given in Eq. (1), λ_i is known as the rationality parameter of player i. By varying it, it is possible to vary player i's ability to respond to the opponent's strategy distribution and the pay-offs obtained under each strategy. Accordingly, the rationality parameter can be regarded as a measure of a player i's rationality. It has been shown that as $\lambda_i \rightarrow \infty$, the equilibrium gets closer to the Nash equilibrium, and as $\lambda_i \rightarrow 0$, the player operates in a totally random (thus irrational) fashion (Goeree et al. 2008). Within this range, the rationality parameter produces equilibria of boundedly rational players. Therefore, we can use this QRE logit function to derive the equilibrium probability distributions of players who operate under non-perfect rationality. Though there exists other models to model the non-optimal rationality of players, such as the near rationality model (Christin et al. 2004), they do not provide a versatile method to quantify the bounded rationality of players with a rationality parameter, as the QRE model does. Heterogeneous QRE model (Rogers et al. 2009) suggests that there exists a heterogeneity in the rationality parameters in a population of players. This heterogeneity of rationality is what we exploit to model the variation of social influence in a social network. QRE model has been defined for both the normal-form and extensive-form games (McKelvey and Palfrey 1995, 1998). However, we will only consider normal-form QRE in this study to model the social influence.

In addition to the leader-follower game that is used to model social influence, we also discuss how the proposed boundedly rational social influence model may be applied to generic strategic decision-making scenarios. In order to do this, we apply the proposed model to a network of agents that collectively play the prisoner's dilemma game.

2.3.2 Prisoner's dilemma

Prisoner's dilemma (PD) (Rapoport 1965) game is one of the classical normal-form games that attempt to model self-interested players. Given the following generic payoff matrix, the PD game has the inequality $u_{12}^2 \geq u_{11}^2 \geq u_{22}^2 \geq u_{21}^2$ in its payoffs, supposing that the payoff matrix is symmetric. In other words, the Nash equilibrium occurs when both the players defect. However, the optimum payoffs for both players are gained when both players corporate, thus creating a dilemma. In this work, the prisoner's dilemma was used to model the social influence in the context of strategic games.

3 modelling influence using bounded rationality

Based on the background theoretical knowledge, we present a social network influence model based on game theory and bounded rationality of nodes. We first model the social influence of nodes as a leader-follower game, where there are influencing nodes or 'seeds' and followers operating in a network of players. The seeds continue to operate with a permanent binding to a particular state. This inclination may be due to some external knowledge or an incentive the seed may have from the external environment. In the context of a influencer-follower scenario, the bounded rationality of a follower is the 'rationality of following'. The higher the rationality of a follower with respect to a seed, the higher the probability of it following the state of the seed. In this social influence model, we assume that the rationality parameter of a particular follower is negatively proportional to its distance from the seed. This assumption accounts for the random noise that is accumulated as the followers move further from a seed. This assumption is based on the literature in the fields of social physics, social science and telecommunications, which suggest that the social influence and social interactions are negatively proportional to the distance between the nodes in concern (Latané et al. 1995; Liebrand and Messick 2012; Akerlof 1997; Goldenberg and Levy 2009). Thus, with respect to the leader-follower game discussed in this work, the rationality parameter, which denotes the rationality of following an influencer, can be regarded as being negatively proportional to the distance between the nodes in concern. Even though the existing literature suggests that the correlation between the distance and social influence is an inversely polynomial one, for simplicity, we assume that there exists a negatively linear correlation between the distance between the nodes and social influence.

Based on the rationality of following, we can measure the probability of a follower being at the state of the influencing node or the seed node, using the QRE logit function given in Eq. (2). Thus, our model does not produce a binary outcome where the followers are active or inactive in a binary fashion, rather the result is a probability on which a follower adopts the state of the influencer. In game-theoretic terminology, the follower's probability distribution is a mixed strategy equilibrium, where the two strategies would be whether to adopt the strategy of the seed or not. Thus, we can apply the QRE logit function given in Eq. (2) to the leader-follower game with boundedly rational followers. Therefore, the follower probability $p_{n,s}$ of adopting the active state s of the seed could be obtained using the Eq. (3).

$$P_{n,s} = \frac{e^{\beta_{n,i}.U_s.P_{i,s}}}{e^{\beta_{n,i}.U_s.P_{i,s}} + e^{\beta_{n,i}.U_{-s}.P_{i,-s}}} \tag{3}$$

where $P_{n,s}$—probability of the follower node n being at state s (active state), $\beta_{n,i}$—following rationality of node n with respect to node i, U_s—utility of adopting the state s, $P_{i,s}$—probability of the influencer i being in state s (this is always 1), $P_{i,-s}$—probability of the influencer i being not in state s (this is always 0) and U_{-s}—utility of not being in state s (inactive state).

