The influence of topology and information diffusion on networked game dynamics

Dharshana Kasthurirathna

A thesis submitted in fulfillment
of the requirements for the degree of
Doctor of Philosophy

Complex Systems Research Group
Faculty of Engineering and IT
The University of Sydney

March 2016

Declaration

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person nor material which to a substantial extent has been accepted for the award of any other degree or diploma of the University or other institute of higher learning, except where due acknowledgement has been made in the text.

Dharshana Kasthurirathna

19th March, 2016.

Abstract

Dharshana Kasthurirathna Doctor of Philosophy
The University of Sydney January 2016

The influence of topology and information diffusion on networked game dynamics

This thesis studies the influence of topology and information diffusion on the strategic interactions of agents in a population. It shows that there exists a reciprocal relationship between the topology, information diffusion and the strategic interactions of a population of players. The structure of a population of players is abstracted by the topology and the information flow of the networks of players while the dynamics are denoted by the strategic interactions of the players in the population. While topology represents a static structure, the information flows are used to model a more dynamic and volatile structure of the population. In order to evaluate the influence of topology and information flow on networked game dynamics, strategic games are simulated on populations of players where the players are distributed in a non-homogeneous spatial arrangement. Game theory, network science and information theory are the three pillars of science used to build the underlying theoretical basis in this research.

A study of evolution of the coordination of strategic players is the first part of this research where the topology or the structure of the population is shown to be critical in defining the coordination among the players. Next, the effect of network topology on the evolutionary stability of strategies is studied in detail. The evolutionary stability of a strategy determines its ability to withstand potentially competitive strategies. Based on the results obtained, it is shown that network topology plays a key role in determining the evolutionary stability of a particular strategy in a population of players. Then, the effect of network topology on the optimum placement of strategies is studied. Using genetic optimisation, it is shown that the placement of strategies in a spatially distributed population of players is crucial in maximising the collective payoff of the population. This further suggests that the topology of the social structure is critical in determining its networked game dynamics.

Exploring further the effect of network topology and information diffusion on networked games, the non-optimal or bounded rationality of players is modelled using topological and directed information flow of the network. While network topology defines a more static form, information flows are used to model a more volatile and dynamic form of the population. These models are then applied to demonstrate how the scale-free and small-world networks emerge in randomly connected populations of players who operate under bounded rationality. It is also shown that the strategic interactions with multiple equilibrium states are directly affected by network topology. Thus, the topological and information theoretic interpretations of bounded rationality suggest the topology, information diffusion and the strategic interactions of socio-economical structures are cyclically interdependent.

Acknowledgements

As with most PhD journeys, mine had its fair share of hurdles, obstacles and confusions. However, as time passes, one begins to see more clearly as the clouds of uncertainty and unpredictability slowly fade away. I'm both relieved and humbled to reach the end of this long journey, filled with a nervous excitement on what the future might bring. This journey would not have been possible if not for the sacrifices, support and guidance of many people who actively contributed to the successful completion of my thesis. Thanks to them, I feel I made the fullest use of this endeavour of creating new knowledge. I owe my heartfelt gratitude to those who were by my side, patiently and resiliently, throughout this journey. Let me attempt to acknowledge their efforts.

First of all, I would like to thank my supervisors, Dr Mahendra Piraveenan and Proffessor Liaquat Hossain, for the guidance they provided me throughout this journey. Dr Piraveenan always made sure that I did not fall short of harnessing my full potential, throughout my candidature. His insightful suggestions made sure that I could have publications of significant impact through meaningful research. This thesis would not have materialised if not for his meticulous contributions in shaping and refining the ideas that originated through the countless discussions we had. I would also like to show my gratitude to Professor Mikhail Prokopenkov, Dr Michael Harre and Dr Shahadat Uddin of the Complex Systems Research group for their invaluable support and guidance that they offered to me whenever I reached out to them for help. I'm also grateful to Professor Sanjay Chawla of the School of IT for his comments and suggestions that helped me immensely to formulate my ideas. Further, I was deeply inspired by the epistemological theory of 'Constructive Relativism'[65] by Dr. Nalin De Silva, in forming the philosophical foundation for some of the key concepts discussed in this thesis.

The work environment I had in the university was productive and entertaining. I would like to thank Yamuna Subramanium, Upul Senanayake, Gnana Kumar, Arif Khan, Rabiul and Andrew Leula for their support and unwavering friendship which enabled me to endure all the hard times. I would fail my duty if I don't thank the administrative staff of the faculty including but not limited to Catherine, Christine, Madelon, Linda and Maria.

None of this would have been possible without the lecturers from the University of Moratuwa and University of Colombo, who inspired me to pursue research. My sincere gratitude goes to Dr Chamath Keppetiyagama, who has always been supportive and accessible supervisor. I'm grateful to the staff of the Department of Computer Science and Engineering, University of Moratuwa for all they've done for me. Tracing back to my school days, I'm forever grateful to the wonderful panel of teachers I had in Royal College, Colombo.

With research, you often tend to live in your own world filled with questions, but time to time, you need to break that bubble to proceed further. For me, my friends have always been that and more. I'm thankful to Rasika Wimalasena, Hiran Milinda and Nilusha Nanayakkara for always providing a home for me whenever I needed one. I'm thankful to Kithsiri Ehelepola for understanding me and supporting me when I needed it the most. During my candidature, many friends helped me and I'm grateful to all of you. You all deserve to be mentioned by name, however I'm unable to do that and I hope you'll forgive me for that.

Finally, I cannot thank my loving wife Yashasvi and my dear parents and family enough for standing by me all the time and making it possible for me to pursue this PhD. Had it not been for you, I dread to imagine where I would be and the fact that you are always there for me no matter what, makes me strong.

To everyone who helped.

Publications

The following publications and manuscripts-under-review have resulted from the candidature for this degree:

1. D. Kasthurirathna, M. Piraveenan, S. Uddin, "Modelling networked systems using the Topologically Distributed Bounded Rationality Framework", In Review.*

2. D. Kasthurirathna, M. Piraveenan, "Emergence of scale-free characteristics in socio-ecological systems with bounded rationality" *Nature - Scientific Reports*, Vol. 5, no. 10448, 2015.*

3. D. Kasthurirathna, M. Harre, M. Piraveenan, "Influence modelling using bounded rationality in social networks" *IEEE/ACM International Conference on Advances in Social Networks*, pp. 33–40, 2015.*

4. D. Kasthurirathna, M. Piraveenan, M. Harre, "Influence of topology in the evolution of coordination in complex networks under information diffusion constraints" *The European Physical Journal B*, vol. 87.1, pp. 1–15, 2014.*

5. D. Kasthurirathna, M. Piraveenan, S. Uddin, "Evolutionary stable strategies in networked games: the influence of topology", *Journal of Artificial Intelligence and Soft Computing Research*, vol. 5.2, pp. 83–95, 2015.*

6. D. Kasthurirathna, H. Nguyen, M. Piraveenan, U. Senanayake, "Optimisation of strategy placements for public good in complex networks", *Proceedings of the 2014 International Conference on Social Computing*, p. 1, ACM 2014*.

7. D. Kasthurirathna, M. Piraveenan, "Topological stability of evolutionarily unstable strategies". 2014 IEEE Symposium on Evolving and Autonomous Learning Systems (EALS), pp. 35-42, 2014*.

8. U Senanayake, P Szot, M Piraveenan, D Kasthurirathna, "The performance of page rank algorithm under degree preserving perturbations". IEEE Symposium on Foundations of Computational Intelligence (FOCI), pp. 24–29, 2014.

9. G. Thedchanamoorthy, M. Piraveenan, D. Kasthururathna, U. Senanayake, "Node Assortativity in Complex Networks: An Alternative Approach", *Procedia Computer Science*, vol. 29, pp. 2449–2461, 2014.

10. D. Kasthururathna, M. Piraveenan, "Cyclic Preferential attachment in complex networks", *Procedia Computer Science*, vol. 18, pp. 2086–2094, 2013.

11. D. Kasthururathna, M. Piraveenan, "Centrality and composition of four-node motifs in metabolic networks", *Procedia Computer Science*, vol. 18, pp. 409–418, 2013.

12. G Thedchanamoorthy, D. Kasthururathna, M. Piraveenan, "Standard deviations of degree differences as indicators of mixing patterns in complex networks", *IEEE/ACM International conference of Advances in Social Network Analysis and Mining*, pp. 1202–1208, 2013.

13. D. Kasthururathna, M. Piraveenan, M. Harre, "Evolution of coordination in scale-free and small-world networks under information diffusion constriants", *IEEE/ACM International conference on Advances in Social Networks Analysis and Mining*, pp. 183–189, 2013.*

14. D. Kasthururathna, A. Dong, M. Piraveenan, I.Y. Tumer, "The failure tolerance of mechatronic software systems to random and targeted attacks", *ASME 2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, pp. V005T06A036–V005T06A036, 2013.

15. D. Kasthururathna, A. Dong, M. Piraveenan, I.Y. Tumer, "Network robustness and topological characteristics in scale-free networks", *IEEE Conference on Evolving and Adaptive Intelligent Systems (EAIS)*, pp. 122–129, 2013.

16. M. Piraveenan, S. Uddin, K.S. Chung, D. Kasthurirathna, "Quantifying encircling behaviour in complex networks", *IEEE Symposium on Computational Intelligence in Cyber Security*, pp. 9–14, 2013.

17. M. Piraveenan, S. Uddin, K.S. Chung, D. Kasthurirathna, "On the influence of topological characteristics on robustness of complex networks", *Journal of Artificial Intelligence and Soft Computing Research*, vol. 3.2, pp. 89–100, 2013.

The papers marked with an asterick (*) have directly contributed to this thesis.

Contents

List of Figures

List of Tables

Nomenclature

Notation

$P(a)$	Probability of the event a
$P(a,b)$	Probability of the event $a \bigcap b$
$P(a\|b)$	Probability of the event a given event b

Typefaces

X, Y, Z	Variable names
x, y, z	Specific values taken by the variables X, Y, Z
$\mathbf{X}, \mathbf{Y}, \mathbf{Z}$	Sets of variables
$\mathbf{x}, \mathbf{y}, \mathbf{z}$	Values to $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$

Abbreviations

TE	Transfer Entropy
AIS	Active Information Storage
MI	Mutual Information
GRN	Gene Regulatory Network
BA model	Barabasi Albert model
QRE	Quantal Response Equilibrium
NE	Nash Equilibrium
TDBR	Topologically Distributed Bounded Rationality
BR	Bounded Rationality
ER Random Networks	Erdős-Rènyi Random Networks

Variables Used

r	Assortativity of a network
N	Number of nodes in a network
K	Degree of a node (the number of links of a node). In the case of directed networks, this is typically used to denote the degree of the target node.

γ	Scale free exponent of a network

Chapter 1

Introduction

We are all connected and we all make decisions. These two factors are not only true for humans, but also all living beings. Further, these two qualities may be attributed to all autonomous systems, such as robots and even distributed sensors. Thus, connectivity and decision-making may be the two most critical components of a biological and even an artificial existence. Interconnectedness and decision making are interwoven aspects of any social structure. For example, a person is judged by the friends they have. To refine this further, it could be said that a person's behaviour and actions are judged by the friends they have. On the other hand, a person's friends or associates are determined by the behaviour or actions of that person. The nature of this reciprocal relationship between the interconnectedness and the decision making of individuals or autonomous agents in general is the central research question that drives this thesis.

Strategic decision-making scenarios among spatially distributed players are abundant in real-world socio-economic systems. Depending on the relationships that are formed among strategic players, their decisions may vary drastically. In turn, the strategic interactions that are formed among players may affect the networks that they form over time. In the following section, three different scenarios where strategic decision making and the interconnectivity among populations of player are described, in order to justify the motivation for this work.

1.1 Motivating examples for networked game analysis

1.1.1 Online auctions

Electronic commerce continues to grow at a rapid pace[19]. Online auctions are one of the most successful forms of electronic commerce. The rapid growth of these markets can be attributed to three factors. The first is that the online auctions provide a less costly way for buyers and sellers on locally thin markets, such as specialised collectibles, to meet. For instance, in May 1997, nearly $500,000 worth of Beanie babies was sold on eBay, totaling 6.6% of eBay's total sales. The second factor is that online auction sites substitute for more traditional market intermediaries such as specialty dealers in antiques, sports cards and other collectibles.

The third and perhaps the most relevant attribute with respect to this work is that online auctions provide a collaborative framework for exchanging information among the bidders. Most online auctions sites have large message boards which can be used to share information among bidders and get an idea of the behavioural patterns of the other players in the auctioning environment. Thus, the connectivity and information exchange among players or bidders play a key role in the strategic decisions made by them in placing bids.

Game theory has been extensively used to model auctions, including online auctions. Generally, players are modelled as buyers (bidders) and sellers. The action set available for each player is the set of bid functions. Under game theoretic models, auctions are studied under two broad categories: private value auctions and common value auctions. In private value auctions, each bidder knows his or her value for the object and the bidders differ in their values for the object. Examples of items that are auctioned under this category are memorabilia and consumption items. Broadly speaking, this is the game theoretic auction model followed in online auctions. In common value auctions, the item has a single though unknown value and the bidders differ in their estimates of the true value of the object. Examples of such auctions are the frequency spectrum auctions and drilling auctions. In both of these auctions models, the implicit information exchange among bidders and sellers is critical. The social networks that exist among these players have a key role in this implicit information exchange. In turn, such information exchange determines the 'rationality' of the decisions made by the players in the auctioning environment. Thus, studying the effect of the topological structure and the information flow among players in an auctioning environment is critical in accurately modelling and predicting the auctioning games.

1.1.2 Political campaigns in social media

Political campaigns are increasingly making use of online social media campaigns. A well-known example of the usage of social media in politics is the presidential campaign of Barack Obama in 2008[60]. He had a landslide victory, partially due to the huge support that he gained from online social media. Another example of how social media can affect the outcome of an election was observed in 2006, USA midterm elections. 85% of all candidates running for the senate had online presence and 32% posted their advertising content on Facebook.

There are various arguments about the role of social media in political campaigns. Bennett [34] suggests that the organisational weakness of online social networks may provide a core resource for new political groups with scarce resources. On the other hand, Bimber [36] argues that while online social networks may increase the fragmentation of political systems, it may not entirely erode the influence of the political elites, institutions and organised groups. It is widely accepted that online social networks are changing the way political advocacy and activism are practised, given that they make many aspects of lobbying, campaigning and organising more effective and efficient [34, 95, 106].

Another aspect of social media that has recently gained much interest is its ability to shape public opinion and even organise social groups to bring about political change. The so called Arab spring, which is a series of public uprisings that occurred in middle-east countries in 2011-12, is a perfect example where the strength of social media contributed to topple a government and bringing about the transition of political power by rallying public support and shaping public opinion via social media. In one study, Howard et al. [110], analysed over three million tweets, gigabytes of Youtube videos and thousands of blog posts and found that social media played a key role in facilitating the public protests and overthrowing the governments in Tunisia and Egypt during the Arab Spring. The key findings of that study were that:

1. Social media played a central role in shaping the political debates in the Arab Spring;

2. A spike in online revolutionary conversations often preceded major events on the ground; and

3. Social media helped spread democratic ideas across international borders.

Thus, it is evident that the social interactions facilitated by social media have opened up a revolutionary trend in political campaigning. On some occasions, the social media campaigns that organise political protests and political movements have not been based on advanced online portals such as Facebook, but on simple text messages on mobile phones. For example, during the impeachment of the Philippine president Joseph Estrada in 2001, a crippling protest was arranged in Manila by a text message that was circulated among the public[229].

In organising public protests, the power of social media is extremely effective. As the decentralised nature of message forwarding and information-sharing on social media makes it extremely difficult for centralised government structures to control. Also, the speed and effectiveness of information-sharing via social media can be extremely versatile in organising public protests.

Game theory has extensively been used to model the social interactions in political campaigns. For instance, Becker [31] proposes a theory of competition among political pressure groups. He suggests that political equilibrium depends on the efficiency of each group in producing pressure, the effect of additional pressure on their influence, the number of persons in different groups and the dead-weight cost of taxes and subsidies. Storm [243] proposes a behavioural theory of competitive political parties, which he suggests as a unified theory of the organisational and institutional factors that constrain party behaviour in parliamentary democracies. Behavioural game theory has been applied to online social media to model public interactions in a political setting. For instance, the Colonel Brotto game, which models the allocation of limited resources by two presidential candidates in a political campaign, has been deployed and analysed on Facebook [136]. The analysis has concluded that the results from the Facebook based Brotto game are consistent with the previous studies of the game. Although there have always been strategic interactions in political campaigns, social media provides an outlet where these interactions can be quantified and measured. It should be also noted that in a population, not everybody responds in the same manner to the campaigns. Thus, there is a heterogeneity of the rationality of players in a political campaign or a protest campaign, if these campaigns are modelled as behavioural games. Therefore, political campaigns that take place over populations of players connected in a spatially distributed structure provide an interesting avenue of research where the topology and information flow among players is critical in predicting the outcome of the game.

1.1.3 Financial markets and social behaviour

Stock markets are an important asset in financial markets. One of the key research questions related to stock markets is whether stock prices can be predicted. Recent advances in the socio-economic theory of finance, behavioural economics and behavioural finance may be critical in predicting the behaviour of the stock markets. Recent research suggests that the information extracted from social media may be vital in predicting the changes in various economic and commercial systems[48].

There are four aspects to consider in a stock options market that is socially constructed: the behavioural postulates of the market, models of micro-networks, models of macro-networks, and consequences of the network structure. The behavioural postulates refer to the basic assumptions of the nature of the market actors. Micro-networks are the structure of the egocentric networks, where networks are formed from the perspective of individual actors. Macro-networks depict the overall structure of the market that emerges from the micro-network formations [20]. Consequences are the effects of the market networks on the price determination.