We add random noise to the followers with the assumption that the rationality of a node of following the influencer is negatively proportional to the distance from the seed or the influencer. Thus, $\beta_{n,i} \propto \frac{1}{d_{n,i}}$ where $d_{n,i}$ is the distance along the shortest path from the influencer i to node n along the shortest path. As the follower moves further from the seed, the rationality parameter reaches 0, making them behave randomly. If the followers are placed closer to the seed, then there is a higher rationality and thus a higher probability of following the state of the seed. Another important factor to note is that not only the distance from the seed, but also the reward or utility of adopting the state too plays a significant role in determining whether a follower adopts the state of the seed. Accordingly, we may extend the single-seed model given in Eq. (3) to a more general scenario where there are multiple seeds, as shown in Eq. (4).

$$P_{n,s} = \frac{\sum_{i=1}^{N} e^{\beta_{n,i}.U_s.P_{i,s}}}{\sum_{i=1}^{N} e^{\beta_{n,i}.U_s.P_{i,s}} + \sum_{i=1}^{N} e^{\beta_{n,i}.U_{-s}.P_{i,-s}}} \tag{4}$$

where N is the total number of influencers in the network. In the above model, each node has a separate rationality parameter for each influencer, based on the distance to them. Thus, it captures the varying network distances from each influencer to more accurately predict the status of the follower.

This implies that a population that is closely knitted has a higher tendency of following a seed compared to a population that is sparsely connected. Further, small-world networks (Albert and Barabási 2002) tend to amplify the effect of social influence as they have relatively low average path lengths (Albert and Barabási 2002).

4 modelling social influence under opposing influencers using bounded rationality

The model we introduced could be used to model the influence of a single type of influencers. Yet, in most real-world social influence scenarios, there are conflicting interests at play. There are influencers with a negative influence as well as positive influence, with respect to a particular node state. In addition to that, there could be instances where there are multiple influencers which are at cross-purposes from each other. A good example of this is political campaigns where there are more than two candidates running. Thus, we can easily extend the above model to account for two opposing types of influencers of states S_1 and S_2. Equation (5) shows the equation of a social influence model where there are multiple and conflicting influencers. It is important to note that this model may be extended to capture the influence of more than two types of influencers, even though in this study we limit our scope to two opposing types of influencers.

observing the evolution of populations of players (Fogel 1993; Santos et al. 2006). The evolutionary dynamics of the populations help to understand how each strategy performs under different network topologies (Santos et al. 2006). If we assume that certain nodes continue to stick to a particular strategy irrespective of its environment, due to some external knowledge or influence external to the network, then we can model them as influencers, while the rest of the population can be regarded as followers that get affected by those influencing nodes. For instance, when the prisoner's dilemma game is played over a network,

$$P_{n,S_1} = \frac{\sum_{i=1}^{N} e^{\beta_{n,i}.U_{S_1}.P_{i,S_1}} + \sum_{j=1}^{M} e^{\beta_{n,j}.U_{-S_2}.P_{j,-S_2}}}{\sum_{i=1}^{N} e^{\beta_{n,i}.U_{S_1}.P_{i,S_1}} + \sum_{i=1}^{N} e^{\beta_{n,i}.U_{-S_1}.P_{i,-S_1}} + \sum_{j=1}^{M} e^{\beta_{n,j}.U_{S_2}.P_{i,S_2}} + \sum_{j=1}^{M} e^{\beta_{n,j}.U_{-S_2}.P_{i,-S_2}}} \tag{5}$$

Here, P_{n,S_1}—probability of the follower node n being at state S_1, $\beta_{n,i}$—following rationality of node n with respect to the influencer i, $\beta_{n,j}$—following rationality of node n with respect to the influencer j, P_{i,S_1}—probability of the influencer i being in state S_1 (this is always 1), $P_{i,-S_1}$—probability of the influencer i not being in state S_1 (this is always 0), P_{i,S_2}—probability of the influencer j being in state S_2 (this is always 1), $P_{i,-S_2}$—probability of the influencer j not being in state S_2 (this is always 0), U_{S_1}—utility of adopting the state S_1, U_{-S_1}—utility of not being in state S_1, U_{S_2}—utility of adopting the state $S_2$and U_{-S_2}—utility of not being in state S_2.

The rationality parameters with respect to each influencer are dependent on the distance of the node in concern from each of the influencers. Note that the followers can take either of the two states S_1 or S_2 under the influence of the two types of influencers. However, it does not account for a neutral state where the followers may not follow either of the two types of influencers. If a neutral state is considered, then the numerator should only contain the exponent of S_1, as in that case a node being influenced to be in state—S_2 does not mean it would automatically adopt S_1. Further, it is possible to extend the same model to take into account multiple types of influencers and not just two opposing types, since every influencer state can be regarded as a possible strategy a follower could adopt with heterogeneous rationality levels.

5 Modelling strategic influence using bounded rationality

In addition to social influence, we can use the same approach to measure the influence of strategic decision-making scenarios. Game-theoretic models are often used in

depending on the topological position and arrangement of each node, different nodes adopt cooperation or defection (Santos et al. 2006). However, if we assume that the cooperation or defection tendency of each follower is affected by the influence by the seeds that stick to a particular strategy, then we can model the social influence of strategic games using heterogeneous bounded rationality and quantum response equilibria.