In the ideal stock market, players or actors are assumed to be perfectly rational. In reality, market actors may be described more realistically by two behavioural assumptions suggested in the transaction cost approach. Those are[262]:

- The recognition that human agents are subjected to bounded rationality; and
- The assumption that at least some agents are given to opportunism.

These two postulates are extremely relevant to the stock options market. The floor participants, such as the stockbrokers are aware of their limitations in receiving, processing and responding to market information. Thus, the exchange of key information plays a vital role in determining the rationality of the decisions made by these players. For example, noise and the physical separation of potential trading partners have been cited as two major obstacles in the efficient communication of offers to buy and sell. Additionally, in a dynamic market environment, a floor participant may not be able to fully survey all potential trading partners.

What makes the information exchange in informal social networks is even more critical in decision-making in stock options is the uncertainty of the markets [48]. Because stock markets are designed to be extremely competitive, they naturally create an environment of extreme uncertainty for brokers and traders. Stock price volatility and market size also contribute to the instability of the market, contributing to the relative significance of information exchanged through informal social networks.

Game theory has been used extensively to model and predict the stock markets and financial markets in general. For instance, evolutionary game theory has been effectively used to model the stock market mechanisms using the minority model [271]. In this model, after everybody has chosen a side independently, those who are in the minority side would win. Another evolutionary game that has been proposed to model stock markets is the prototype trading model. In this model, each player is initially given the same amount of capital in two forms; cash and stock, while all trading consists of switching back and forth between cash and stock. Each player would have a strategy that makes recommendation for buying and selling a certain amount of stock for the next timestep.

While game theory could be used to effectively model stock trading in stock markets, the players in a stock market are boundedly rational due to the limitations of the information flow in their social networks. These two factors make a strong case for considering the network structures and information flows in modelling financial markets as network-based games.

1.2 Significance of the topic

As evident from the case studies discussed above, the intertwining of social networks and strategic decision making scenarios are abundant in real life. When populations of players interact with each other strategically, the variation of rationality among them plays a key role in the outcome of the decisions and the status of the population as a whole. The variation of the topological features also affects the exchange of information among the players. Depending on the population that is observed, different topologies are critical in determining how the players interact among each other in a strategic game.

While the scale-free topology has been observed in most real-world populations, its significance is seldom explored in strategic games played in populations of players. The impact of scale-free networks and random networks on strategic games is an interesting research question that needs to be explored in order to accurately predict the behaviour and the outcome of strategies. Further, in doing this the effect of network topology on the evolutionary outcome of the games can be investigated.

Another aspect that is intertwined with network topology and information flow is the rationality of players in a population. The effect of network topology and information flow on determining the rationality of nodes is an important research question that is critical in predicting the outcome of strategic interactions in populations of players; thus, these aspects of strategic games are addressed in this work.

In order to broadly capture the above aspects of the strategic decision-making of autonomous agents, this research studies the topological and information theoretic aspects of network based games. The applications of such analysis varies across different disciplines ranging from politics to entertainment, as suggested by the above-mentioned case studies and examples.

1.3 Research question

This research is built of three pillars of science: network science, game theory and information theory. These three pillars are interwoven to model real-world socio-economic systems, which consist of strategic players that are topologically distributed and operate under information diffusion constraints. Since these these disciplines are interrelated in some way, the research question of this study incorporates the theoretical aspects of all three disciplines. In particular, we focus on the two aspects of Network science, which is network topology and information diffusion. Network topology is the structure or the organisation of nodes in a spatial arrangement. Information diffusion deals with the flow of information through the network. We examine the question on how does the network topology and information diffusion in a network affect a strategic game played over that network. It is important to note that the discussion is not limited to evolutionary games although evolutionary games are a critical component of it. Instead, evolutionary games as well as the equilibrium aspects of network games from a micro to macro perspective are looked at. This broad research question is broken down into the following sub questions.

1. How does the network topology and information diffusion of a population of players affect the evolution of coordination in the population of players?

2. How is the evolutionary stability of strategies affected by the network topology of a population of players and what are the other topological effects that are critical in determining the evolutionary stability of a strategy?

3. What is the optimum method to distribute the contending strategies in a heterogeneous network of players in order to maximise the common public good of the population?

4. How can the bounded rationality of players be modelled using their topological behaviour? What are the emergent topological properties that arise from such a model and what are the possible applications of such a topological model of bounded rationality?

5. How can the bounded rationality of players be modelled based on the directed information flow and what are the topological and functional implications of such a model?

The subsequent chapters from chapter 3 to chapter 8 address each of these questions in detail.

1.4 The Structure and the Methodology of the thesis

This thesis is structured as follows. Chapter one introduces the thesis and provides a background and the motivation for this work, including an introduction to the research question and the structure of the thesis. Chapter two discusses the background knowledge of the work in the fields of network analysis, game theory and information theory. It also provides the basis for the construction of the research question.

This work follows a quantitative and experimental research methodology. More specifically, findings are based on simulated results based on network and game theoretic models. Qualitative analysis is used to interpret the results and also justify the models used. Thus, broadly speaking, the research methodology used in this work is a combination of quantitative and qualitative research methodologies. We incorporate real-world network data into these models to further validate the results, whenever applicable. Further, we make extensive use of statistical methods as correlation analysis to determine the possible strength of relationship among network properties such as scale-free correlation and clustering coefficient, where applicable.

While this is the broad research methodology used in this work, each chapter elaborates on specific research techniques used in the context of that work. The chapters from chapter three to eight can be sub-divided into two segments. The chapters three to five study the effect of networked game dynamics from different perspectives such as the effect on the evolution of coordination and the evolutionary stability of strategies. The second segment, which expands from chapter six to eight, is based on the modeling of bounded rationality based on network topology and information diffusion.

Accordingly, the third chapter discusses the influence of network topology in the evolution of coordination in network-based games. This is further analysed with respect to the information diffusion in networks. Quantitative analysis of results generated from experimental simulations were used in this chapter. In order to simulate the coordination game played on a population of nodes, an ensemble of scale-free networks, small-world networks, hierarchical-modular networks, Erdős-Rènyi random networks, and lattices were used. Exploring the evolution of coordination on such varying set of topologies help to study the affect of topology more broadly, instead of limiting the study primarily to the Erdős-Rènyi random networks and Scale-free networks[225].

The fourth chapter discusses the effect of network topology on the evolutionary stability of strategies. In order to evaluate this, different strategies were evolved in well-mixed and scale-free topologies. The strategies are then evolved over a number of timesteps to observe which strategy dominates the population and which strategy dies out. Effectively, this work extends on the work done by Adami and Hintze [4], by taking into account the heterogeneity of networks as a factor of evolutionary stability. Based on the observations gathered, the topological effect on the evolutionary stability of strategies is introduced as the 'topological stability' of strategies.

The fifth chapter focuses on the optimisation of strategy placement in order to maximise the common public good of a population. For this purpose, several well-known strategies are considered and the question is posed that given a pair of strategies present in a network, which strategy should occupy the hubs in order to maximise the total pay off or the common good of the society. Evolutionary optimisation is used as the method for evaluating the effectiveness of strategy placement. This is a novel approach introduced to optimise the placement of strategies in order to maximise their collective payoff.

The sixth chapter proposes a topological model of bounded rationality that suggests there is a topological interpretation of bounded rationality in populations of players and thereby proposes a topological model of bounded rationality. Based on the notion that there is a correlation between network topology and node rationality, a topological model of bounded rationality is proposed. In order to validate this model, a random network is evolved to optimise the rationality to observe how the topology of the network evolves. In addition to that, different network topologies, such as well-mixed, random and scale-free are compared to observe which topology produces the interactions with highest level of rationality. The methodology adopted here is a novel approach of modeling and quantifying rationality.

The seventh chapter discusses the potential applications of the topologically derived bounded rationality. Three applications are considered: the peer-to-peer network formation, network security and peer-to-peer routing applications. In all of these applications, the topologically distributed bounded rationality model is shown to produce results that match the real-world observations. Thus, these results are shown to be as validation of the applicability of the topologically distributed rationality model in populations of strategic players.

Next in chapter eight, the rationality is modelled as based on the information transfer among nodes, instead of the topology of the nodes. Using random boolean networks as the underlying information transfer model, the transfer entropy is applied as a dynamic bounded rationality measure in populations of strategic players. The implications of such an interpretation of rationality are discussed, with respect to random boolean networks. When the chaotic nature and the complexity of the random boolean networks are high, it is shown that the information transfer based rationality is at its peak, suggesting that the complexity of networks is a result of their tendency to optimise the rationality of interactions. The methodology used to model rationality as an information theoretic measure is a novel approach adopted in this work.

As previously mentioned, while the research methodology followed in each of the chapters varies, they follow the broad methodology of a simulation-based quantitative methodology with qualitative interpretations. A more detailed elaboration of the research methods followed to address each subquestion can be found within each chapter.

The chapter nine presents the summary of conclusions derived from each of the studies performed in this work.

1.5 Major Research Contributions

Following is a list of major research contributions of this work. These contributions are distributed over the research areas of network science, game theory and information theory.

- Both network topology and information diffusion are critical in determining the prevalence of coordination in networked populations.
- Noise and time-lag of payoff information adversely affect the evolution of coordination in a networked population.
- It is the peripheral hubs and drive the coordination in a population of players.
- Systems that are small-world but not scale-free are likely to evolve into being dominant in coordination and sustain it under difficult information-diffusion conditions.
- Three basic factors determine the topological stability of strategies in a non-homogeneous network; network topology, the evolutionary process and the initial distribution of the strategies.
- In order to maximise the collective utility of a networked population, the more dominant strategies need to occupy the hubs.
- The function of socio-economic systems in terms of bounded rationality is affected by the form or topology of socio-economic systems.
- When evolutionary pressure is applied on social systems to optimise their strategic interactions, scale-free and small-world features emerge.
- The topology of a socio-economic structure affects the function of the network in terms of its capacity to facilitate multiple equilibria.

- The topology and the strategic interactions of socio-economic structures are cyclically interdependent.
- The directed information transfer quantified using Transfer entropy may be effectively used to quantify the bounded rationality of players in a networked population.
- Complexity that emerges in the real-world systems may be a result of their tendency to optimise the strategic interactions.

Chapter 2

Background

This work is based on three fundamental fields of science; Complex systems/Complex networks, Game theory and Information theory. As the focus is on studying the influence of topology and information transfer on the strategic games played among spatially distributed players, all these three fields of science are relevant in building up the background for this thesis. Following sections discuss each of these fields of study within the scope of this research.

2.1 Complex Systems and Complex Networks

Complex Systems is a field of science that is used to study how the components of a system and their relationships give rise to the collective behaviour of the system[23]. In other words, complex systems are systems where the collective has properties that cannot be derived by aggregation of the constituents. Complex systems science has sprung out of multiple fields of study such as non-linear dynamics, statistical mechanics, information theory, computational theory, behavioural psychology and evolutionary biology[165]. Most of the real-world complex systems, such as the networks formed on top of the relationships that are formed among people, the neurons that are connected in the brain, the cells that are connected in the body all can be modelled as complex networks. However, modeling complex systems as complex networks may lead to a certain level of loss of information as the nodes in a network are often modelled as abstract and homogeneous entities[194], while actual components in a complex system are mostly heterogeneous.

There are three main approaches used in the study of complex systems[23]. Namely, (1) How interactions give rise to patterns of behaviour, as in homophily[156] (2) The space of possibilities or the possible patterns that can occur, and (3) The formation or growth of complex systems through evolution and pattern formation[25]. Currently, all three approaches are being pursued extensively by the research community. In this work, all these approaches have been taken as appropriate.

Even though complex systems do not necessarily have a formal definition, there are certain key characteristics that can be used to identify a complex system. Following are three such characteristics that are common to almost all complex systems.

1. Emergence
2. Interdependence
3. Self-organisation

2.1.1 Emergence

Emergence is the advent of the properties in a system in a large-scale or macro view that may not be evident in the micro level. An example of a system where emergence is prevalent is the world wide web (WWW)[10]. The Internet may consist of computers, servers, routers and other myriad components may have specific characteristics and behaviours of their own. However, the WWW its own dynamics and features that may only be evident in a large scale macro view. Also, the WWW would have its own growth trends and patterns that are evident in the large scale view. Emergence is only applicable for details that are important for the large scale view, as not all details that may be present in a system may have a relationship with the large scale, macro perspective of the system. More formally put, emergence refers to a system's global behaviour that arises from the collective actions of the simpler components[165]. It is this feature of complex systems that gives rise to their non-linear characteristics. One example for this emergent behaviour is the emergence of coordination in a social network even when the micro-level interactions may be self-centered[5].

2.1.2 Interdependence

What makes complex systems 'complex' is that its parts are interdependent. Because of this interdependence of components, the traditional approaches of modelling such as discrete event simulation and the modular approach may not be applicable in complex systems. Different kinds of interdependence can be observed. There may be systems where the individual components are not strongly coupled or connected with the rest of the systems, such as with the case of circuit-breaker systems. In some complex systems, such as biological systems, the removal or the failure of a single component may affect the entire system. Thus, identifying the nature of interdependence is each system is critical in modelling their behaviour accurately.

2.1.3 Self-organisation

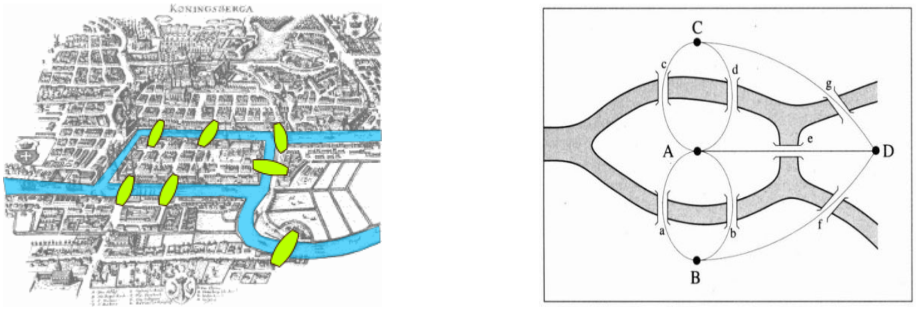
Perhaps self organisation is the single most significant characteristic shared by all complex systems. Self organisation can be described as the spontaneous appearance of large scale organisation through limited interactions among simple components[160]. As denoted by this definition, it is a feature that arise as a result of emergence and the interdependence of components. The best examples for self-organisation can be observed in nature. Extremely large systems such as galaxies, solar systems to minute systems such as cells are all display self-organisation behaviour. The antonym of self organised systems may be designed systems, which are man made. Unlike designed systems, self-organising systems do not have a central design and the components organise themselves through local interactions. One of the key motivations for studying the self-organising behaviour of natural systems is that it may pave way to designed systems that have the ability to self organise and evolve over time, in other words building 'designed self-organising systems'. While this may be a distant goal, certain attributes and behaviours of self-organising systems can be incorporated into designed systems. A good example is the usage of the particle swarm optimisation[206] model where local interactions of autonomous rule-bound agents give rise to collective and global solutions to computationally difficult problems.

2.2 Complex Networks: Modeling Complex systems as networks

Though complex systems cover a broad range of real-world systems, there is no common consensus on modelling and analysing them. However, complex systems invariably consist of components and their interactions. Thus, a complex system can be approximated as a network of components. One of the key issues that has to be resolved here is the loss of heterogeneity of nodes, since networks encompass nodes that are essentially abstract and homogeneous in nature[194].

It was the pivotal work of Leonard Eular published his article 'Seven bridges of Konigsberg' that gave birth to modern graph theory[35]. In this work, he demonstrated that 'one cannot come back to the starting point by traveling through all the bridges once and once only'. Although the study of networks originally sprung out of graph theory, later on it started to grow as a field of its own while borrowing concepts from other fields of science such as statistical mechanics and Artificial Intelligence. A significant milestone in the field of network science is when Erdős and Rènyi did their influential proposition of Random graphs[81]. Recently, there has been a considerable interest in the study of networks in many disciplines ranging from communication systems to political science. This has paved way to the birth of a new science called 'network science', where the focus is on studying the holistic and global properties of networks that are common across myriad disciplines. In a way, this is a divergence from the standard reductionist approach used in modern physics where a system is studied by analysing it's components and constituents. In network science however, more attention is given to the collective and synergistic properties of the components that consist in the system. It has to be noted that sociologists have been conducting significant amount of quantitative studies of social networks over decades[94, 258]. However, it is quite recently that mathematicians and physicists and computer scientists have gained momentum in developing abstract models of networks.

With the advent of the world wide web, the interest of networks have gained surging interest. Online social networks have made it possible for individuals from across the world to form connections and build relationships that may not have been possible if it wasn't for the leaps in technological advances in the recent years. Studying the growth and evolution of these systems need novel approaches that may help to utilise such networks in



(a) The original map of the Konigsberg bridges

(b) The graph constructed based on the Konigsberg bridges

Figure 2.1: Eulers seven bridges of Konigsberg[2, 1]

the most productive manner. A good example for this is utilising the online social networks for direct and indirect marketing of goods and services. On the other hand, it may also help to avoid potential dangers and undesirable factors that could spread via networks. For example, if the most influential members within the social network are identified, it may help to crack down a terrorist network, preventing potential acts of terrorism. How game theory and network science can be intertwined is discussed extensively in the book, 'Networks, Crowds and Markets' by Easley and Kleinberg[78].

Following are some of the key questions that networks scientists have been and are still trying to address[165]:

- What topological measures can be used to characterise the properties of networks?
- What are the properties that are common across different kinds of real-world networks? What are the domain specific properties of real-world networks?
- How to design algorithms to study the properties of networks?
- How do these properties of networks affect the information diffusion, robustness and failure tolerance of networks?
- Given a network with certain set of properties, what is the most optimum way to search for a particular node in the network?

This work too tries to address some of these broad research questions within the adopted context. In order to fully appreciate these questions and their implications, it is necessary to have a more detailed view on networks and their properties and attributes.

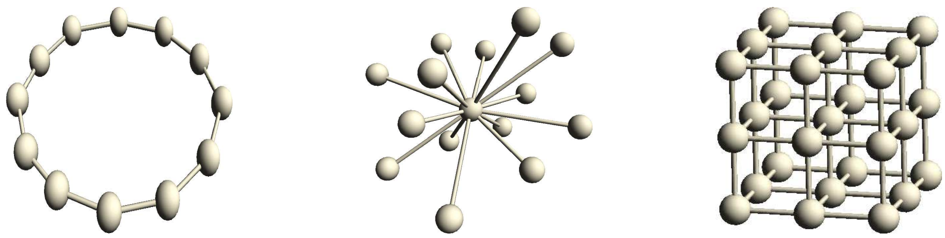
2.2.1 Networks

A network is a representation of a system where the components are represented as nodes or vertices and their interconnections are represented as links or edges[72]. Though this a fairly simple representation of a system, it is extremely powerful and flexible in capturing the static and dynamic properties of the system in concern.

A network could consist of nodes and links of different types. On the other hand, there could be networks with same abstract type but different attributes attached to them. A example would be a social network where attributes such as age, sex and income would be specific attributes attached to each node. Links could also have properties attached to them. For instance, in a social network, the type of relationship could be a property of the link. The properties of links could be of either scalar or discrete types. For instance, the sex of an individual in a social network is a discrete attribute while the weight of a node could be modelled as a scalar attribute. However, by nature a network representation is an abstract representation where nodes and links are represented as abstract entities. This abstraction may be beneficial in identifying the common properties over different types of networks, as representing too much specifics could be detrimental in identifying the abstract properties of a network. On the other hand, it could cause the loss of important information that are specific to each node. It is up to the network scientist to compromise on the level of abstraction and the provision of node-specific information in modelling networks.

According to the system that needs to be represented, different types of networks can be utilised. The links of a network may be directed, such as in road traffic networks, or undirected such as the network of neurons in the brain. A network could have weighted or unweighted links. For instance, the power line network would have load capacities attached to each power line, which can be considered as a 'weight' attached to the network. One of the most common network models is knows as 'Bipartite networks', which is used to model the relationships between two types or classes of objects, such that the two types of objects are fully connected with each other. For instance, a bipartite graph could be used to map the graduate students with a faculty.

The 'topology' of a network is the spatial structure of the nodes and their interconnections. Unlike the scalar or discrete attributes that may be attached to the nodes or the links, the network topology carries implicit information that could be node specific or globally relevant to the entire network. Topology of a network can be analysed using graph theoretical and statistical methods to identify these properties of nodes and the network. One of the key advantages of using network topology for network analysis is that it can be done without having access to the concrete scalar or discrete properties of nodes and links. Thus, a network scientist may derive useful information about the system that is represented using the network through topological analysis, even if the nodes and links represented are completely abstract. This makes topological analysis a powerful tool that could be applied to networks across myriad domains. The Fig. 2.11 represents some of the most common and trivial network topologies that could be used to model a system[246].



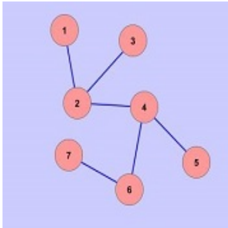
(a) Ring (b) Star (c) Lattice

Figure 2.2: Basic topologies

Though the graphical representation of a network may be useful to a human observer to capture its global characteristics and patterns, simple representations of networks are necessary in network analysis. One of the most common methods of representing a network is by using an adjacency matrix. An adjacency matrix is a $N \times N$ matrix, where the network would consist of N nodes. The matrix values could either consist of link weights or a boolean flag to represent the presence or absence of a link. The other most common representation is a link list where each item in the list would be the source node and the destination node of each link, if the network is directed. In this work, both these approaches are extensively used to perform the network analysis. Fig. 2.3 depicts a simple undirected network with the corresponding adjacency matrix and the link list.

2.3 Real-world Complex Networks

While complex network properties have been observed in a plethora of networks under heterogeneous domains, real-world complex networks can be divided into four broad cat-



(a) Graphical representation

	1	2	3	4	5	6	7
1	0	1	1	0	0	0	0
2	1	0	0	1	0	0	0
3	0	1	0	0	0	0	0
4	0	1	0	0	1	1	0
5	0	0	0	1	0	0	0
6	0	0	0	1	0	0	1
7	0	0	0	0	0	1	0

(b) Adjacency matrix

1	2
2	3
2	4
4	5
4	6
6	7

(c) Link list

Figure 2.3: Different representations of networks

egories. Namely, Social networks, information networks, technological networks and biological networks[177]. Even though the domains of these networks may vary, complex network features and mathematical models are relevant across these domains.

2.3.1 Social Networks

A social network consists of a set of people or groups with certain patterns of interactions or contacts among them[104]. The friendships among individuals, business relationships among companies and collaborations among scientists are all examples of social networks. Social networks are one of the most dynamically changing and evolving forms of complex networks that can be observed in the real-world. Further, they can be modelled with the mathematical models such as the scale-free model and the small-world model. One of the key advantages of studying social networks from a complex networks perspective is that it enables harnessing the decades of qualitative and quantitative studies done by social scientists, studying social networks. The first examples of such studies include the study on factory workers in late 1930s[218] and the mathematical models developed by Rapoport[215], who was the first to stress the significance of degree distribution in studying networks. One of the difficulties that existed in studying traditional social networks is that the difficulty in measuring and quantifying the social interactions accurately. With the advent of the Internet and the World wide web however, a branch of social networks have gained prominence in the form of online social networks. While these networks are becoming more and more relevant from applications ranging from online marketing to shaping public political opinion[229, 244], they enable the network scientists to more objectively quantify the social interactions and extract the network topological information. This has made online social networks an invaluable tool in network analysis, which has resulted in a new rigour in study of social networks from a complex networks perspective[164, 138].

2.3.2 Information Networks

Information networks differ from social networks in that they are formed among information based entities, not individuals or organisations. A classic example of an information network is the citation network of academic papers[70]. These networks contain accurate and objective information about the network topology and content, making them an attractive choice in network analysis. Citation networks are acyclic, since paper citations are bound by a chronological order.

Perhaps the most ubiquitous information network available is the world wide web, which is essentially a network of web pages. It is a unique network of unparalleled size and growth rate, posing challenging questions on searching and ranking for nodes. The information network of WWW has facilitated creation of algorithms and techniques are have later been adopted in the field of complex network analysis in general. A good example of such an algorithm is the Page-rank algorithm, which was initially used to rank web pages by Google[192], and subsequently has been applied in complex network analysis in general[49, 87]. Some of the lesser known information networks include the citation network of patents[114] and online peer-to-peer resource sharing networks[147]. While Peer-to-Peer resource sharing networks have their own growth algorithms, at the macro level they have been observed to demonstrate complex networks characteristics[118].

2.3.3 Technological Networks

Technological networks are man-made networks that are designed to distribute a commodity or a service. Interestingly, these networks are designed, though they evolve and grow in to self-organising networks behaviour and other complex network characteristics. A good example of such networks is the power-grid, which is used to distribute electrical power. Numerous studies have been done on the network analysis of the power grid[7]. Wireless sensor networks, mobile device networks and the Internet are other well-known technological networks[44]. These networks differ from information networks in that a node would be represented by a physical entity, rather than an information based entity. Another important characteristic specific to technological networks is that their topology and structure is indirectly affected by the geography of their distribution. Further, the technological networks such as the Internet have several layers of inter-operable networks such as the router network and the Internet service provider network.

2.3.4 Biological Networks

Biological networks are an important subset of complex networks that have opened several multi-disciplinary research avenues. One of the most well-known biological networks is the network of metabolic pathways. Substantial body of work exists in the network analysis of metabolic pathways[205, 116]. Gene regulatory networks are another important subclass of biological networks. These networks try to capture the expression of a gene in a network structure. Random boolean networks have been effective in modelling the gene regulatory networks[11], using which important questions on the origin and nature of life are addressed. Food webs and animal networks too fall into the category of biological networks, which help biologists to extract information about animal behaviour using network topological analysis[77]. Networks of the brain or physical neural networks help to uncover the structure and functionality of the brain in a network perspective[79].

2.4 Network properties and measures

Graph theory and statistical mechanics have contributed with myriad network properties and measures that are used to analyse networks. There are both local and global properties that are used to analyse networks. Local measures are those that are bound to a particular node or a link. On the other hand, global properties enable quantifying the network-wide properties that a network may possess. Based on these measures it is possible to categorise networks and even predict their behaviour. While network science literature consists of numerous properties and measures defined to analyse networks, this section only discusses those measures that are directly relevant to this work.

2.4.1 Centrality Measures

Perhaps the most common problem that arises with respect to networks is how to identify the most influential or prominent nodes in them. This problem is relevant in numerous scenarios ranging from preventing the spread of contagious diseases to protecting a computer network from malicious attacks and viruses[56, 161]. Centrality measures are those measures that are used to quantify the prominence of a node within a network. Following are some of the most common centrality measures that are used in network analysis.

Degree Centrality

Perhaps the most common and intuitive centrality measure available in network analysis is the Degree centrality. Degree refers to the number of connections or links that a node may have. For instance, in Fig. 2.3[a], the degree of node 1 would be 1 while the degree of node 4 would b 3. If the network is directed, the *in-degree* and *out-degree* of each node can be considered, depending on whether the incoming links or outgoing links are measured. Degree is a local measure that can be a topological property of a node.

Betweenness Centrality

While degree is a local measure that can be used to measure centrality, it fails to capture the global influence in measuring a node's prominence is a network. Betweenness centrality of a node is defined as the number of shortest paths going through the node in concern, considering the shortest paths that connect any two given nodes in the network. The formal definition of betweenness centrality would be[222]:

$$BC(v) = \frac{1}{(N-1)(N-2)} \sum_{s \neq v \neq t} \frac{\sigma_{s,t}(v)}{\sigma_{s,t}} \tag{2.1}$$

Here, $\sigma_{s,t}$ is the number of shortest paths between the source node s and the target node t. $\sigma_{s,t}(v)$ is the number of shortest paths between source node s and target node t that lies through node v. Betweenness centrality is most relevant in situations where the information flow within a network is taken into consideration, such as in traffic networks.

Closeness Centrality

Closeness measures the average distance to other nodes in the network, from the node in concern. In essence, it's a measure of the time that it takes to spread the information from a particular node to the other nodes in the network. While it is closely related to betweenness centrality, closeness more relevant in situations where a node acts as a generator of information rather than a mere mediator. The Eq. 2.2 denotes the formal definition of the closeness centrality[222].

$$CC(v) = \frac{1}{\sum_{i \neq v} d_g(v, i)} \tag{2.2}$$

Here, $d_g(v, i)$ denotes the shortest path (geodesic) distance between nodes v and i. The average is inverted so that the node that is closest to the other nodes will have the highest closeness centrality.

Eigenvector Centrality

A recently proposed centrality measure, eigenvector centrality measures a node's influence in a network by taking into account the influence of its neighbours. Google's page-rank algorithm[191], that is used to rank web pages according to their relevance can be regarded as a variant of the eigenvector centrality. The eigenvector centrality[38] assumes that the centrality score of a node is proportional to the sum of the centrality scores of the neighbours. As such, it is defined iteratively. If the centrality scores of nodes are given by the matrix $\mathbf{X}$ and the adjacency matrix of the network is $\mathbf{A}$, then x can be defined iteratively as,

$$\mathbf{x} \propto \mathbf{Ax} \tag{2.3}$$

i.e

$$\lambda\mathbf{x} = \mathbf{Ax} \tag{2.4}$$

The centrality scores are obtained by solving this matrix equation. It can be shown that, while there can be many values for λ, only the largest value will result in positive scores for all nodes [182].

2.4.2 Degree Distribution

As mentioned earlier, the degree of a node in a network is the number of edges connected to that node. Suppose p_k is the fraction of nodes in the network that has the degree k.

Thus, p_k would be the probability that a node chosen randomly would have the degree k. A plot of p_k of a given network can be generated by making a histogram of the degrees of the nodes. This histogram is called the degree distribution[177]. Degree distribution of a network can be thought as an abstract representation of a network. However, it does not provide a unique one-to-one mapping of a network as networks with varying mixing patterns may have similar degree distributions[180]. In a directed network, the in-degree and out-degree distributions may be considered depending on whether it's the in-degree or out-degree that is used to plot the histogram. In practice the degree distribution is plotted in logarithmic scale to highlight its characteristics. Depending on the degree distribution plot of a network, it may be characterised into different categories. These categories are discussed further in section 2.5.

2.4.3 Clustering coefficient

Also referred to as the 'transitivity' of a network, clustering coefficient measures the 'clustering' or group behaviour of the network. This is an important property that has been observed in real-world networks and particularly in social networks[163]. This pattern emerges when the neighbours of a node tend to make connections among themselves. In social network terminology, this may occur when the two people make connections via a mutual friend. In network topology terminology, clustering is the presence of heightened number of triangles in the network[177]. It can be quantified using the clustering coefficient C as follows:

$$C = \frac{3 \times number\,of\,triangles\,in\,the\,network}{number\,of\,connected\,triples\,of\,nodes} \tag{2.5}$$

Defined in this manner, clustering coefficient is a local measure, which is seldom referred to as local clustering coefficient. Clustering coefficient can also be measured as a global measure by averaging the local clustering coefficient values over the entire network. This global measure is particularly relevant in signifying the Small-world network model[260, 9, 177], as described in section 2.5.3.

2.4.4 Average Path length

Average path length is the average distance along the shortest paths that exists among all possible pairs in a network. It can be regarded as a measure of efficiency in spreading information over a network. The formal definition of the average path length would be,

$$l_G = \frac{2}{n.(n-1)} . \sum_{i \neq j} d(v_i, v_j) \tag{2.6}$$

Here, $d(v_i, v_j)$ denotes the shortest distance between the nodes v_i and v_j, and n is the number of nodes in network G. Average path length is also an important measure that is used to define the Small-world network model [177], which will be explained in detail in section 2.5.3. If the network is disconnected, the average path length calculation would be infinite, as certain nodes may not be reachable by other nodes. To avoid this issue, the average path length can be calculated for all connected components and then the average of those values can be considered as the average path length of the entire network.

2.4.5 Assortativity

One of the interesting questions that can be asked about networks is that which nodes would tend to pair up with others. Social science and epidemiology has put forth *homophily* [156] as a possible answer to this question, suggesting that nodes tend to connect with similar nodes. Network scientists use the term ‘assortative mixing’ or ‘assortativity’ to define this mixing pattern. Even in technological networks such as in the network of Internet Service Providers (ISPs) within the Internet, assortative mixing has been observed[177], suggesting that this property may be relevant in networks in general, and not just in social networks.

Assortative mixing can occur based on any property of a node. A very common example of assortative mixing in social networks is mixing by race. Further, such mixing can occur based on sex, age and even income groups. However, when generic networks are analysed based on network topology, topological measures become more relevant in analysing assortative mixing. One of the key advantages of using a topological measure to study assortative mixing is that it enables it to be quantified and compared and contrasted among networks. Often, it is the degree of nodes that is used to quantify assortativity, as it is the most intuitive and inherent property of a node.

Assortative mixing can be quantified using the Assortativity coefficient. It is calculated by measuring the Pearson correlation coefficient of degrees between lined pairs of nodes. The Eq. 2.7 is used to calculate the assortativity coefficient in a network[180, 247].

$$\rho = \frac{1}{\sigma_q^2}\left[\sum_{jk} jk\left(e_{j,k} - q_j q_k\right)\right] \tag{2.7}$$

Here, j and k denote the degrees of a given pair in the network, while q_k and q_j represent the *remaining degree distribution.* The remaining degree distribution is a slight variant of the degree distribution discussed in section 2.4.2, where the degree of each node is considered excluding the link that connects the pair. The term e_{jk} denotes the *joint probability distribution* of the remaining degrees of the two nodes that are connected by the link, which would specify the probabilities at which the given remaining degrees would recur within the network. For an undirected network, this property would be symmetric, and would follow the sum rules $\sum_{jk} e_{jk} = 1$ and $\sum_j e_{jk} = q_k$. The assortativity coefficient would have the range [-1:1]. A positive assortativity would mean that the nodes with similar degrees would have a higher tendency to connect with each other, while negative assortativity would indicate that it is the nodes with dissimilar degrees that tend to pair-up in the given network. An assortativity coefficient value close to 0 would indicate that there is no clear correlation of mixing patterns with the remaining degrees of nodes. It has been observed that collaboration networks and social networks tend to demonstrate positive assortative mixing while natural networks such as food webs and neuron networks show negative assortativity[180]. Thus, mixing patterns quantified using assortativity is a useful measure that can be used to analyse and classify networks.

There have been other variants of assortativity that have been proposed that defines assortativity as local measure instead of a global measure and using alternative link properties to define assortativity [199, 248]. However, these measures would not be explained in detail in this section as it is beyond the scope of this work.

2.4.6 Network Resilience

Network resilience is a network's ability to maintain its functionality and connectivity against node removal[128, 129]. This property has been subjected to a great deal of scrutiny, particularly in the field of epidemiology[170]. Further, applications of this property can be found in fields such as computer networks, technological networks and financial networks[15, 120].

2.5 Network models

Three general models of networks can be found in network science literature: *random*, *small-world* and *scale-free* [165]. These three models are identified by their degree distributions, global properties such as the average path length and clustering coefficient, and how they evolve over time. These models can either be used to model and approximate real-world networks. Further, studying such abstract reference models may help network scientists to predict the characteristics and even manipulate the real-world networks that they are identified with. It has to be noted that the network models that are discussed here are display properties that emerge due to self-organisation behaviour and not designed network topologies such as the star or ring topology.