Equations (6, 7) depict the two logit functions based on QRE with an influence-rationality parameter to derive the probability of cooperation in a networked PD game. The pay-offs given are the payoff values represented in the generic payoff matrix in Fig. 1.

$$p_{1,c} = \frac{e^{\beta_{1,c}(p_{2,c}(u_{111}+(1-p_{2,c})(u_{121})))}}{e^{\beta_{1,c}(p_{2,c}(u_{111}+(1-p_{2,c})(u_{121})))} + e^{\beta_{1,d}(p_{2,c}(u_{211}+(1-p_{2,c})(u_{221})))}} \tag{6}$$

$$p_{2,c} = \frac{e^{\beta_{2,c}(p_{1,c}(u_{112}+(1-p_{1,c})(u_{212})))}}{e^{\beta_{2,d}(p_{1,c}(u_{112}+(1-p_{1,c})(u_{212})))} + e^{\beta_{2,d}(p_{1,c}(u_{122}+(1-p_{1,c})(u_{222})))}} \tag{7}$$

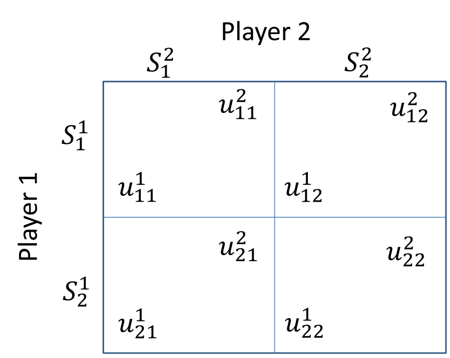

Fig. 1 The payoff matrix of a generic normal-form game

Here, $p_{1,\mathrm{c}}$ and $p_{2,\mathrm{c}}$ are the probability of cooperating for players 1 and 2, respectively. The rationalities $\beta_{1,\mathrm{c}}$ and $\beta_{1,\mathrm{d}}$ can be used to quantify the influence on player 1 on cooperating and defecting, respectively. These influences are determined by the distances from the influencers or seeds adopting each strategy. Similarly, $\beta_{1,\mathrm{d}}$ and $\beta_{2,\mathrm{d}}$ signify the influence-based rationalities of defection for players 1 and 2, respectively. Formally put,

$$\beta_{n,\mathrm{c}} \propto \sum_{i=1}^{N_\mathrm{c}} 1/d_{n,i_\mathrm{c}} \tag{8}$$

where $\beta_{n,\mathrm{c}}$ is the influence-based rationality of cooperating in node n, d_{n,i_c} is the distance of node n from the influencer i (which is a pure cooperator), and N_c is the total number of cooperator influencers within the network. The influence-based rationality of defecting could be calculated in a similar manner. Thus, each follower captures the influence of the cooperator and defector influence nodes within the network, through the distance-based bounded rationality for each strategy. Similar to the influence game, this model can be extended to incorporate multiple types of influencers with multiple strategies.

6 Optimising influence using bounded rationality models

In this section, we propose a method to place the influencers in order to maximise their influence on the population, based on the bounded rationality-based influence models that we have proposed. We mainly look at two scenarios. One is where the network has only a single type of influencers and the requirement is to select the placement of influencers to maximise their influence. This is termed as the influence maximisation problem in the literature (Kempe et al. 2003). The other is the scenario where the network has two kinds of opposing influencers. Supposing the network is already occupied with one type of influencers, we need to identify the optimum way of placing the rivalling set of influencers, so that the influence of the originally placed influencers is minimised. These optimisations are applicable to the influencer-follower game that we discussed earlier and also more general strategic decision-making situations.

Firstly, let us consider a scenario where there is only one type of seeds or influencers in a network in an influencing game. Since the bounded rationality of following is inversely proportional to the distance of the followers from the influencing node, the influence is maximised when the influencers are placed closer to the followers. The natural candidate to locate that placement is the closeness centrality of the network, since closeness centrality is used to identify the nodes that have average minimum shortest path distance to the other nodes within the network. Equation (9) depicts the equation for calculating the closeness centrality of a node. When the influencers are placed according to the closeness centrality of network, the influence on the rest of the network is at the peak, when there is only one type of positive influence at play.

$$C_\mathrm{H}(x) = \sum_{y \neq x} \frac{1}{d(y,x)} \tag{9}$$

where $d(y, x)$ is the distance along the shortest path from node x to node y.

Next, we will look at what is the most optimum method to place the conflicting influencers when there are multiple opposing influencers in operation. Assuming that the original influencer or influencers are already placed in a network, there are two factors that affect the effectiveness of the opposing influencers. Those two forms of negative influence are as follows:

active negative influence (A_n)—the distance from the opposing influencers to the follower nodes in the network;

passive negative influence (P_n)—the blocking effect of opposing influencers by their relative placement in the network topology.

Following are the formal definitions of the active and passive negative influence components.

$$A_{n_i} \propto d_{ij}, \quad j \in J \tag{10}$$

where j is an opposing influencer and J is the set of all opposing influencers. A_{n_i} is the active negative influence component on node i that is following the influencers, and d_{ij} is the distance of the following node i from the influencer j.

$$P_{n_i} \propto BC_{ik}, \quad k \in K \tag{11}$$

where K is the set of all original influencers. BC_{ik} is the betweenness centrality of node i with respect to the set of original influencers.