2.5.1 Random networks

Random networks were initially proposed by Erdős and Rènyi in their pivotal work 'On random graphs'[81]. Erdős and Rènyi proposed a model to generate random graphs, where a graph is defined by n labelled nodes connected by e links, where links are chosen randomly from $\frac{n(n-1)}{2}$ possible edges[9]. Hence, the total number of graphs that could be generated would be $C^{e}_{\left[\frac{n(n-1)}{2}\right]}$. While random networks display elegant mathematical properties, most characteristics of they deviate significantly from those of real-world networks. Thus, random networks are often used as an abstract reference model rather than an actual approximation of the real-world networks. Within the scope of this work, random network model is used as a reference model in comparison to the scale-free and small-world network models.

As mentioned before, the properties of the random network model show marked differences from real-world networks. For instance, random networks do not show clustering or grouping behaviour. On the other hand, empirical results from real-world networks, such as collaboration networks and social networks give relatively higher clustering coefficient values that are not comparable with the clustering coefficients[9, 183] obtained from random networks with similar number of nodes.

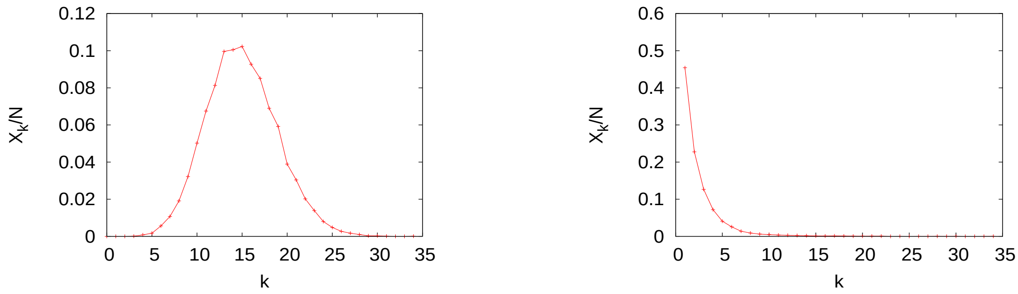
The other key variation that random networks display from real-world networks is in its degree distribution. While random networks display Poisson degree distributions, the degree distributions of most real-world networks are of Power-law degree distributions. The properties of the power-law degree distribution would be discussed further in section 2.5.2. The degree distribution of a random graph generated using the Erdős-Rènyi model would be of the following form, where p_k would be the probability of a node having degree k, where n would be the size of the network. The Fig. 2.4[a] shows a typical Poisson degree distribution of a random network.

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \simeq \frac{z^k e^{-z}}{k!} \tag{2.8}$$

Still, the random graph model proposed by Erdős and Rènyi is significant since it paved way to the concepts of 'growth', 'evolution' and 'emergence' of network models. When generating a random network, edges are accumulated in a successive and iterative fashion. One important discovery of the random graph model is that most of the significant properties of random graphs appear quite suddenly, making it the first 'complex network model' where emergence of properties is prevalent.

2.5.2 Scale-free networks

Scale-free networks are those networks that consist of a power-law degree distribution with a long tail. Thus, these networks would be deviating significantly from the Poisson degree distribution of the random networks. The Fig. 2.4[b] shows a typical power-law degree distribution in comparison to a Poisson degree distribution of a random network. Unlike random networks, most real-world networks observed do have scale-free networks, making this model much more relevant in studying real-world networks. In a random network, vast majority of nodes would have the same number of links. However, in a scale-free networks



(a) Random

(b) Scale-free

Figure 2.4: The degree distributions of a random network generated using the Erdős-Rènyi model and a Scale-free network generated using the preferential attachment growth model. The x-axis is the degree k and the y-axis represents the number of nodes with degree k, X_k divided by the size of the network N. The degree distribution of the random network shows a Poisson degree distribution while the scale-free network shows a power-law degree distribution. Both networks contained 10,000 nodes. The scale-free exponent γ in the scale-free network was measured to be 2.73.

there would be significant variations in the number of links that a node has. There would a only a few number of heavily connected nodes or "hubs", while a vast majority of the nodes would be connected these hubs via a relatively smaller number of links. For instance, in social networks, typically there would be highly connected individuals while most others would be connected to them. As such, these networks would not display 'scaling' as the grow, hence earning the name 'scale-free networks'. Formally put, a scale-free network would have the degree distribution p_k could be expressed using the following power-law relationship.

$$p_k \sim k^{-\gamma} \tag{2.9}$$

Here γ is called the 'scale-free exponent', which is a measure of the 'scale-freeness' of the network. For most real-world networks, this value is observed to be falling within 2 and 3[9].

Barabási and Albert proposed the 'Preferential-attachment growth model' to generate scale-free networks in 1999[25]. Apart from being a growth model, it has also been used as an explanation on why real-world networks demonstrate scale-free behaviour. Prior to them, Simon and Price [232, 211] had proposed the notion that power-law degree distributions in networks arise due to the tendency that "rich get richer". Several variations of the preferential attachment model has also been proposed[123]. Barabási and Albert argued that the networks evolve under two main contributing factors. Namely, *growth* and *preferential attachment.*

1. Growth

   The property of growth implies that networks have an inherent tendency to expand and growth. Within the context of the Barabási and Albert model, growth is a process that happens linearly such that in each timestep, a node is added with m links where the links are made with the already existing nodes in the network.

2. Preferential attachment

   Under preferential attachment, the probability $\prod$ that a new node will be connected to node i would be dependent on the degree k_i of node i, such that,

   $$\prod(k_i) = \frac{k_i}{\sum_j k_j} \tag{2.10}$$

   where j would be any existing node in the network.

Examples of real-world networks that demonstrate scale-free behaviour are WWW, collaboration networks, social networks, networks of the brain[27] and software networks[252]. Considering the heterogeneity of the domains, it is truly remarkable that scale-free model could have such wide applicability.

2.5.3 Small-world networks

Stanley Milgram[158] was one of the first to demonstrate that social networks demonstrate the *Small-world effect*, where most the nodes in a network appear to be connected via a relatively short path through the network, irrespective of the network size. He devised an experiment where letters were passed among the first-name acquaintances of the participants and measuring the number of intermediate passers that was involved in the successful delivery of the letters. It was observed that the letters that were delivered were passed through at most six people. This experiment paved way to the concept of *six degrees of separation.*

Even before Milgram, the existence of the small-world effect had been speculated. One of the most remarkable references to this can be found in the short story by the Hungarian writer Frigyes Karinthy[119], which was published in 1929. Pool and Kochen had attempted to propose a mathematical basis for the small-world effect[66]. Subsequently, it has been noted that the small-world effect is not just specific to social networks but can be observed in networks in general[29, 167].

The small-world effect can be particularly important when considering the spread of information in a network. It suggests that there could be rapid spread of information, may it be a disease, or a gossip, through a network irrespective of the network size. In a computer network, small-world effect could be utilised to reduce the number of hops or intermediate computers that are required to transfer a packet of data from the source to destination. Thus, it is a very important feature of networks that can be utilised when interacting with networks.

More recently, mathematical interpretations of small-world effect have been proposed, instead of the qualitative definitions suggested by the earlier work on this model. One such definition is that in such networks the average path l, would scale proportional to the logarithm of network size n. Further, small-world networks demonstrate high clustering coefficient suggesting that the grouping behaviour would actually useful in reducing the average path length of the network. Hence, small-world effect is a relative measure that is typically characterised by the higher clustering coefficient and the lower average path length. Based on these properties Watts and Strogratz[260] proposed their model of small-world networks. It is used throughout this work to generate small-world networks.

Watts-Strogatz model

The model proposed by Watts-Strogatz to generate small world networks consists of two basic steps.

1. Start with order - Start with a ring lattice with N nodes, where every node is connected to its first K neighbours ($\frac{k}{2}$ on either side).

2. Randomly rewire the edges in the lattice with probability p. By varying p it is possible to change the transition from an ordered lattice to a fully random network.

For $p = 0$ the resulting network would be a regular lattice while for $p = 1$ the resulting network would be a completely random network. For low but non-zero values of p, the resulting network would have high clustering coefficient with few 'long-distance' connection, making the average path length relatively low. Such networks would have the 'small-world effect' with relatively high clustering coefficient and relatively low average path length. Fig. 2.5 depicts a graphical representation of the topological transition that occurs in the Watss-Strogatz model. While relatively easy to implement, the Watts-Strogatz model has the shortcoming of producing degree distributions that are not comparable with most real-world networks[165].

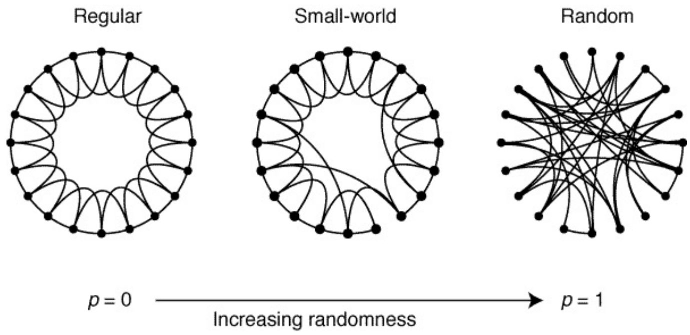


Figure 2.5: The variation from a regular lattice to a total random network in the Watts-Strogatz model[260], when the rewiring probability p is increased. The networks that have relatively higher clustering coefficient and lower average path length would have the small-world effect.

It has to be noted that the small-world and scale-free effects are mutually exclusive[14]. In other words, there could be real-world networks that demonstrate both the small-world and scale-free behavior, such as most social and collaboration networks. However, there could be networks that only demonstrate either of those effects.

A variation of the Small-world networks have been proposed by Newman et al.[178], where instead of rewiring the links, shortcuts are added among the existing network structure.

Both the Preferential attachment model and the Watts-Strogatz model were extensivvely used throughout this thesis to generate the theoretical scale-free and small-world networks, respectively.

In this sub-section of the background, emphasis was given on providing an introduction on the concepts of network science that are relevant in this study. However, without delving into the foundations of strategic interactions among autonomous agents built upon game theory, it is impossible to study the strategic interactions that occur on top of networks of players. Therefore, next, we will look at the science of strategic games, which is the next pillar on which this work is based.

2.6 Game theory

Game theory is the science of strategic decision making among agents. Strategic decisions are decisions that are interdependent, where actions of one affects the other. On the other hand, game can be regarded as the formal study of cooperation and conflict[171]. For game theory to be applied, multiple agents have to interact in a strategic decision making environment. These agents may be individuals, firms, governments or combinations of those. The models and concepts provided in game theory is used to formulate and analyse strategic decision making scenarios. Following are the basic concepts and terminology that are used in game theory to define strategic decision making scenarios.

- Game - The particular strategic decision making scenario that is being modelled. This may range from a stock trading in the share market to placing a bid in an online auction.
- Player - The agents that are involved in making the strategic decisions.
- Strategy - Actions that are available for each player.
- Payoff - The return or utility assigned to each action taken by each player.

2.6.1 History

Game theory originated as a branch of mathematics and micro-economics. The earliest known game theoretic analysis was done by Antoine Cournet in his study of Duopoly in 1938[141]. Mathematicians Emile Borel and Von Neumann[155] formulated a formal theory of games in their respective works. However, the inception of the modern field of game theory could be regarded as the monumental volume *Theory of games and economic behaviour*, by Von Neumann and the economist Oskar Morgenstern. This work provides much of the terminology and concepts that are in use in the field of game theory.

Perhaps the most important concept that modern game theory is based on is the concept of Nash equilibrium, which was proposed by John Nash in 1950[174]. He proposed that finite non-cooperative games always have an equilibrium point at which all agents choose actions that are best for them given their opponents' choices. In this work, the concept of Nash equilibrium and its variations are used extensively, as explained in the subsequent sections. Even though game theory originated as a branch of micro-economics, later on it has been applied to myriad fields such as political science, psychology, sociology and even biology[61, 172]. This may be due to the fact that the prevalence of strategic decision making scenarios in all these domains, where game theoretic concepts may be applied to model and analyse such scenarios.

With the rise of computer and communications technology, game theory has found many novel applications in areas such as online auctions, network routing and frequency bandwidth allocation. For instance, Google cooperation's 'AdSense' uses game theory to bid on keywords that are intended to be used for targeted advertising[115]. In engineering, there is growing interest in using non-cooperative game theory due to the possibility of designing large-scale systems that globally regulate their performance and functionality in a distributed and a decentralised manner. Modelling a system as a set of sub-systems that compete for a limited resource is relevant in most engineered systems, where game theory is an attractive tool of choice. Examples of such applications include congestion control of network traffic, optimising network routing and power allocation in wireless sensor networks[13, 241]

Following sub-sections explain some of the basic concepts and models identified in contemporary game theory. In particular, several well-known classifications of strategic games are discussed.

2.6.2 Cooperative and Non-cooperative game theory

Cooperative game theory[41] focuses on how coalitions could be formed among players to maximise the utility of each player in concern. As such, cooperative games can be regarded as a situation where the cooperative behaviour is enforced on players by factors external to the game or the environment itself. Cooperative game theory is more appropriate to legal and political situations where the relative amount of power held by each player has to be taken into account. Bargaining is an example where cooperative game theory can be applied, where the relative strengths of two parties involved is critical in arriving at a solution. Formally, a cooperative game consists of a finite set of N players, called the *grand coalition*, and a characteristic function $v\colon 2^N \to R$ from a set of all possible coalitions of players to a set of utilities that satisfies $v(\theta) = 0$.

Non-cooperative game theory is on the other hand is concerned about the strategic decision making by individual players. These games are called *non-cooperative* since each player tends to pursue its own interests which may conflict with the interests of others. In the non-cooperative game paradigm, the details of the ordering and timing of players' choices are critical in determining the outcome of the game. It is important to note that cooperation can arise in non-cooperative games as well, however it is not so by design but due to emergent behaviour. Non-cooperative game theory can be used to define most of the decision making scenarios in the real-world, such as in evolution of biological species, negotiations in the power distribution[82, 68] and forecasting in financial markets. In this work, we would be mainly focusing on the non-cooperative games, where individual, self interested and rational agents would interact with each other in strategic decision making environments.

Non-cooperative games can be represented in two basic forms depending on the consideration of the time dimension, known as Normal-form and Extensive-form game representations.

2.6.3 Normal form and extensive form games

Normal form game[109] descriptions define strategic decision making scenarios where the players play their moves simultaneously. In other words, normal form games do not take into account the temporal element in decision making. Even if there's a temporal difference in the moves taken by the players, if there isn't a flow of information among them, such scenarios may be represented as normal form games.

A normal form representation may consist of the following components.

1. A finite number of players.
2. A strategy set assigned to each player.

3. A payoff function, which assigns a payoff to each player based on his strategy and the strategies of the other players.

A Normal form or a simultaneous move games are used to define scenarios where the players are not aware of each others actions when choosing their strategies. It could be that the players play the moves simultaneously or that they don't have access to the information of the opponent's strategies. Thus, even when there is a time difference in the moves, if the players don't have the knowledge of the opponent's moves, it can be considered as a normal form game. If the number of players engaged is two and if the list of strategies are limited to a few elements, the outcome of the payoff function can be represented in a matrix, which is referred to as the *payoff matrix*. Payoff matrix depicts the two players, their strategies and payoffs. Following figure depicts a sample payoff matrix.

Player 1 \ Player 2	L	R
U	1, 3	2, 4
D	1, 0	3, 3

Figure 2.6: Sample payoff matrix

According the to payoff matrix given in Fig. 2.6, player 1 has two different strategies: Up (U) and Down (D). Player 2 has two different strategies, left L and right R. According to the given payoff matrix if player 1 chooses strategy U and player 2 chooses strategy R, the outcome is (2,4), which suggests that the payoff of player 1 is 2 and player 2 is 4. Examples of normal form games include classical games such as prisoner's dilemma game[216] and Coordination game, which will be discussed further in the subsequent sections.

Extensive form games[197] represent strategic decision making scenarios where the players play their strategies consequently. In other words, the temporal sequence of events are taken into consideration. Typically, an extensive form game is represented as a game tree. Each level of the tree represents a temporal state of the game and each node represents a stage of the game. Examples of games that can be represented in the extensive form include board games such as Chess and Auction games, where the sequence of actions are important. A game in extensive form may be analysed directly or could be converted into a equivalent strategic form game and then analysed.

Fig. 2.7 depicts a sample game tree of an extensive form game. According to the given game tree, the payoffs of player 1 would be 1 and the player 2 would be 2, given that the player 1 players the strategy D and subsequently player 2 chooses the strategy U'. It is important to note that the flow of information is what is critical for the decision making of the players in the extensive-form games, rather than the actual flow of time. If the information flow is not intact, the game could effectively be regarded as a normal-form game, even if the actions are taken at two different instances of time.

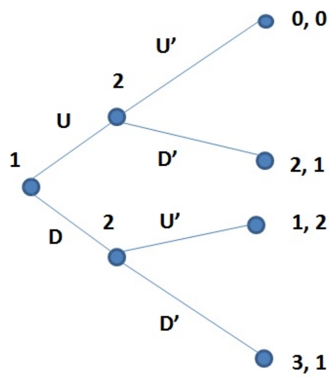


Figure 2.7: An extensive form representation of a game

Extensive form games can further be sub-divided into **perfect** and **imperfect** information games. An extensive-form game has perfect information if each player is perfectly informed of all the events that have previously occurred when making a decision. Board games such as chess and tic-tac-toe are examples of perfect information games, where each player is aware of the steps that are taken up to the point of making a decision. Further, only one player makes a move at a given time, thus there are no simultaneous moves. Backward induction[16] method is used to solve a game with perfect information, by reasoning backwards in time.

However, in most real-world strategic decision making scenarios, players do not have full access to the information that is relevant to their choices. Extensive form games with imperfect information are used to model such scenarios. The information that is relevant in such scenarios would be the type of the players, their strategies and their respective payoffs. Normal-form games can be regarded as a special case of imperfect information extensive-form games where a player has absolutely no knowledge of the previous move of the opponent.