The first form of negative influence could be measured using the closeness centrality of the opposing influencer. We call it an active factor since the opposing nodes directly influence the other nodes to follow them, based on how close the opposing influencers are to the followers. The second form of negative influence is relevant since the social influence is spread through the network topology. If a node is occupied by an opposing influencer, then it does not take part in spreading the influence of the original influencers, unlike the regular nodes. From the perspective of the original set of influencers, the opposing influencers cease to exist in the network, in their quest to spread their influence. Thus, simply by being placed in positions where the distance from the original influencers to the followers could be increased, the opposing influencers can minimise the influence of the original set of influencers. In other words, the opposing influencers are best placed in 'between' the original influencers and the followers of the network. Thus, another well-known centrality measure, which is the betweenness centrality, can be employed to identify the topological positions where the influence of the original influencers is minimised. However, we need not calculate the betweenness centrality values of all nodes in the network to identify these positions. Only the betweenness centrality values for the original influencers are sufficient to identify the topological positions where their influence can most effectively be interfered with. We call this the passive negative influence since the opposing influencers reduce the influence of the original influencers simply by passively occupying the high betweenness centrality nodes.

By combining these active and passive negative influence components, we can derive a measure that could be used to identify the positions where it is most effective to place the negative influence nodes. Thus, we can propose an algorithm to optimally place the negative influencers in a network where there are already existing positive influencers.

Algorithm 1: Optimum influence arrangement of negative influencing seeds, when a network is already occupied with positive influencing seeds.

Data: Network topology, Opposing influencers
Result: Optimum arrangement of opposing influencers
1 $\forall n, n \in N$, calculate and assign $BC(n, s), s \in S$;
2 $\forall n, n \in N$, calculate and assign $CC(n)$;
3 $OC(n) = \frac{BC(n,s)+CC(n)}{2}$;
4 Order the nodes according to descending order of OC(n);
5 Place the opposing seeds in the descending order of OC(n);

Here, n is a node in the set of nodes N. $BC(n, s)$ is the betweenness centrality of node n with respect to the original set of influencers S. $CC(n)$ is the closeness centrality of each node n. $OC(n)$ denotes the combined measure of placing the opposing nodes, which is the average of $BC(n, s)$ and $CC(n)$.

6.1 Complexity analysis

Influence maximisation is an optimisation problem whereby we try to optimally place the seeds in order to maximise the influence spread. Under the independent cascade model and linear threshold model, the influence maximisation becomes an NP-hard problem (Kempe et al. 2003). Therefore, in order to solve this problem, it is necessary to employ a greedy algorithm to find the optimum seed arrangement. The greedy algorithm of optimising influence spread outperforms the degree and centrality-based heuristics (Kempe et al. 2003, 2005). Still, it requires an approximation using the Monte Carlo simulations of the influence cascade model over a number of interactions to obtain an accurate estimation of the influence spread. Thus, it may be inefficient in a sufficiently large network with multiple seeds involved.

Using the proposed bounded rationality-based influence model, we can propose an exact algorithm to identify the optimum set of influencers without depending on heuristics. The time complexity of finding the optimum placement of a single type of influencers is $O(VE)$ where V is the number of vertices and E is the number of edges in the network, which is the time complexity of calculating the closeness centrality. The time complexity of optimising the negative influence when there are opposing influencers is $O(VE) + O(V_s E)$, where V_s is the initial set of seed nodes. Based on the proposed influence model, it is possible to measure the actual optimisation problem in polynomial time. Also, instead of an approximation, it is possible to derive an exact solution using the bounded rationality-based influence model.

7 Methodology

In this section, we present the methodology used to perform simulations to demonstrate the applicability of the bounded rationality-based influence model. First, we compare three different network topologies, scale-free, ER random and well-mixed, to see how they could facilitate the influence spread under a bounded rationality influence model. The networks considered were synthetic networks that contained 1000 nodes with the average degree of four. The Scale-free network was generated using the Barabasi–Albert model (1999). To compare networks, we placed a seed at the hub of each network and measured the average probability of following the seed, over the network. The network with the highest average probability of following is the one that facilitates most influence. The two states that we considered were either active or inactive states where active refers to adopting the state of the influencer. However, since the QRE model gives a probability distribution of whether a node follows the influencer or not, the outcome is not a binary-state distribution, rather a probability distribution of being in the active state. The payoff of adopting the state of the seed is set to be 1 and not adopting the state is set to 0.

$$p_{n_1} = \frac{e^{(\beta_{n_i} \cdot p_i^1 .1)}}{e^{(\beta_{n_i} \cdot p_i^1 .1)} + e^{(\beta_{n_i} \cdot p_i^0 .0)}} \tag{12}$$

Here, p_{n_1} is the probability of node n being in state 1, where state 1 denotes being active and 0 denotes being inactive. β_{n_i} is node n's 'rationality of following' with respect to the influencer i, which is dependent on node n's distance along the shortest path from the influencer i. Therefore, $\beta_{n_i} = c/d_{n_i}$ where c is a constant and d_{n_i} is the distance of node n from node i along the shortest path. 1 and 0 are the pay-offs of states 1 and 0, respectively.

Next, in order to test how the payoff of following affects the probability distribution of following the seed, we performed the same experiment while varying the payoff of the active state. The same scale-free network that was generated using the Barabasi–Albert model (1999) was used for this simulation, and the variation of the average probability of following over the payoff of the active state was observed. Since real-world social networks have been observed to be closely resembling the scale-free topology (Albert and Barabási 2002), the synthetic network was used for all the subsequent experiments.

Then, we observed the optimum method to reduce the influence of the hub by placing opposing influencers in the network. To test this, we considered two different scenarios on the scale-free network that was used for the previous experiments. In the first one, the hub is placed with the original influencing node and four opposing influencers are distributed according to different configurations. In the first configuration, they are placed in the remaining hubs in the order of the degree. In the next two configurations, the opponents are placed in the order of betweenness from the hub and the closeness centrality of the network, respectively. In the 4th configuration, the opponents are placed in the order of the combined measure that we presented in Algorithm 1. We compared and contrasted the variation of negative influence by observing the adoption probability under these four configurations.

The same experiment was then repeated for multiple conflicting influencers on both sides. The original set of influencers were randomly distributed, and four opposing influencers are placed according to the four configurations mentioned above. This experiment was repeated over fifty iterations to account for the effect of randomness in the initial configuration of influencers. We assume that the followers adopt either the active or the negatively active states; thus, they would not be in an inactive state. It should be noted that an inactive state could also be incorporated by extending the QRE model that we use. The pay-offs of following each type of influencers were set to 2 and 1, respectively. In each type of influencer, the payoff of not following the influencer is a negative payoff of -2 and -1, respectively. Equation (13) shows how the probability of a node being in state s, which is the state of the original set of influencers, can be calculated using QRE and distance-induced bounded rationality.

$$p_{n_s} = \frac{\sum_{i=1}^{N} e^{(\beta_{n_i} \cdot p_i^s .2)} + \sum_{j=1}^{M} e^{(\beta_{n_j} \cdot p_j^s .-1)}}{\sum_{i=1}^{N} e^{(\beta_{n_i} \cdot p_i^s .2)} + e^{(\beta_{n_i} \cdot p_i^{-s} .-2)} + \sum_{j=1}^{M} e^{(\beta_{n_j} \cdot p_j^s .-1)} + e^{(\beta_{n_j} \cdot p_j^{-s} .1)}} \tag{13}$$

Here, p_{n_1}—probability of node n adopting state s, β_{n_i}—rationality of following the influencers of state s where N is the number of such influencers and β_{n_j}—rationality of following the influencer of state $-s$ where M is the number of such influencers, 2 and -2 are the pay-offs of either following or not following the influencers of state 1, and 1 and -1 are the pay-offs of either following or not following the influencers of state $-s$.

This particular experiment was repeated on a real-world Wiki-vote network of seven thousand nodes (Leskovec and Krevl 2014), which has a scale-free topology. This was done in order to compare the probabilities of following the influencers in a real-world network, under the four different configurations considered.

Further, we placed the opposing strategies in an alternating fashion and tested the placement of opposing strategies according to the above-mentioned four metrics. This way, we can obtain the sum of the locally optimal solutions of the placement of opposing strategies. Then, we compared and contrasted the optimum strategies of placing the opposing influencers, under the locally optimal and globally optimal contexts.

Next, we looked at how the strategic game-based influence operates in a bounded rationality-based influence model. We used the scale-free network that was used in the previous experiments to conduct this particular experiment as well. Specifically, we tested the optimum method to reduce the influence of already existing influencers. We use the prisoner's dilemma game with four cooperators distributed randomly in the network while four defectors try to negatively influence to minimise cooperation in the network. The rationality of cooperation and defection is negatively proportional to the distance to each influencer. As with the previous experiment, we compare different opponent placement strategies with the optimum placement strategy that we discussed in 1. The pay-offs of the PD game were set such that $u_{111}, u_{122} = 4, u_{121}, u_{212} = 0, u_{122}, u_{211} = 5$ and $u_{221}, u_{222} = 1$. Equations (6, 7) are used to calculate the probability of cooperation in each iteration. For each node, the rationality of cooperation and defection are calculated by taking into account the cumulative effect of influencers of each type. For example, for node n, the rationality of cooperation is $\sum_{i=1}^{N} \frac{c}{d_{n_i}}$, where c is a constant and N is the number of cooperators in the network, while d_{n_i} is the distance to each cooperator from the node.

These experiments enable us to evaluate how the bounded rationality-based influence modelling can be applied in a influencer-follower scenario or a strategic game scenario, where there are a dedicated set of influencers or seeds and the rest of the population is following them. It should be noted that although we consider only two types of rivalling influencers or strategies, the bounded rationality-based influence model could be expanded for multiple types of influencers and strategies as well.

8 Results

Table 1 shows the comparison of the average following probability under the three network topologies considered. As shown in the table, the scale-free topology facilitates the highest average following probability compared to the ER random and lattice topologies. This is due to the fact that a scale-free network of a comparative size and average degree may have a lower diameter compared to a ER random or a lattice network.