While acknowledging the relevance and applicability in extensive form games in a wide-range of domains and applications, this works focuses mainly on normal form games applied in a network context.

Complete and incomplete information games

Another important categorisation that can be made regarding games is with respect to the information availability to players. In complete information games, all players are completely informed of all other players' payoffs and all possible strategy profiles they may undertake. Both normal-form and extensive-form games may fall into this category. For instance normal form games like prisoner's dilemma and extensive-form games such as chess can be regarded as complete information games.

In games with incomplete information[102], players may not have common knowledge of the game being played. Following may be some of the aspects on which the players may not have common knowledge.

- Payoffs
- Who the other players are
- What are the possible strategies available
- What are the utilities attached to each strategy
- The knowledge that the opponent has of the game and of the player in concern

Due to the complex and dynamic nature of the information flows, most real-world networks fall into the category of the incomplete information games. Following are some of the examples where incomplete information games may be used to model the strategic decision making scenarios.

1. In price competition scenarios, firms may be aware of their own costs, but not the costs of others.

2. Firms involved in R&D might have knowledge about their own product, but may not know whether there are similar products being developed by the other firms.
3. A government may decide on a tax policy without having the prior knowledge of the ploys that people may develop to avoid paying taxes.
4. The climate change agreements among countries may vary based on their beliefs of the effect of climate change.

The players in an incomplete information game may have *private* and *public* information components that they utilise in making strategic decisions. This heterogeneous of information about the opponents and the game may lead to non-optimal or bounded *rationality* of the players. Capturing this heterogeneity of rationality of the players in a network is a critical research question, that would help to predict the behaviour of agents accurately. This is one of the key research questions that we try to address in Chapters 6,7 and 8.

2.6.4 Pure strategy and Mixed strategy games

In a pure strategy game, players would have a complete definition on how the players would play the game. It provides a deterministic strategy for each player when the opponent plays a move. The **strategy set** of a pure strategy game is the set of pure strategies available to that particular player. In other words, the pure strategy game is a game where the 'strategy space', that is the set of strategies available for the player, coincides with the 'action space', which is the set of actions that could be taken by the player. Classical prisoner's dilemma game and the Coordination game are defined as pure strategy games.

In mixed strategy games[255], there's an assignment of a probability to each strategy. As such, the strategies are not chosen deterministically. The probability distribution of a mixed strategy game is continuous, enabling a player to select from an infinite number of mixed strategies. A pure strategy game can be regarded as a special case of the mixed strategy game, where one strategy is selected, the probability of it being selected is always 1 and the probability of other strategies being selected would be 0.

The payoff matrix given in Fig. 2.8 demonstrates the matching pennies game, which is an example of a mixed strategy game. In such a game, each player would choose its strategy simultaneously based on a coin toss. Thus the probability distribution of selecting a strategy for each player would be [0.5, 0.5].

		Player 2	
		Head	Tail
Player 1	Head	1, -1	-1,1
	Tail	-1,1	1,-1

Figure 2.8: Matching pennies game where mixed strategies are used.

In this work both pure and mixed strategy games are simulated among spatially distributed players.

2.6.5 Nash Equilibrium

A dominant strategy is a strategy that would always provide the best possible payoff irrespective of the strategy taken up by the opponent. However, most strategies do not fall into this category and thus it is necessary to consider the equilibrium states to consider the most appropriate strategy to adopt. In 1950, John Nash[174] suggested the concept of Nash equilibrium, which has become the pivotal concept in game theory ever since. Nash equilibrium suggests that there exists a profile of strategies such that each player's strategy is the best response against the equilibrium strategies of other players. Here, the best response is the strategy that gives the highest payoff. In other words, at *Nash equilibrium*, the players cannot improve their payoffs unilaterally.

A formal definition for Nash equilibrium can be given as follows. Let (S, f) be a game with n players, where S_i is the strategy set of a given player i. Thus, the *strategy profile* S consisting of the strategy sets of all players would be, $S = S_1 \times S_2 \times S_3 .... \times S_n$. $f = (f_1(x), ..., f_n(x))$ would be the payoff function for $x \epsilon S$. Suppose x_i is th strategy profile of player i and x_{-i} be the strategy profile of all players except player i. Thus, when each player $i \epsilon 1, ..., n$ chooses strategy x_i that would result in the strategy profile $x = (x_1, ..., x_n)$, giving a payoff of $f(i)$ to that particular player. A strategy profile $x^* \epsilon S$ is in Nash equilibrium if no unilateral deviation in strategy by any single player would return a higher utility for that particular player. Formally put,

$$\forall i, x_i \epsilon S_i : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*) \tag{2.11}$$

A strategic game may have multiple equilibria. According to the above definition the game represented by the payoff matrix in Fig. 2.9 would have equilibrium pairs where the two players select the equilibria $(U_{1,1}, U_{2,1})$ and $(U_{1,2}, U_{2,2})$.

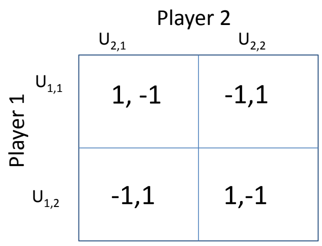


Figure 2.9: Game with multiple Nash equilibria

While the equilibria in the above game are *pure Nash equilibria*, since the strategies involved are pure strategies, Nash equilibria has been shown to exist in Mixed strategy games as well[173]. In Mixed strategy games, it is the expected payoff under a given strategy distribution that is considered in identifying the mixed strategy nash equilibrium. For instance, in the *matching pennies* shown in Fig. 2.8, the mixed strategy Nash equilibrium would be [0.5,0.5] for both players, where the expected payoff of each player would be 0.

Next, a brief explanation is given on the two games that are extensively used throughout this work as network-based games; The prisoner's dilemma game and the Coordination game.

2.6.6 Prisoner's dilemma game

Prisoner's dilemma game[216] is a classical normal form game defined between two players. The name is derived from a hypothetical decision making scenario between two prisoners. There is no judicial evidence against either of them, except if one prisoner testifies against the other. If one of them testifies, he will be awarded with immunity from prosecution, whereas the other will serve a longer sentence. If both testify, the punishment will be less. However, if both of them do not testify, their mutual sentences will be substantially reduced. Thus, the Nash would occur when both players testify against each other. However, the optimal strategy for them is not to testify. This is the 'dilemma' that they are faced in the prisoner's dilemma game.

Fig. 2.10 depicts the canonical payoff matrix of a prisoner's dilemma game. The 'cooperation' is identified as the situation where the prisoner doesn't testify and 'defection' is where the prisoner testifies against the other. In the given canonical form, the payoff inequality $T < R < P < S$ would hold in a prisoner's dilemma game.

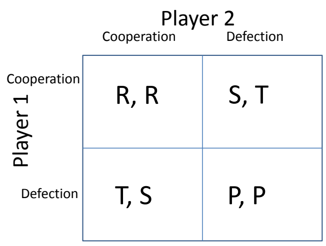


Figure 2.10: Payoff matrix of a prisoner's dilemma game

Prisoner's dilemma game is extensively used to model decision making scenarios where the individual 'defections' at the expense of others lead to overall less desirable outcomes. Examples of such situations include arms races, price wars among corporations, litigation instead of settlement and environmental pollution[149, 43]. Prisoner's dilemma game is extensively used in modelling populations of players in social networks, particularly in the form of an iterative game[17, 83].

Iterated prisoner's dilemma game

When the prisoner's dilemma game is player over many iterations and when the previous actions of the players affect the next selection of each player, it is called the iterated prisoner's dilemma game[17]. In pure strategy games, a player may adopt the strategy of an opponent, based on the payoffs of previous interactions, while in mixed strategy games the strategy probability distribution of a player may alter based on the previous interactions with the opponents. Iterated prisoner's dilemma game is widely applied in populations of players. Even though the Nash equilibrium of a single-shot prisoner's dilemma game is mutual defection, it has been observed that cooperation may emerge as the dominant strategy in an iterated prisoner's dilemma game in a population of players[196]. This is in apparent conflict with the Darwinian world-view of the prevalence of the fittest who are continuously in conflict with each other. It has been suggested that this peculiar phenomenon could be a result of the scale-free topology of most of the real-world populations of players[223]. It is a perfect example of the critical nature that network topology players in determining the outcome of strategic interactions in a population of players, which is part of the motivation for this work. In the existing literature, the iterated prisoner's dilemma game has been hardly used as a means of measuring public good. It is usually the Public goods game that is used for this purpose. But in a voluntary network of self-interested agents, maximizing the public good under the iterated prisoner's dilemma game may be more realistic. Thus, in Chapter 5, the iterated prisoner's dilemma game along with genetic optimisation is used to determine optimum placement of strategies in a non-homogeneous network, in order to optimise public or common good.

n-Memory strategies

In iterated prisoner's dilemma games, the players consider the history of interactions between the opponent in choosing the next strategy. N memory strategies consider the interactions occurred in the previous N steps, when making the next move. Memory-one strategies are a special subclass of N memory strategies where only the immediate previous interaction with the same opponent is considered in deciding the next move. The resulting mixed strategy distribution is a probability distribution, that is conditional to the previous set of interactions by the same players. Perhaps the most commonly known memory-one strategy is the tit-for-tat strategy[184], where a player would always respond with the same action that its opponent used in the previous iteration. In iterated prisoner's dilemma game, a memory-one strategy would consist of four probabilities depending on the previous set of interactions of the two players. Those four probabilities would be [CC, CD, DC, DD] where C and D would denote cooperation and defection in the previous interaction by each player, respectively. By varying these 4 probabilities, it is possible to come up with infinite number of memory-one strategies. There are well-known memory-one strategies such as the Zero-determinant (ZD) strategy and the Pavlov strategy, which have specific conditional probability functions attached to them. There has been a recent interest in the evolutionary stability of the Zero Determinant strategy[4]. However, there hasn't been a sufficient discussion on the effect of network topology on the evolutionary stability of strategies. Thus, in Chapter 4, we adopt memory-one strategies such as the ZD strategy, in an attempt to determine the topological effect of the evolutionary stability of strategies.

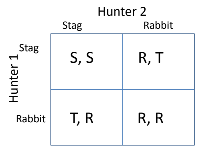
2.6.7 Coordination game

In this work, Coordination game[157] is extensively used, particularly in Chapter 3 to evaluate the effect of network topology on coordination behaviour. Coordination game is a classical game defined in game theory literature that has two pure Nash equilibria. Coordination game is used to model scenarios which the agents involved can mutually gain higher utility by coordinating or making decisions that are mutually consistent. In other words, what is beneficial for a single player would be beneficial for all. Thus, the players engaged in a coordination game may not have the 'dilemma' that the players in the prisoner's dilemma game would have. However, it should be noted that the two Nash equilibria in coordination game are mutual cooperation and mutual defection, where cooperation and defection are the two strategies available. It is mutual cooperation that gives the highest utility to the agents involved although mutual defection too is a Nash equilibrium solution.

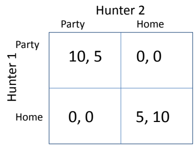
There are several variations of coordination games defined. Following are some of the variations of the coordination game utilised in this work. These variations exist due to the relationship among the payoffs given in the payoff matrix. Stag-hunt game, Battle of the sexes game and the pure coordination games are the three variations of the coordination game that are mainly adopted in this work. Following figures depict sample payoff matrices that depict the payoff relationships in each of those games.

Each of these games are woven around an interesting premise that describe a particular strategic decision making scenario. For instance, the stag-hunt game refers to a scenario where two hunters hunt for a stag or a Hare. In order to hunt a stag, both of them need to coordinate, thus coordination gives the highest payoff. If either of them chooses to hunt a hare, the other hunter would get zero utility as it is impossible for a single hunter to hunt a stag. If both hunters hunt a hare, then they would get an equal but lesser payoff. On the other hand, the battle-of-the-sexes game describe a strategic decision making scenario

(a) Pure coordination game



(b) Stag-hunt game



(c) Battle of the sexes

Figure 2.11: Variations of the coordination game

where a husband and a wife would decide on whether to go to a party or staying at home, depending on their preferences. These payoff matrices can be effectively used to describe a strategic decision making scenario where coordination would give a higher mutual payoff compared to defection. A typical example where coordination game is applied is discussed in Roger McCain's book "Game Theory: A non-technical Introduction to the analysis of strategy"[152]. He describes a scenario where two are driving down the same road that will soon meet each other. Supposing these is not centralised traffic direction is available, the only way each car can avoid collision is by *coordinating* with each other. We noticed that there was a lack of rigorous theoretical analysis on how the network topology and information diffusion facilitates coordination in a population of networked players, in the existing literature. Thus, in Chapter 3 we attempt to evaluate the cumulative effect of network topology and information diffusion on a population of networked players who are collectively playing the coordination game.

So far in this subsection, we discussed the history and background and game theory that are relevant to this work. Next, we would look at some of the fundamental assumptions made in game theory. These assumptions are central to this research, since providing a network based interpretations to them is a key contributions of this work.

2.7 Rationality of self-interested players

In strategic games, being *rational* is closely associated with being self-interested. In a game theoretic context, *self-interested behaviour* is the tendency of a player to maximise its own payoff. While this self-interested behaviour may not always be the case with autonomous agents in a strategic game in the real-world, it is a fundamental assumption in game theory. In particular, Nash equilibrium assumes that players make decisions with perfect rationality. In literature on game theory and strategic decision making, rationality is often discussed, although there isn't a precise definition of it. One possible definition of rationality is that a player is rational if[195];

1. the player consistently acts to improve its payoff without the possibility of making errors,

2. the player has full knowledge of other players' intentions and the actions available to them.

3. the player has infinite capacity to calculate a priori all possible refinements to the Nash equilibrium of the game in its attempt to find the best possible strategy available.

Thus, if a game involves only rational agents, each of whom assume that the other players too are rational, then the theoretical predictions of Nash equilibrium would match with the actual outcome of the strategic decision making scenario.

2.7.1 Bounded rationality

Even though perfect rationality is a fundamental assumption in game theory and the theoretical predictions of Nash equilibrium, in most real-world decision making scenarios among real-world players, such perfect rationality is hardly observed. It was originally Herbert A. Simon[231] as an alternative basis for mathematical modelling of decision making. Decision making by real-world strategic players is almost always constrained by three limiting factors; the cognitive capacity of the agent, the information available for the agent to make a decision and the time available to compute the most appropriate decision[88]. These factors are due to the limitations of the factors that are essential for a player to be perfectly rational, as discussed in the previous sub-section. These limiting factors may be observed in human players and even non-human autonomous agents. This non-optimal and limited rationality is known as the 'bounded rationality' of self interested players.

The notion of bounded rationality does not mean that players make irrational decisions. On the contrary, players do strive to make rational decisions, yet they may not always do so. In other words, the players in a strategic decision making environment are inclined to make 'better' decisions, even though they may not always make the 'best' decision. As a result, there would be an error component attached to the decisions made by the self-interested players in strategic environments. This error component may be due to the lack of information available, limitations of the cognitive capacity and the time constrains in making decisions. Within the context of this work, it is assumed that all players that are spatially distributed have equal computational time and cognitive capacity in making decisions and the only limitation would be the availability of information about the game and the other players in the environment. This may enable modelling bounded rationality from the perspective of network topological properties and information transfer, where such externally observable characteristics may be used to deduce the bounded rationality of players. In Chapters 6, 7 and 8, we attempt to model the distribution of bounded rationality of players in a networked population, using network topology and information transfer among players.

Many different models of bounded rationality have been proposed. These models can be thought of as generalisations of the Nash equilibrium, where errors in decision making due to bounded rationality is considered in predicting agent behaviour and the outcomes of the games. Most of these models have originated in the fields of political science or economics, due to the inherent complexities of human interaction in those fields. Following are some of the commonly known bounded rationality decision making models.

2.7.2 Generalisations of Nash equilibrium

For non-cooperative games, Nash equilibrium is the pivotal concepts which predicts an equilibrium state in existence for all non cooperative strategic scenarios. However, this prediction is based on the notion of perfect or optimum rationality. According to the argument of bounded rationality however, there exists non optimal rationality of players in making decisions. Thus, it is necessary to come up with alternative equilibrium models that facilitates for the bounded rationality. These models can be regarded as extensions or generalizations of Nash equilibrium. There's a noise element introduced in each of the bounded rationality equilibrium models to account for the errors introduced by the bounded rational players.

Several approximation models of Nash equilibrium are found in the literature. The most prominent of them are the Bayes-Nash equilibrium model, noisy introspection model, the ϵ-Equilibrium and the Quantal Response Equilibrium(QRE) modes. These equilibrium models attempt to accommodate for the error or the noise component in decision making by bounded rational players.

2.7.3 Bayesian-Nash equilibrium

The first extension to Nash equilibrium that we consider is the Bayesian-Nash equilibrium [67, 113]. Though it is not directly related to bounded rationality, Bayesian-Nash equilibrium is applicable in games where there is incomplete information. The incompleteness of information may lead to bounded rationality of players making them choose non-optimal decisions in a stochastic manner. In most game theoretic situations, the agent in concern is unsure about the preferences or intentions of others. The existence of incomplete information in a game introduces additional strategic interactions and also raises questions related to 'learning'. Following are some of the common examples of games with incomplete information;

- Bargaining - The amount that the other party is willing to pay is unknown to the agent.
- Auctions - How much should an agent pay for an object that the agent wants, knowing that the other agents too compete for the same object?
- Market competition - Firms that are unaware of the exact cost of their competitors.
- Signalling games - How to infer information about the others agents from the signals they send, such as in a job interview process.

- Social learning - How to leverage the decisions of others in order to make better decisions.