Figure 2 depicts the variation of the following probabilities in a scale-free network, when the seed is placed at the hub and when the payoff for adopting the state of the seed is increased. As shown in the figure, there is a clear positive correlation between the payoff of the active state and the influence spread. Thus, this shows that not only the topology of the network and the positioning of the seeds, but also the payoff of the active state is critical in determining the spread of influence.

Next, we depict the results when two conflicting types of influencers are placed in a scale-free network. The original influencer is placed in a hub, and four opposing influencers

Table 1 Average probability of following the seed in different topologies

Network topology	Average probability of following
Scale-free	0.95
ER Random	0.89
Lattice	0.51

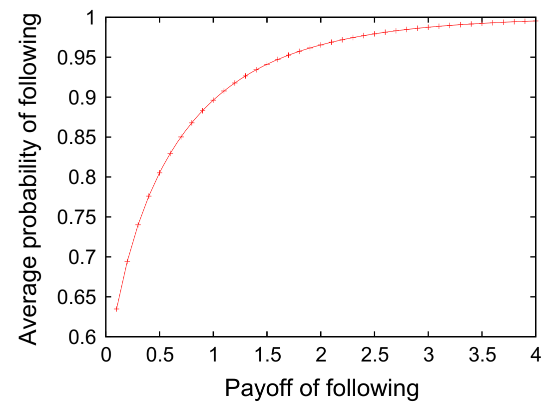

Fig. 2 Variation of the probability of following against the payoff of the active state

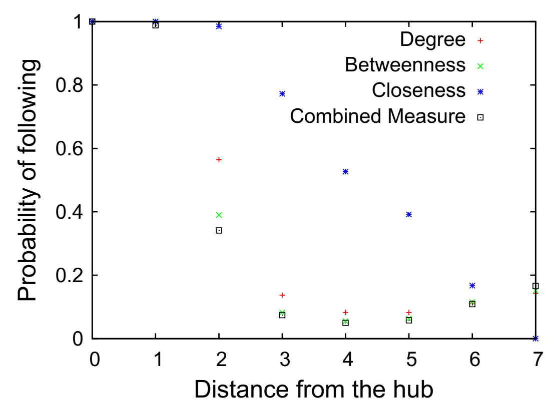

Fig. 3 Variation of the probability of following against the distance from seed placed at hub in a scale-free network. Four opposing seeds are placed in the order of (1) degree, (2) betweenness with respect to the hub, (3) closeness centrality and (4) combination of both of 2 and 3

are placed according to four different configurations. Namely, they are placed according to the degree centrality of the network, betweenness centrality from the hub, closeness centrality of the network and a combination of the 2nd and 3rd measures (as discussed in Algorithm 1). Figure 3 shows the comparison when the influencing seed is placed at the hub in opposition to four counter influencers in the same four configurations. As shown in the figure, the combined measure method proposed in Algorithm 1 gives the best results in terms of maximising the negative influencer, thereby reducing the average probability of following. Table 2 shows the average probability of following when the opposing seeds are placed in the four different configurations. The results reiterate that it is the negative seed placement method discussed in the Algorithm 1 that provides the optimum reduction of the influence from the original seed.

Figure 4 depicts the variation of following probability against the average distance from the influencing seeds

Table 2 Average probability of following the seed in different topologies

Opposing influencer configuration	Average following probability
Degree	0.29
Betweenness (hub)	0.21
Closeness	0.79
Combined measure (2 and 3)	0.19

The opposing seeds are placed in four different configurations

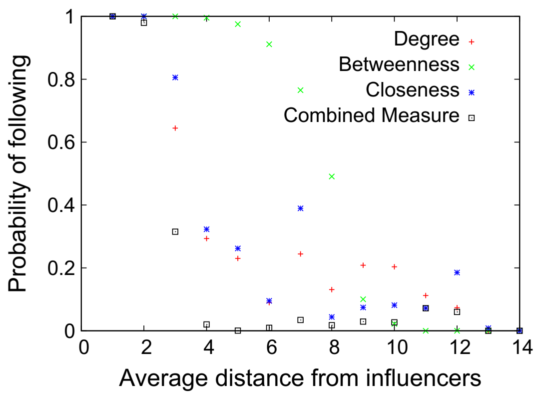

Fig. 4 Variation of the probability of following against the average distance from seeds placed at random positions in a scale-free network. The opposing seeds are placed in the order of (1) degree, (2) betweenness with respect to the original influencers, (3) closeness centrality, (4) combination of both of 2 and 3. The results are averaged over fifty independent runs

Table 3 Average probability of following the randomly placed seeds in different topologies

Opposing influencer configuration	Average following probability
Degree	0.33
Betweenness (seeds)	0.60
Closeness	0.33
Combined measure (2 and 3)	0.21

The opposing seeds are placed according to four different configurations

when multiple original influencers are placed randomly. In order to normalise for the effect of the randomisation of the positions of the original seeds, the results were averaged over fifty independent runs. As depicted in the figure, it is the combined measure used to determine the placement of the opposing seeds that mitigate the influence of the original set of influencers most effectively. This is further confirmed by the comparison of average probabilities of following, given in Table 3.

The same set of experiments were run on a Wiki-vote network of seven thousand nodes (Leskovec and Krevl 2014), in order to test the effectiveness of the proposed

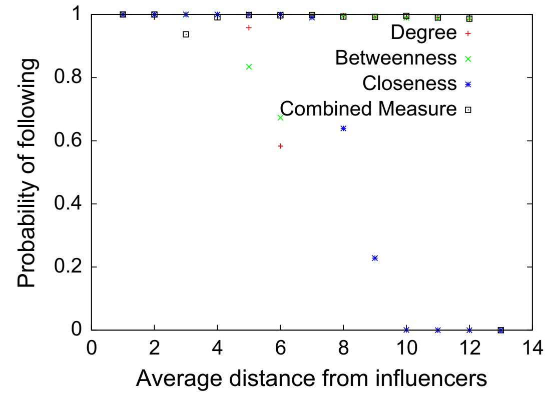

Fig. 5 The variation of the probability of following against the average distance from seeds placed at random positions in a scale-free network. The opposing seeds are placed in an alternating fashion in the order of (1) degree, (2) betweenness from the original influencers, (3) closeness centrality and (4) combination of both of 2 and 3. The results are averaged over fifty independent runs

Table 4 Average probability of following the randomly placed seeds in different topologies

Opposing influencer configuration	Average following probability
Degree	0.96
Betweenness (seeds)	0.95
Closeness	0.79
Combined measure (2 and 3)	0.99

The opposing seeds are placed according to four different configurations in an alternating fashion

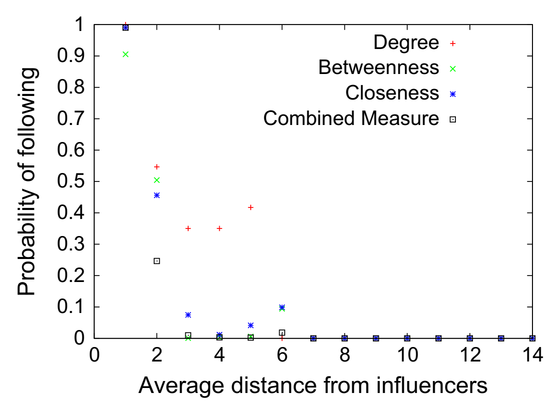

Fig. 6 Variation of the probability of following against the average distance from seeds placed at random positions in the Wiki-Vote network. The opposing seeds are placed in the order of (1) degree, (2) betweenness with respect to the influencers, (3) closeness centrality and (4) combination of both of 2 and 3. The results are averaged over fifty independent runs

counter-influencing technique using a real-world network. The probabilities of following against the average distance from the influencing seeds for the Wiki-vote network are shown in Fig. 6. Table 5 compares the average probabilities

Table 5 Average probability of following the randomly placed seeds in different topologies in the Wiki-Vote network

Opposing influencer configuration	Average following probability
Degree	0.71
Betweenness (seeds)	0.19
Closeness	0.06
Combined measure (2 and 3)	0.01

The opposing seeds are placed according to four different configurations

Table 6 Average probability of coordination in different topologies when the cooperator seed is placed at the hub

Opposing strategy configuration	Average probability of cooperation
Degree	0.226
Betweenness (hub)	0.209
Closeness	0.335
Combined measure (2 and 3)	0.206

The defector seeds are placed in four different topologies

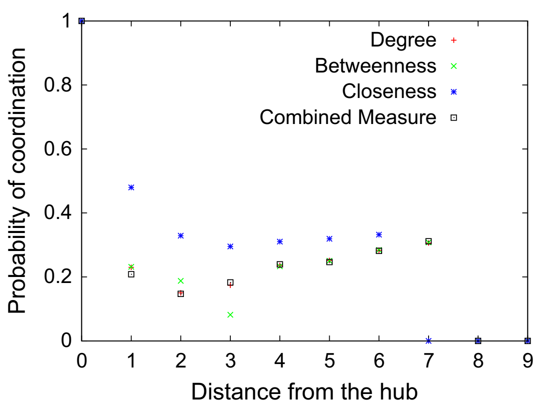

Fig. 7 Probability of coordination against the distance from the coordinator seed placed at hub in a scale-free network. Four defector seeds are placed in the order of (1) degree, (2) betweenness with respect to the coordinator, (3) closeness and (4) combination of both of 2 and 3

of following, in the Wiki-vote network. As these results from the real-world Wiki-vote network too shows, the combined measure performs best out of the four metrics used to place the opposing seeds.

Figure 5 shows the variation of the probability of following against the average distance from the original influencers, when the opposing strategies are placed in an alternating fashion. Thus, this is the locally optimal solution to place the opposing strategies. According to Figure and Table 4, which shows the average probability of following, it is evident that placing the opposing strategies based on closeness centrality gives the optimal solution under the locally optimal scenario, compared to the other metrics of placing the opposing strategies.