Bayesian games are modelled by introducing *Nature* as a player of the game. Nature would randomly assign a player with a type according to a probability distribution function of the players' type space. A Bayesian game would consist of,

- A set of players I
- A set of pure strategies for each player i: S_i
- A set of types for each player i: $\theta_i \epsilon \Theta$
- A payoff function for each player $i : u_i(s_1, ..., s_I, \theta_1, ..., \theta_I)$
- A probability distribution p$(\theta_1, ..., \theta_I)$ over types

The knowledge of all these components assumed to be the **common knowledge** in a Baysian games, which is refereed to as the 'common-prior assumption'. Based on this assumption, at Bayesian Nash equilibrium, each type of player chooses a strategy that maximises the expected utility given the actions of all types of other players and the player's 'beliefs' about the other players. Following is the formal definition of the Bayesian Nash equilibrium.

The strategy profile s is a Bayesian Nash equilibrium if for all $i \epsilon I$ and for all $\theta_i \epsilon \Theta_i$,

$$s_i(\theta_i) \epsilon \arg\max_{s_i' \epsilon S_i} \sum_{\theta_{-i}} p\left(\theta_{-i} | \, \theta_i\right) u_i \left(s_i' s_{-i}(\theta_{-i}) \theta_i, \theta_{-i} \right) \tag{2.12}$$

The expected payoffs that are calculated using the **Bayes rule** are optimised to identify the strategy distribution at the Bayesian-Nash equilibrium. It has to be noted that while the Bayesian Nash equilibrium tries to incorporate the incomplete information that a player may have about the other players' types, it does not directly differentiate the heterogeneity of *rationality* among players.

2.7.4 ϵ Equilibrium

ϵ-Equilibrium[214, 55], also called the near-rationality equilibrium is a generalization of Nash equilibrium based on the deviations of the payoffs from the Nash equilibrium predictions. Given ϵ is a non-negative parameter, a game is considered to be in ϵ Equilibrium if for any player it is not possible to gain more than ϵ in expected payoff by unilaterally deviating from his strategy. Nash equilibrium could be regarded as a specific instance of ϵ equilibrium where the value of ϵ would b 0. Following is the formal definition of ϵ equilibrium. In the near rational equilibria a player who is not perfectly maximising her utility cannot improve her payoff by a substantial amount by playing her Nash equilibria more accurately. Though the payoff losses for a player is quite small, the equilibria derived often represent substantial departures from the respective Nash equilibrium prediction. Models of near rationality equilibria may be appropriate for the description of empirical phenomena.

ϵ equilibrium concept relaxes conception of a perfectly rational player to a model where each player is satisfied to get closer to, but not necessarily arrive, at its best response to the other players' strategies. In a ϵ Equilibrium, no player can increase its utility by more than ϵ by choosing another strategy. Thus, an ϵ equilibrium is identified by identifying a strategy for each player so that her payoff is within ϵ of the maximum possible payoff given the other players' strategies. The formal definition of the ϵ equilibrium is as follows;

Definition 2.7.1. ϵ - Equilibrium $\sigma^{\epsilon} = (\sigma_1^{\epsilon}, ..., \sigma_n^{\epsilon}) \epsilon \sum$ comprises a mixed-strategy ϵ equilibrium of a game G if, for all $i \epsilon N$, for all $\sigma_i^{'} \epsilon \sum_i$, and a fixed $\epsilon > 0, u_i(\sigma_i^{'}, \sigma_{-i}^{\epsilon}) - u_i(\sigma_{-i}^{\epsilon}) \leq \epsilon$.

A pure strategy ϵ-Equilibrium is a vector of pure strategies, $s^{\epsilon} \epsilon S$, that satisfies the equivalent condition. According to the given definition, $\epsilon = 0$ would reduce the ϵ-Equilibrium to Nash equilibrium. Therefore, ϵ-Equilibrium can be regarded as a generalised form of Nash equilibrium.

Even though ϵ-equilibrium provides a convenient model of providing for the errors in decision making, it does not provide a mechanism for quantifying the rationality of players in a rationality parameter. Hence, we adopt the Quantal Response Equilibrium model which consists of a rationality parameter that could be used to model the bounded rationality of each player.

2.7.5 Quantal response equilibrium

Probabilistic choice models are often used to incorporate stochastic elements in to the analysis of individual decision, which was proposed by McKelvey and Palfrey[153, 91] These models are based on Quantal response functions such as the logit and probit functions. Quantal response functions provide an analogous approach to model noisy players. The Quantal response functions, on which the Quantal response equilibrium is based on, have the unique feature where the deviations from optimal decisions are negatively correlated with the associated costs. In other words, in a when players behave according to a quantal response function, they are more likely to adopt a better choice compared to a worse choice, though they may not always adopt the best choice. Formally put, quantal response function maps the vector of expected payoffs from available choices into a vector of choice probabilities that is monotone in the expected payoffs.

In a strategic decision making environment, a player's beliefs from adopting different strategies are indirectly determined by its beliefs about the opponents' actions. Thus, beliefs determine the expected payoffs, which in turn would generate choice probabilities according to a quantal response function. In equilibrium, the Quantal response functions imposes the requirement that the beliefs should map the equilibrium choice probabilities.

Quantal response equilibrium can be regarded as a more generalised form of Nash equilibrium. QRE converges to the Nash equilibrium when the quantal response functions become very steep and approximate best response functions.

Compared to the bounded rationality and imperfect information based equilibrium models discussed so far, the QRE model has three distinct advantages when it is used as a tool for quantifying bounded rationality. Firstly, the probability of mistakes depends on the payoff differences between actions, so mistakes are less likely when there is a lot at stake for players. Secondly, it is a consistent equilibrium theory in the sense that th subjects' responses take into account the mistakes of other players as well. Thirdly, it provides a *rationality parameter*, particularly when the logit and probit functions are used, that is useful in quantifying the individual or collective rationality of each players.

QRE model has been defined for both Normal-form and extensive form games. Since this work makes extensive use of the Normal-form QRE model, it will be explained in more detail compared to the other equilibrium models. Following is the formal definition of the normal-form QRE model.

Let $G = (N, S_1, ..., S_n, \pi_1, ..., \pi_n)$ be a normal form game, where $N = 1, ..., n$ is the set of players, $S_i = s_{i1}, ..., s_{iJ(i)}$ is player i's set of strategies and $S = S_1 \times ..... \times S_N$ is a set of strategy profiles, and $\pi_i\colon S_i \rightarrow R$ is player i's payoff function. Further, let $\sum_i = \delta^{J(i)}$ be the set of probability distributions over S_i. An element $\sigma\epsilon \sum_i$ is a mixed-strategy, which is mapping from S_i to $\sum_i$, where $\sigma_i(s_i)$ is the probability that player i chooses pure-strategy s_i. Let $\sum = \sum_1 \times .... \times \sum_N$ be the set of mixed-strategy profiles. Given a mixed-strategy profile $\sigma\epsilon \sum$, player i's expected payoff is $\pi_i(\sigma) = \sum_{s\epsilon S} p(s)\pi_i(s)$, where $p(s) = \pi_{i\epsilon N}\sigma_i(s_i)$ is the probability distribution over the pure-strategy profiles induced by σ.

Let P_ij denote the probability that player i selects strategy j. The main argument behind QRE is that the strategies with higher expected payoffs are more likely to be chosen, although the best strategy is not necessarily chosen with probability 1. Thus, a *quantal response function* can be defined as follows;

Definition 2.7.2. Quantal response function:

$P_i\colon R^{J(i)} \rightarrow \Delta^{J(i)}$ is a **regular quantal response function** if it satisfies the following four axioms.

- Interiority: $P_{ij}(\pi_i) > 0$ for all $j = 1, ..., J(i)$ and for all $\pi_i\epsilon R^{J(i)}$.
- Continuity: $P_{ij}(\pi_i)$ is a continuously differentiable function for all $\pi_i\epsilon R^{J(i)}$.
- Responsiveness: $\delta P_{ij}(\pi_i)/\delta\pi_{ij} > 0$ for all $j = 1, ..., J(i)$ and for all $\pi_i\epsilon R^{J(i)}$.
- Monotonicity: $\pi_{ij} > \pi_{ik}$ implies that $P_{ij}(\pi_i) > P_{ik}(\pi_i)$ for all $j, k = 1, ..., J(i)$.

These four features are critical for the validity and applicability of the model. Interiority ensures that the model is consistent with all possible data sets, which is important for empirical applications of the model. Continuity suggests that the arbitrary and small changes in the payoffs do not cause jumps in the choice probabilities. Responsiveness suggests that if the expected payoff of an action increases, the choice probability must also increase. Monotonicity implies that an action with a higher expected payoff is chosen more frequently than an action with a lower expected payoff.

Suppose $P(\pi) = (P_1(\pi_1), ..., P_n(\pi_n))$ to be regular if each P_i satisfies the above regularity axioms. Since $P(\pi)\epsilon\sum$ and $\pi = \pi(\sigma)$ is defined for any $\sigma\epsilon\sum$, $Po\sigma$ defines a mapping from $\sum$ to itself. Thus, a *regular Quantal Response Equilibrium* can be defined as follows;

Definition 2.7.3. Quantal response equilibrium Let P be regular. A **Regular Quantal Response Equilibrium** of a normal-form game G is a mixed-strategy profile σ^* such that $\sigma^* = P(\sigma^*)$.

Logit QRE

The most commonly applied quantal response functions are logit functions, producing a logit-Quantal response equilibrium (LQRE). In a logit-QRE, the players' strategy strategies are chosen according to the probability distribution;

$$P_{ij} = \frac{exp(\lambda_i EU_{ij}(P_{-i}))}{\sum_k exp(\lambda_i EU_{ik}(P_{-i)})} \tag{2.13}$$

where P_ij is the probability of player i choosing the strategy j. $EU_{ij}(P_{-i})$ is the expected utility of player i choosing strategy j given other players are playing according to the probability distribution P_{-i}. The parameter λ_i is known as the *rationality parameter*, which would be discussed further in the subsequent chapters, where it is interpreted based on network topology and the information transfer among players in a network. As $\lambda_i \to 0$, the player would be 'completely irrational', behaving randomly, and as $\lambda_i \to \infty$, the strategic interaction approaches Nash equilibrium. The rationality parameter is often set arbitrarily to match the empirical observations. However, in this work, an attempt is made to quantify it using network topological measures and information transfer measures.

Following is an example where the Logit-QRE is applied in a generalised matching pennies game. Suppose there are two players engaged in a generalised matching pennies game where the row player has to choose from top (T) or bottom (B) and the column player has to has to choose from left(L) or right(R). Row wins a penny while column loses a penny if the outcome is (top,right) or (bottom, left) and Column wins a penny otherwise. Thus, the row player's expected payoff for Top (U_T) is a function of the column player's probability of choosing Right (p_R). The expected payoff would thus be, $U_T(p_R) = p_R-(1-p_R) = 2p_R-1$. Similarly, $U_B(p_R) = 1 - 2p_R$.

The Fig. 2.12 depicts the step functions associated with the choice probabilities of the two players. The smoothed curves represent the appropriate quantal response functions. If the Top-Right outcomes is changed such that the Row player gets 9 and the Column player gives -1, it would shift the Row player's best response line as given in the dotted line. The dotted curved line shows the respective quantal response function. The new Nash equilibrium, which is at the intersection of the step functions gives p_T as 0.5, while the respective QRE based p_T is higher than that, as shown in the figure. This kind of shift in choice probabilities due to the variations of a player's own-payoffs do coincide with the empirical observations and laboratory experiments[97, 90, 219], which demonstrates the applicability of the QRE model.

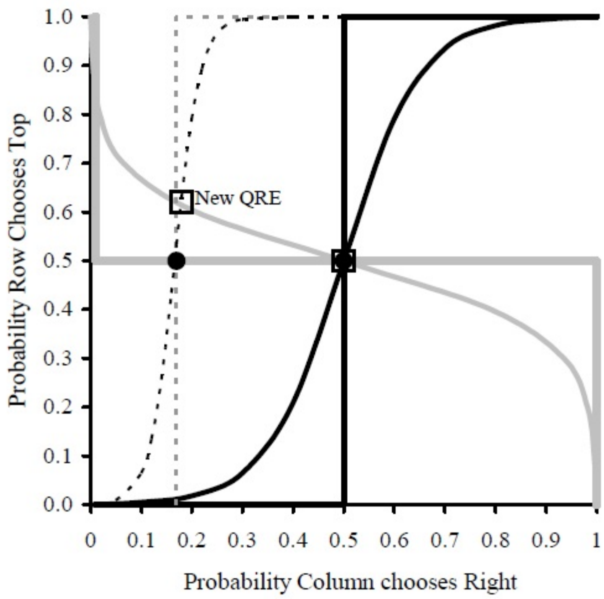


Figure 2.12: Step and quantal response functions of a generalised matching pennies game.[91]

The QRE choice probabilities of the generalised matching pennies game can be derived using the logit function based calculations given in Eq. 2.14 and 2.15. Similar choice probability calculations based on the logit-QRE functions are used throughout this work.

$$p_T = \frac{exp(\lambda_R[X+1]p_R - 1)}{exp(\lambda_R[X+1]p_R - 1) + exp(\lambda_R[1-2p_R])} \tag{2.14}$$

$$p_R = \frac{exp(\lambda_C[1 - 2p_T])}{exp(\lambda_C[1 - 2p_T]) + exp(\lambda_C(2p_T - 1))} \tag{2.15}$$

Here, p_T and p_R would provide the probabilities at which the Row player would select the Top option and the column player would select the Right option. λ_R and λ_C are the rationality parameters of the Row player and the column player, respectively. As these parameters are increased, the response functions become more responsive to the payoff differences. As the rationality parameters reach infinity, the logit functions converge to the step functions shown in Fig. 2.12.

Quantal response equilibrium has been proposed for extensive form games as well[154]. In the extensive form QRE, players follow the Bayesian rule and calculate the expected payoffs based on the QRE strategies of the other players involved. While QRE in extensive form would have numerous applications ranging from auctions to bargaining problems, in this work we would only focus on QRE and its rationality parameter in its normal form.

2.8 Evolutionary game theory

The game theoretic concepts discussed so far have mainly been on classical game theory, where the games are 'static' and they are played among two players. However, in most real-life decision making scenarios, the games are played among populations of players that interact with each other. Examples of such games are auctions, stock trading options and political campaigns. Evolutionary game theory originated from the adoption of game theory into the field of evolutionary biology. Evolutionary game theory[235, 261] improves on traditional game theory by providing dynamics describing how a game would evolve over time in populations of players. John Maynard Smith in his book *Evolution and the Theory of Games, 1982*[236] introduced the mathematical basis of *Evolutionary game theory* (EGT). Even though the inception of the field is bound with evolutionary biology, EGT can also be applied in studying the evolution of a population of players over time, such as in sociology and political science.

Evolutionary biology is based on the idea that an organism's genes largely contribute to its observable characteristics. Thus, genes determine the fitness of an organism in a given environment. Evolutionary game theory argues that the success of an organism depends on its interaction with other organisms within a population. Thus, the fitness of an organism would be dependent on its interaction with other organisms in that population. In game theoretic terminology, the interactions of players would be the strategies and the fitness of each strategy would be the payoff. As the population evolves over time, certain strategies would be dominant while some other strategies would be extinct. Thus, evolutionary game theory provides a mathematical basis to model the evolutionary process using concepts of game theory. On the other hand, it opens a new dimension for game theorists to observe how the evolution of strategies in games would happen over time in populations of players. Some of the critical questions asked in EGT include; which populations/strategies are stable? When to individuals adopt other strategies? and would it be possible for mutants to invade a given population?

2.8.1 Evolutionary stability

Evolutionary stability of strategies or Evolutionarily Stable Strategies (ESS)[245, 107] is an important concept in evolutionary game theory. If a strategy is evolutionarily stable if, when the entire population is following that strategy, a small invading group with a different strategy would eventually die-out over multiple generations. These invaders could either be migrants from a different population or mutants from within the same population. More formally put, evolutionary stability of a strategy can be defined as,

- The fitness of an organism in a population is the expected payoff it receives from interacting with a random number of opponents within the population.
- Strategy T would be invading strategy S at the evolutionary step x, for some small positive number x, if an x fraction of the underlying population adopts the strategy T and a 1-x fraction of the underlying population uses S.
- Strategy S is considered to be evolutionary stable if there is a relatively small positive number y such that when any other strategy T invades S at any level x ¡ y, the fitness of the organism playing S is strictly greater than the fitness of the organism playing T.

While Nash equilibrium is a static equilibrium, Evolutionary stability of strategies signify a dynamic equilibrium that takes in to account the space and time dimensions of population of players. In the context of this research, the effect on the evolutionary stability of strategies by the topological aspects of a population of players is studied. Following example demonstrates the applicability of evolutionary stability of strategies.

To explain the concept of ESS further, let us consider a following hypothetical game among populations of players, called the Hawk-Dove game. In this game, two groups of players called Hawks and Doves compete for a resource of a fixed value V. The **Hawk** strategy demonstrates aggressive behaviour, not stopping until injured or the opponent backs down, while the **Dove** strategy is retreating immediately if one's opponent initiates aggressive behaviour. The game can be defined by making the following assumptions of the two strategies.

1. When two players with the Hawk strategy meet, conflict would result and the two players are equally likely to get injured.
2. The cost of conflict reduces the individual fitness by some constant value C.
3. When a hawk confronts a dove, the dove would retreat and the hawk would obtain the resource.
4. When two doves meet, the resource would be shared equally among them.

The payoff matrix given in Fig. 2.13 summarises these assumptions.

$E(D, D)$, which is the expected payoff when two doves meet each other is $V/2$, is less than the expected payoff when a Hawk meets a Dove, $E(H, D) = V$. This would mean that *Dove* is not a evolutionarily stable strategy. On the other hand, $E(H, H) = 1/2(V - C)$ and $E(D, H) = 0$. Thus, H would be an ESS if $V > C$. However, if $V < C$, neither H nor D would be regarded as an ESS. ESS has been defined for mixed strategies as well.

The effect of network topology on the evolutionary stability has hardly been studied. In Chapter 4 of this thesis, we study the topological effect on the evolutionarily stability of strategies, focusing on pure strategy networked games.