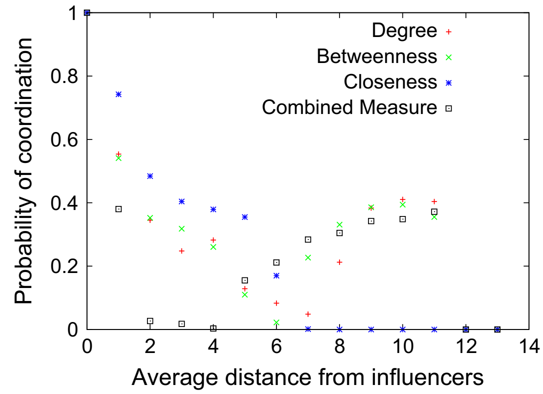

Fig. 8 Variation of coordination against the average distance from coordinators that are randomly placed in a scale-free network. Four opposing seeds are placed in the order of (1) degree, (2) betweenness with respect to the coordinators, (3) closeness and (4) combination of both of 2 and 3

Table 7 Average probability of coordination when the cooperator seeds are randomly placed in different topologies

Opposing strategy configuration	Average probability of cooperation
Degree	0.36
Betweenness (seeds)	0.35
Closeness	0.44
Combined measure (2 and 3)	0.25

The opposing seeds are placed according to four different configurations

Next, we present the results for the same set of experiments repeated for a strategic decision-making scenario, where the PD game is played over a scale-free network. Figure 7 shows the results for the scenario when the coordinator seed is placed at the hub, and the defecting seeds are placed according to the four different configurations discussed above. Similar to the influencer-follower game, the strategic decision-making scenario too is most affected when the opposing strategies are placed according to the combined measure. Table 6 shows the average probability of cooperation in those four types of configurations of placing the defectors. The placement of opposing strategies purely based on the betweenness centrality from the hub adopting the pure coordinator strategy too facilitates an effective reduction in cooperation in the overall network.

Figure 8 depicts the variation of cooperation against the average distance from multiple cooperator seeds that are placed randomly. The defector seeds are placed according to the four configurations discussed previously. Here too, the combined method of placing the opposing strategies make the highest reduction in the cooperator strategy in the network, further emphasised by the average cooperator probabilities shown in Table 7.

9 Conclusion and future work

In this work, we propose a novel social influence model based on the bounded rationality of agents in a social network. First, we model the social influence as an influence game where there are two types of players, influencers and followers. The followers are assumed to be following the influencers stochastically, and their likelihood of following is computed based on their bounded rationality, which is inversely proportional to their distance from the influencers. Based on this model, we show that scale-free networks facilitate social influence compared to ER random and lattice networks.

We then extend this model to scenarios where there are multiple and opposing seeds. We propose a method to optimally place the negative seeds to minimise the influence of the original set of positive influencers. In this method, the opponents are placed according to the order of a combined centrality measure, which is the average of the betweenness centrality from the original seeds and the closeness centrality of the nodes within the network. We demonstrate that in general, placing the opponents in the order of betweeness centrality from the original seeds, thereby 'interfering' with their influence on the rest of the network, is more effective than choosing the nodes that have higher closeness centrality within the entire network to place the rivalling influencers. Moreover, the proposed combined centrality measure performs best in maximising the negative influence, when the original seed is placed at the hub, or when multiple original seeds are randomly distributed. This particular outcome is further validated from the results obtained using the Wiki-Vote real-wold network.

We then extend this influence model to strategic decision-making scenarios in a social network. Here, each node is assumed to have a level of rationality related to following each particular strategy, and that rationality is negatively proportional to the distance from each seed with that strategy within the network. This approach enables us to model the network with heterogeneous rationalities in nodes that are dependent on the number of seeds and the distances from them. Using this model, we could demonstrate that as with the influence game, strategic games like the prisoner's dilemma game could be simulated in a social network with bounded rationalities that are influenced by the seeds that have permanently adopted a particular strategy. The followers adopt their respective strategies based on the bounded rationality of following each strategy and their respective payoffs. Based on this model, we demonstrate that the most efficient way to counter an existing strategy is to place the seeds with the opposing strategy in the order of the combined centrality measure of betweenness from the original strategy seeds and the closeness centrality of the network. However, when the opposing strategies are placed in an alternating fashion, that is, when the local optimal is considered in placing the opposing strategies, it is the closeness centrality that gives the best performance in inhibiting the influence of the original set of strategies.

While this work is primarily based on simulated experiments, a formal proof of the proposed optimisation technique proves that the proposed technique gives the optimal solution. However, in this work, we limit our scope to the simulated experiments while attempting to formulate a formal proof could be a possible extension to this work.

To our understanding, this is the first attempt to model the social influence using bounded rationality and the QRE model. The applications of such a model could be myriad; especially, it allows the computation of social influence in a computationally efficient manner. Moreover, as it is based on game-theoretic principles, it allows the payoff of following an influencer or adopting a strategy to be a key variable in the modelling, which is not present in the standard social influence models. Countering the influence of an existing network is a critical problem that may have many applications in scientific, social and political networks. Thus, the combined centrality measure of placing opponent nodes and strategies may be quite useful in negatively affecting existing social influence. This model can be even applied to approximate social influence in scenarios where there are multiple types of influencers and strategies, such as in a political campaign. Further research is needed to explore the applicability of this model in real-world social networks.

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