2.8.2 Replicator dynamics

The replicator dynamics model[109] provides a convenient mathematical framework for evaluating the evolution of a particular strategy in a population of players. Consider

	Hawk	Dove
Hawk	½(V-C), ½(V-C)	V, 0
Dove	0,0	V/2, V/2

Figure 2.13: Payoff matrix of the Hawk-Dove game

a population of n types where $x_i(t)$ denotes the frequency of type i. The state of the population at time t would be given by the vector $x(t) = x_1(t), ..., x_n(t)$.

Suppose the individuals in a population meet randomly and then engage in a symmetric game with payoff matrix A. Then $(Ax)_i$ is the expected payoff for an individual of type i and $x^T Ax$ is the average payoff in the population of state x.

Thus, the evolution of x over time would be described by the replicator equation;

$$\dot{x}_i = x_i[(Ax)_i + x^T Ax] \tag{2.16}$$

The replicator equation describes a selection process where more successful strategies would tend to spread in the population. We utilise replicator dynamics as a theoretical benchmark to test for the evolutionarily stability of strategies simulated on networked players.

2.8.3 Networked Games

Social networks are important in many facets of strategic decision making. The social networks affect most decisions that people make, such as whom to vote for and which products to buy, as such decisions are influenced by the choices of their friends and acquaintances. Games on networks, where an agent chooses an action and then the payoffs of each player is determined by those of his or her neighbours is a special perspective that may be used to model such collective decision making scenarios. There may be also other applications that involve strategic decision making and networks of relationships, such as exchange or trade on networks (networked markets). These sorts of analyses tend to be much more particular in their structure. This structure may be represented by a specific bargaining protocol or timing on interactions.

While most of the network game interactions are modelled as the repetitions of two-player interactions, there have been numerous attempts to model the games in a more integrated manner with the underlying network. For instance, numerous attempts have been made to define the payoffs of a network as a function of the network topology. For instance, Galeotti et al.[85] modelled the public goods game in such a manner where the game parameters and roles were derived from the network topology.

Initial work done on games on populations were done among well-mixed populations where all players are connected to each other in a fully connected network. However, in the real-world populations, the players are not connected in a well-mixed network. Instead, they tend to arrange themselves in spatially distributed topologies. These spatial topologies may adhere to different spatial structures such as the scale-free or small-world structures. The spatial restrictions of networks are often instrumental in determining the evolutionary stability of a strategy. For instance, it has been shown that coordination is evolutionarily unstable in a well mixed population though it may be evolutionarily stable in a scale-free population[224].

There are mainly two approaches taken in studying games on networks. One is to look take a top down approach and try to calculate the nash equilibrium of a game on a network. For instance, Bramoullè and Kraton have attempted calculating the Nash equilibria of the Public Goods game in networked players[40]. This is not a trivial task as the advent of network structure would mean that there could be multiple equilibria present, even in the simplest form of the games and network structures. Moreover, the complexity of equilibria would continue to grow as the network grows in size. However, this is still a major avenue of research taken up by the game theory community who are interested in networked populations of players.

The other approach is to take a bottom-up approach and observe how the micro level interactions among players would facilitate the emergence of topological or game theoretic features in the population. This is the approach mainly undertaken in the field of evolutionary game theory and that is the approach that is used in this work. Further, studies have been conducted on modelling the Public goods game in a topologically distributed environment where network topological features affect the contribution of each players[80, 96]. Public goods game[103] studies scenarios where individuals contribute an individual resource, such as money or network bandwidth, for a collective benefit. Modelling the individual contributions using network topological features would be beneficial in identifying the free riders and the variations of contributions in the Public goods game, in applications such as collective file sharing in the Internet. Since major part of this work is woven around the evolutionary stability and the bounded rationality of individual nodes, adopting a bottom-up approach in modelling and simulating networked games would be more appropriate in this work.

The primary reason for networks and social networks to be prominent in strategic decision making of humans is that their behaviour is sometimes emotional. Also, their decisions can sometimes be based on concepts of fairness and reciprocity. Humans are also bound by their reasoning capabilities. In order to study how agents behave in social and economic situations, numerous empirical studies have been conducted. This results in the sub-field of study known as Behavioural game theory. Behavioural game theory examines how humans behave in numerous game theoritic settings. However, most of these empirical studies are confined to relatively small number of persons, which makes the credibility of the outcomes of such research questionable. On the other hand, the usage of online social networks to gather the human behaviour data in game theoretic settings can be useful as they overcome this particular limitation posed by the logistics [136].

This concludes the background on game theoretic concepts used in this work. The next sub-section would explore some key concepts of information theory that are relevant in this study. The theoretical interpretation of information theory and information transfer is an integral part of this work, as it is suggested as a means of quantifying the distribution of bounded rationality of a network of players.

2.9 Information theory

Concepts of information theory[64] are used in this work to quantify to bounded rationality of agents in a game, from the perspective of information transfer. Information theory is the science of quantification of information. It is widely used in several applied fields of science and technology such electrical engineering, telecommunications engineering and computer science. Claude Shannon[228] was instrumental in developing information theory as a field of science, particularly relating to his work on signal processing. While the concept of information is too broad to be captured by a single definition, for probability distributions the concept of *entropy* has been proposed and been used as a measure of information. Entropy measures the uncertainty of a random variable. In the information theoretic context, this quantification of uncertainty is the measure of level of information contained in a random variable.

2.9.1 Entropy

The formal definition of entropy is as follows. Let X be a discrete random variable with alphabet A and the probability mass function $p(x) = Pr(X = x), x \epsilon A$. Then the entropy $H(X)$ of a discrete random variable X would be[63],

$$H(X) = -\sum_{x \epsilon X} p(x) log\ p(x) \tag{2.17}$$

If the log value of the above equation is 2, then the resulting entropy would be expressed in bits. Note that entropy is a function of the distribution of X. Thus, it does not depend on the actual values taken up by X, but only on the probabilities. Following are several extensions of entropy that are used to measure information in different dimensions.

2.9.2 Joint entropy and conditional entropy

Joint entropy is the definition of entropy extended to a pair of random variables. Thus, the *joint entropy* $H(X, Y)$ of a pair of discrete random variables (X, Y) with a joint distribution $p(x, y)$ can be defined as,

$$H(X) = -\sum_{x \epsilon X} \sum p(x)_{y \epsilon Y} p(x,y) logp(x,y) \tag{2.18}$$

This can also be expressed as,

$$H(X,Y) = -Elogp(X,Y) \tag{2.19}$$

Conditional entropy, on the other hand is defined as the expected value of the entropies of the conditional distributions, averaged over the conditioning random variable. That is, if $(X,Y) \sim p(x,y)$, then the conditional entropy $H(Y|X)$ is defined as,

$$H(Y|X) = \sum_{x \epsilon X} p(x) H(Y|X=x) = -E_{p(x,y)} log_p(Y|X) \tag{2.20}$$

Based on these definitions, it can be shown that the entropy of a pair of random variables is the entropy of one plus the conditional entropy of the other.

Relative Entropy and Mutual Information

Relative entropy is a measure of the distance between two probability distributions of two random variables. The relative entropy $D(p \parallel q)$ is a measure of the inefficiency of assuming that the distribution is q when the true distribution is p. The relative entropy is also known as the *Kullback-Leibler Divergence.* Formally put, the Relative entropy between two probability mass functions $p(x)$ and $q(x)$ is defined as,

$$D(p \parallel q) = \sum_{x \epsilon X} p(x) log \frac{p(x)}{q(x)} = E_p log \frac{p(X)}{q(X)} \tag{2.21}$$

Relative entropy is always non-negative and it is zero only if $p = q$. However, it does not give the true distance between probability distributions since it does not satisfy the triangle inequality. In this study, a variation of the KL-divergence is used as a metric that satisfies the triangle inequality.

Based on relative entropy, it is possible to measure the amount of information that one random variable contains about another random variable. This measure is called the *mutual information*, which is the reduction in the uncertainty of one random variable due to the knowledge of the other. Consider two random variables X and Y with a joint probability mass function $p(x, y)$ and marginal probability mass functions $p(x)$ and $p(y)$. Thus, the *mutual information* $I(X; Y)$ would be the relative entropy between the joint distribution and the product distribution $p(x)p(y)$;

$$I(X;Y) = \sum_{x \epsilon X} \sum y \epsilon Y p(x,y) log \frac{p(x,y)}{p(x)p(y)} \tag{2.22}$$

$$= E_p(x,y) log \frac{p(X,Y)}{p(X)p(Y)} \tag{2.23}$$

It should be noted that *mutual information* is not a directed measure meaning that it does not quantify the directed information flow from one random process to another.

Network entropy

While Entropy is a measure of the information content of a system, Solè and Valverde[239] have proposed a measure of information content of complex network, utilising the topological information and the node states of the network. This measure can be conveniently mapped to measure the information content of a network based game where the node states would be the states of the game or the strategies of the game. For instance, the 'co-ordinator' and 'defector' strategies can be considered to be states of a prisoner's dilemma game played over a network of players. This measure can be used to observe the evolution of information in a network at a given point in time. The information content $I(q^t)$ of a complex network at time t can be defined as,

$$I(q^t) = \sum_y \sum_z e^t_{y,z} log \frac{e^t_{y,z}}{q^t_y, q^t_z} \tag{2.24}$$

Here, $e^t_{y,z}$ is the proportion of links connecting the nodes with states y and z at time t. q^t_y is the proportion of links with a node of state y at one end and q^t_z is the proportion of links with a node in state z.

2.9.3 Local information dynamics

Information dynamics are typically used to characterise universal computational systems such as the Cellular automata (CA) model[143, 145]. These information dynamics can be further sub-divided into three main components, *information storage*, *information transfer* and *information modification.* In order to quantify these local information dynamics, measures such as the Active information and Transfer entropy have been suggested in the literature[212, 144]. In a distributed network, local information dynamics help to capture the temporal and spatial variations of information flow and storage. In this work, such local information measures are used to quantify the bounded rationality of spatially distributed players distributed in a network.

Information storage

Excess entropy encompasses all types of structure and memory by capturing correlations among all time lengths. In studying local information dynamics, the focus is on the amount of information that is actually *in use* at every local point in time and space. Information storage or memory of an autonomous component in the system tries to quantify the amount of information that it has accumulated over the past, that would be relevant to predicting its future. *Active information storage* is the information in storage that is actually relevant in predicting its next state. Hence, active information is directly comparable with information transfer in each computation. The formal definition of the active information storage of an agent X is the average mutual information between its semi-infinite past and its next state x_{n+1} at the timestep $n+1$.

$$A_x = \lim_{k \to \infty} \left\langle log_2 \frac{p(x_n^{(k)}, x_{n+1})}{p(x_n^{(k)}) p_{x_{n+1}})} \right\rangle \tag{2.25}$$

Here, k would be the finite history length. From a computational perspective, an agent can store information regardless of whether it is causally connected with itself. This is because information storage can be facilitated in a distributed fashion via one's neighbours.

Information transfer

The information transfer between a source and a destination is defined as the information provided by the source about the destination's next state this was not contained in the destination's own past. This measure is particularly important in modelling the bounded rationality of players as it can be used to quantify the directed information flow. It addresses the deficiencies of the mutual information measure, which is a static and directional measure. The information transfer is measured by transfer entropy[257]. By definition, transfer entropy is a directed measure that takes into account the directed information flow from a source to a destination. The transfer entropy from a source Y to destination X is defined has the average mutual information between the previous state of the source y_n and the next state of the destination x_{n+1}, conditioned on the semi-infinite past of the destination x_n.

$$T_{Y \to X} = \lim_{k \to \infty} \left\langle log_2 \frac{p(x_{n+1}|x_n^k, y_n)}{p(x_{n+1}|x_n^k)} \right\rangle \tag{2.26}$$

This formulation is also known as the *apparent transfer entropy.* Apparent transfer entropy measures the effect of information transfer from a single source only. If the information transfer is affected by the interaction of multiple sources, it does not account for that cumulative effect, which is addressed by a more refined version of transfer entropy known as the *complete transfer entropy.* For this work, we mainly focus on the apparent transfer entropy as an approximation of the information transfer between a single source and a destination. In Chapter 8, we utilise the directed and incoming information transfer measured as transfer entropy as an implicit measure of the bounded rationality of a sub-optimal player in a network.

Information modification

Information modification refers to the processing of information into a new form. More formally, it has been interpreted to mean interactions between transmitted and/or stored information which result in a modification of one or the other. While there isn't a widely accepted measure to quantify information modification, *Separable information* has been proposed as a potential measure of it. While information modification is crucial in distributed computation, this work doesn't consider the information modification aspect of players in a networked game, particularly with respect to modelling their bounded rationality.

This concludes the background section where the three theoretical cornerstones used in this work, game theory, network theory and information theory were discussed. These three components of theoretical basis are essential for the subsequent chapters where the research questions introduced in the Introduction are addressed in detail. The next chapter discusses the effect of network topology and information diffusion on networked games, using the coordination game as a basis for strategic interactions.

Chapter 3

The influence of topology on the evolution of coordination in complex networks under information diffusion constraints

3.1 Introduction

The Darwinian world view [30] suggests that it is the competition between others that drives the individuals in a population. However, there exists a dilemma in that coordination is abundant in real-world social structures. This work evaluates the heuristic that the structure or the topology of a population may be critical in determining the coordinating behaviour among strategic players. It is the first research question that we address under the broader question of how the network topology and information diffusion influence the network-based games. This chapter studies the effect of network topology under information diffusion constraints on the evolution of coordination in a population of players. We simulate the coordination game on four well-known classes of complex networks commonly used to model social systems: scale-free, small-world, random, hierarchical-modular, and the well-mixed model. In particular, focus is given on understanding the impact of information diffusion on coordination, and how this impact varies according to the topology of the social system. The simulation results demonstrate that while timelags and noise in the information about relative payoffs affect the emergence of coordination in all social systems, some topologies are markedly more resilient to these effects than others. The findings also show that, while non-coordination may be a better strategy in a society where people do not have information about the payoffs of others, coordination will quickly emerge as the better strategy when people get this information about others, even with noise and time lags. Societies with a small-world structure are most conducive to the emergence of coordination, despite limitations in information propagation, while societies with scale-free topologies are most sensitive to noise and time-lags in information diffusion. Surprisingly, in all topologies, it is not the highest connected people (hubs), but the slightly less connected people (provincial hubs) who first adopt coordination. Our findings confirm that the evolution of coordination in social systems depends heavily on the underlying social network structure[126, 127].

The rest of this chapter is organised as follows. The following section discusses the background for this work. Next section presents the justification of choosing particular classes of networks for simulating the coordination game. The following sections describes the methodology that was employed for the simulations. The results are presented with a discussion on their implications.

3.2 Background

Studying the behavioural or evolutionary dynamics of a population has played a central role in our understanding of emergent phenomena. Such studies have shed light on biological systems from cells [18] to species [237], on international politics [217] and even on the firing of populations of neurons as we try to understand the internal states of another person's mind [267]. Modern approaches in this field began with the work of Von Neumann and Morgenstern [253] in 1944 and were taken up by John Nash who developed one of the most influential ideas in game theory, that of the Nash equilibrium [174]. The Nash equilibrium states that in an incentivised situation between two or more strategic players of a 'game', at least one choice of strategy can be found whereby no other player can achieve a better outcome by unilaterally changing their strategy. This particular notion of equilibrium and its subsequent extensions have profoundly influenced our understanding of game theory as well as strategic interactions more broadly.

However, it quickly became apparent that the notion of a Nash equilibrium had a significant flaw: the cooperation that had been observed between players in behavioural experiments and we observe in everyday life was not an 'Evolutionary Stable Strategy' (ESS) [151], i.e. the theoretical results from applying Nash equilibrium did not reflect empirical observations. While successful methods to address this shortcoming have recently been explored using both finite and infinite population sizes [190], the disconnect between theory and observation was first addressed when the study of spatially extended games was introduced [186, 185, 52, 220]. In these games, interactions between agents were based on their spatial relationship rather than a well-mixed population. Further, interacting strategic agents played games in a two-dimensional space, typically on a lattice, and only the nearest neighbours on the lattice interacted with one another. This is a more physically realistic model than that of a well-mixed population where every agent can potentially interact with every other agent. In this scenario, it was shown that cooperation was a stable strategy and a portion of cooperating agents were able to persist indefinitely in the system, thereby lining up theory with observation.

These results were then extended to other topological spaces by using different network topologies, generalising the idea of a rigid lattice to that of stochastic connections between agents. In these models the relationships between individuals are not described by spatial connectedness but by more abstract connections such as the role of a species in a food web [193] or people connected via a social network [234]. Such generalisations have significantly broadened and deepened the phenomena that game theory has been able to explore.

Networked game theory has progressed significantly since the introduction of the so-called small-world and scale-free topologies into the more general field of network theory more than a decade ago [9, 194, 73, 135]. One of the more significant results to come from this work was a result by Ohtsuki et al. [187] where a general rule for when cooperation is favoured over non-cooperation for different network topologies was developed. Labelling benefits of cooperation as b and the costs of cooperation as c, and given the average $\bar{k}$ number of connections an agent has to other agents, cooperators are favoured when $b/c > \bar{k}$. This very general result holds for a number of broad classes of networks and works as a very good rule of thumb. However, as pointed out in Ohtsuki et al.'s original paper, this analysis is a poor estimate in the case of scale-free networks and further study is needed in order to understand what role is played by scale-free networks (and other well known network topologies) that leads to such special circumstances.

With this background in mind, this research compares the evolutionary game dynamics over four different network topologies commonly used to model social systems: the scale-free, small-world, hierarchical-modular, and Erdős-Rènyi random topologies [81]. We also consider regular lattices with homogeneous topology where relevant, since the well-mixed scenario is a special case of lattice structure, where average degree is network size minus one. The coordination game was adopted for the study (sometimes referred to as a stag-hunt game), with the aim of understanding the evolution of coordination in these topologies. A parameter α is introduced, which is used to vary the strength of strategy selection/introduces noise of a stochastic model that has also been used in other studies of strategic interaction [250].

Parameter variation has been utilised to emulate non-linear responses in evolutionary games [100, 265]. It has been suggested that global optimisation plays a vital role in comparison to local optimisation, especially in crises [99]. Interestingly, changing a parameter in an evolutionary game has been compared to selecting a personality type, and this in turn could lead to new equilibrium concepts [263]. These findings in evolutionary game theory and adaptive systems were employed to study the evolution of coordination in networked evolutionary games.

3.3 Studying games on networks

To analyse the evolution of coordination on realistic network topologies, the well known 'stag-hunt' game was used. It is variation of the coordination game([265, 233, 39]). Since it is a two-player game, it can be simulated pair-wise on neighbours sharing a link on a social network model. In this game, if two players coordinate, they both get the highest reward S (half a Stag), and if one-player does not coordinate while the other does, the non coordinator gets higher reward R (Rabbit), while the coordinator gets a lower reward T (Nothing). If both players do not coordinate, they both get reward R (Rabbit). The game is modelled such that $S > R > T$. The analogy is that half a stag would typically have more meat than a rabbit, and hunting a rabbit is better than going home empty handed.

In the classical stag-hunt game, coordination is needed only to hunt a stag, whereas rabbits can be hunted independently by each person or agent. As such, a coordinator (someone who uses a 'coordinating' strategy) is somebody who intends to hunt a stag, and a non-coordinator (sometimes called a defector, though this term is more appropriate to the prisoner's dilemma game) is someone who intends to hunt a rabbit, in a particular round. This game can be applied in strategic decision making situations where two players can gain higher payoff by coordinating rather than by defecting. The pure strategy Nash equilibria of this game occur when both players coordinate and both players defect. The mixed strategy scenario was not considered in this study.

Following is a brief description within the context of this analysis, on the five types of network models applied.

Scale-free networks: It has been recently shown that many real-world networks are scale-free networks, including technical, biological and social networks [194, 73, 24, 26, 27, 47, 166, 202, 203]. Particularly, many social networks are scale-free and heterogeneous, because there are always people who are more 'famous' and well-connected, while there are many who are relatively isolated. Scale-free networks display power-law degree distributions, described by $p_k = Ak^{-\gamma}U(k/k_{max})$ where U is a step function specifying a cut off at $k = k_{max}$. There are a number of growth models which generate scale-free networks, and prominent among them is the Barabàsi-Albert model [9], which utilises preferential attachment. Due to the prevalence of scale-free features in many online and offline social networks, scale-free networks are good models to study games on social systems, and often used for this purpose in recent literature [225]. Thus, scale-free networks make ideal candidates to study the evolution of coordination of populations of players.

Small-world networks: An equally justifiable model to study the evolution of coordination is the small-world network model. Small-world networks have low characteristic path lengths (compared to network diameter) and high clustering ([260, 139, 179]). The small-world effect on social systems was first and famously demonstrated by Milgram with a network of acquaintances [159] in United States, where he showed that the average number of hops required before a letter addressed to a random addressee within the country reached them was only six: thus the 'six-degrees of separation' [259]. It has since been shown that a range of real-world networks, including social networks, biological networks such as gene regulatory networks, metabolic networks, Protein-Protein Interaction networks, and signalling networks, as well as Internet show the small-world property [73, 221, 12]. Of course, many small-world networks can be scale-free to a certain degree, and vice-versa, but the scale-free and small-world characteristics need not (and often, do not) overlap.

Hierarchical-modular networks: Another category of networks coming into recent prominence is modular and hierarchical-modular networks. It has been recently observed that the hierarchical-modular structure of brain networks enhances the brain's robustness [256]. Similarly, many designed and evolved engineered systems are highly modular [108, 120]. More importantly, hierarchical-modular structure has been observed in human/social networks as well. For example, Ahn et al. [6] studied hierarchical organisation in several social networks. It is also evident that networks of people in the military/defense domain naturally exhibit hierarchical structure[254]. In the end, hierarchy is inherent in the social structure of human beings, coupled with modularity; therefore it makes sense to study how coordination games can be played in hierarchical-modular networks. Hence, this topology was selected as the third topology of interest.

Erdős-Rènyi random networks: Finally, the Erdős-Rènyi random topology was adopted to study the evolution of coordination [73]. Even though such random networks were once used extensively to model distributed systems, researchers have since realised that most real-world networks do not display degree distributions similar to random networks [9]. Yet, random networks are often used as null models to compare against other network models, and they are used for the same purpose in this work.

Well-mixed networks: Traditionally, game theory experiments were simulated on 'well-mixed' populations [225], where every agent was assumed to be connected to every other agent, since then the importance of topology was realised, spawning the research area of networked game theory. We therefore also test some of our results on well-mixed populations for comparison. A network which simulates a well-mixed population is a regular lattice, with average degree of $N - 1$ where N is the number of nodes. We found however, that this average degree has no bearing on the results, and any regular lattice yields qualitatively similar results in the experiments described in this chapter. For simplicity, therefore, we present the results obtained from a regular lattice of average degrees four, eight or twelve, so that it matches with the average degrees of other topologies.

The above mentioned scale-free, small-world, hierarchical-modular, random and lattice topologies were adopted to study the evolution of coordination in social systems.

3.4 Methodology

The study uses quantitative analysis of results generated from experimental simulations were used in this study. In order to simulate the coordination game played on a population of nodes, an ensemble of scale-free networks, small-world networks, hierarchical-modular networks, E-R random networks, and lattices were used. The scale-free networks were generated using a version of preferential attachment [8], varying the average degree of the networks, using the algorithm 1. The small-world networks were generated using the algorithm proposed in Watts and Strogatz [260] using a rewiring probability of $p = 0.5$ (unless otherwise specified), again varying the average degree of the networks. This algorithm was discussed in detailed in Chapter 2. The method described by Sarkar and Dong [226] was followed to produce hierarchical-modular networks, as described in algorithm 2. This method involves 'rewiring' each edge in a perfectly modular network to take away intra-community edges in each module with a rewiring probability p. By varying p, it is possible to obtain networks that have varying levels of hierarchy. The Erdős-Rènyi random networks were generated simply by randomly choosing M pairs among N nodes and connecting them. Generating lattices is trivial. Networks of size $N = 10^3$ were averaged over 100 networks for each parameter configuration. In evolution scenarios, we typically considered $T_e = 1500$ time-steps to be sufficient for the network to achieve steady state. This number was chosen based on the preliminary results.

The payoff matrix of the game was constructed in such a manner that the reward for both parties coordinating S would be a variable β, such that $\beta > 1$. When one node is coordinating and the other is not coordinating, the coordinator would not get any return ($T = 0$) while the non-coordinator would get a return of unity ($R = 1$). Thus, it was possible to manipulate the game environment by varying the single parameter β. Each pair of nodes connected by a link would engage in a single round of coordination game, after which the collective returns for each node p_i would be stored and used to adjust the accumulated payoff, P_i. The algorithm 3 describes the steps followed to simulate the evolution of strategies in a population of players.

At the beginning, players were randomly assigned as coordinators or non-coordinators. After each iteration, the players would adopt the role of the neighbours based on a certain probability. This probability was affected by the current accumulated payoff of each node. In the case of complete information diffusion suppose that two nodes x and y are connected and their current accumulated payoff values are P_x and P_y. These are the payoffs that are accumulated within each node after a certain number of time-steps. If x and y are different in their respective roles (that is, one is a coordinator and the other is a non-coordinator), the probability p that x would adopt the role of y is given by:

$$p = \max\left\{0, \frac{(P_y - P_x)}{k_{max}(R - T)\beta}\right\} \tag{3.1}$$

where k_{max} is the larger of the degree of x, k_x and the degree of y, k_y. This is a model commonly used in recent literature [225] to simulate evolutionary adaptation in a game scenario[1].

This model was modified to quantify information diffusion, by introducing a parameter α that signifies the level of information a node can gather about its neighbours. Therefore, a node may change strategies either (i) randomly (ii) based on information of its neighbours' payoffs. Therefore, the adaptation probability could be calculated by;

$$p = (1 - \alpha)\rho + \alpha \max\left\{0, \frac{(P_y - P_x)}{k_{max}(R - T)\beta}\right\} \tag{3.2}$$

[1]However, note that [225] uses different symbolism, with a payoff ordering where $T > R > S$, and their application is to a prisoner's dilemma game, thus the update rule that has been proposed may seem slightly different at first glance. However, simple analysis will reveal that the rules are essentially very similar.

where ρ is a uniformly distributed random number between zero and one, and represents white noise. Therefore, the higher the α, the more the ability of the system to distinguish real cumulative payoff information from noise.

Later in the chapter, analysis is done on the influence of time-lags on the information diffusion and the emergence of coordination. In order to do that, a time-lag parameter was introduced λ. The payoffs considered are those payoffs which each node had accumulated λ time-steps before the current time, P_x^λ and P_y^λ. As such, the diffusion equation becomes

$$p = (1-\alpha)\rho + \alpha \max\left\{0, \frac{(P_y^\lambda - P_x^\lambda)}{k_{max}(R-T)\beta}\right\} \tag{3.3}$$

Algorithm 1: Generating a scale-free network based on the Barabási-Albert Model[9].

```
Create the first node;
while Network size is less than the expected size do
    Create a new node n;
    forall the node n2 in the network do
        Calculate the probability p of node n joining the node n2 in a way
        proportional to the degree of n2 ;
        Connect node n with n2 according to the probability p;
    Add node n to the network;
```

Algorithm 2: Generating a Hierarchical Modular network.

```
Start with a perfectly modular network;
Randomly rewire the links according to probability p to generate a hierarchical
modular network;
```

Algorithm 3: Simulating the iterated coordination game in a network.

```
Randomly assign coordinators and defectors among the population;
Play the coordination game among the players of the population;
while Number of iterations is less than the expected number of iterations do
    Add the payoffs of the game to the cumulative payoff;
    Randomly select a node n;
    Randomly select a neighbour node n2 of node n;
    Based on the degrees and cumulative payoffs of nodes n and n2, calculate the
    probability p of adopting a neighbour's strategy;
    Generate a random floating point number num;
    if num < p then
        Adopt the neighbour's strategy;
```

3.5 Results

3.5.1 Pre-evolutionary balance

Initially, experiments were carried out to understand the balance between the payoffs of the coordinators and non-coordinators on average, when nodes do not adapt. In order to do that, single shot coordination game were simulated on scale-free, small-world, hierarchical-modular, random and lattice networks, without any evolution. In the case of scale-free networks, one-hundred networks (with a size $N = 1000$ and an average degree $\bar{k} = 4$ in all cases) were used, and the average payoffs for coordinators and non-coordinators were calculated for a range of β values. The results are shown in Figure 3.1. When the same process was undertaken for $T_e = 1500$ rounds, the results were identical: understandably, since, in the absence of evolution, number of rounds would not make any difference, and the results were averaged across networks.

From the results, it is possible to observe that, predictably, the relative payoff for coordinators steadily increases with β. Only when β >2, is the average payoff for coordinators higher than that of non-coordinators. Therefore, the total payoff for a pair of coordinators must be four times higher than the individual return of a non-coordinator (the stag must have four times more meat than the rabbit) for people in a scale-free network to decide to adopt the coordination strategy, in the absence of information about strategies adopted by others and their payoffs. If $\beta = 1.5$ for example, (the stag has three times more meat than the rabbit), it may intuitively seem better to coordinate, since half a stag still has more meat than a rabbit, yet due to the chance of the other node (person) not coordinating, this is not the case. Similar results were observed for the small-world, E-R random and hierarchical-modular networks (not shown), therefore it is also obvious that this result does not depend on the network topology. We also analysed regular lattices of various average degrees (starting from four up to $N - 1$ where N is network size), and found that this result (β >2 for coordinators to have higher relative payoff) holds also for well-mixed populations: which is not surprising, since this simply is a consequence of the relative proportion of coordinators and non-coordinators in the population (50 % each). As such, it is clear that in the absence of evolutionary adaptation, topology does not determine the relative payoffs of coordinators and non-coordinators (when the network concerned is sufficiently large to negate finite-scale effects).

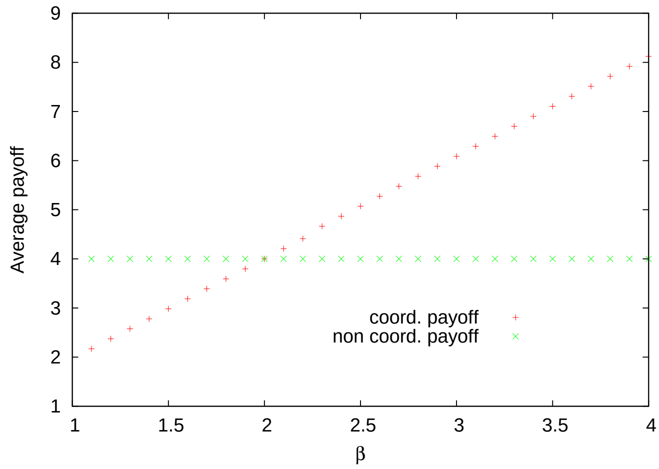


Figure 3.1: The average score of coordinators and non-coordinators in 100 scale-free networks after a single round of coordination game. Note that results for $T_e = 1500$ time-steps were identical (results do not vary with time). The nodes do not have a-priori knowledge about the strategy used by their neighbours. It can be seen that for coordination rewards $\beta \leq 2.0$, the average reward for non-coordinators is higher. Similar results were obtained for small-world, hierarchical-modular, random network, and regular lattice classes.

3.5.2 Evolution of coordination

Next, simulations were done to study the evolution of coordination with the assumption of complete information diffusion among players. Eq. 3.1 was used to simulate the evolution of strategies in all five classes of networks. Three different average degree values ($\bar{k} = 4, 8, 12$ respectively) were used, and utilised the average over 100 networks in each case. The populations were evolved for $T_e = 1500$ time-steps in each network, and the proportion of coordinators during and after the evolution was measured.

Figure 3.2 shows the evolution of the proportion of coordinators for networks of $\bar{k} = 4$, with $\beta = 1.8$ was used. It is observed that in all classes of networks, coordinators begin to dominate after a certain stage. Earlier it was observed that in pre-evolutionary balance, for a β less than two, it is advantageous to be a non-coordinator. However, it appears that in evolved systems where players are aware of the payoffs of their neighbours, it is advantageous to be a coordinator after a certain time frame. This is confirmed by Figure 3.3, which shows the average payoff of coordinators against time-step, for $\beta = 1.8$, and for all five classes of networks. In all cases, this average payoff initially increases with time, though it decreases in some cases once coordinators become a majority. For $\beta \geq 2.0$, it is advantageous from the beginning to be a coordinator, and these results confirm that this is not changed by the evolution of the system.

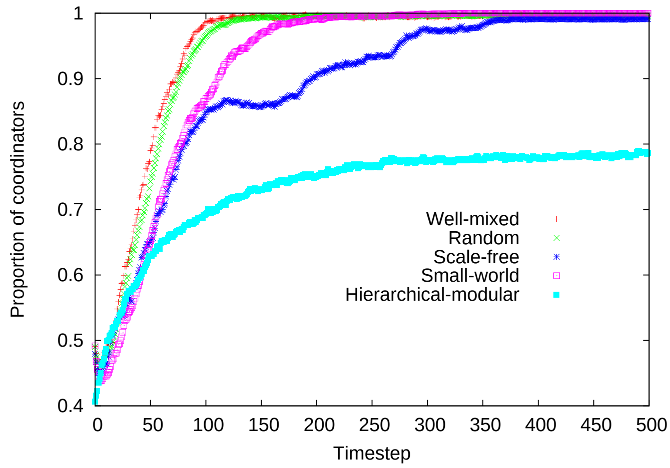


Figure 3.2: Proportion of coordinators against the time-step for the five different types of networks considered. The network size was $N = 1000$ nodes in all cases, and $T_e = 1500$ time-steps are considered (though only up to 500 are shown in figure, since the trend is clear after this point). $\beta = 1.8, \bar{k} = 4$.

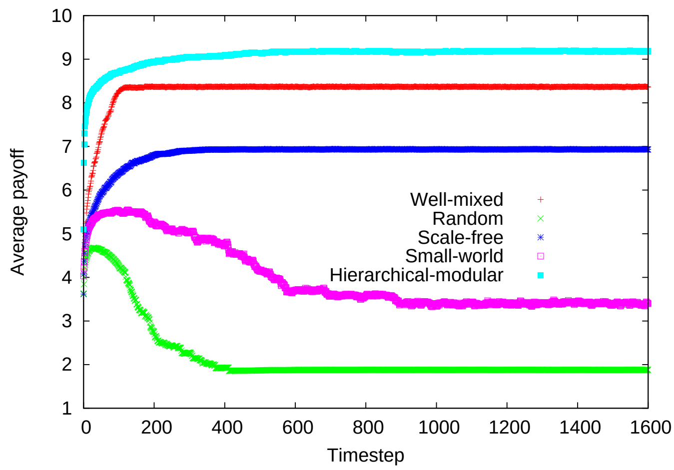


Figure 3.3: Average payoff of coordinators against the time-step for the five different types of networks considered. The network size was $N = 1000$ nodes in all cases, and $T_e = 1500$ time-steps are considered. $\beta = 1.8, \bar{k} = 4$.

Interestingly, from Figure 3.3 it is evident that while for scale-free and hierarchical-modular networks (as well as the well-mixed population), the average payoff of coordinators increases and stabilises with evolution, this is not the case with small-world and E-R random networks. With these networks, the average payoff for coordinators increases, then decreases and stabilises. Considered with Figure 3.2, it is clear that in these two classes of networks, the non-coordinators become extinct after a certain number of time-steps. Thus, in these networks, the lower average payoffs are derived at larger time-steps when

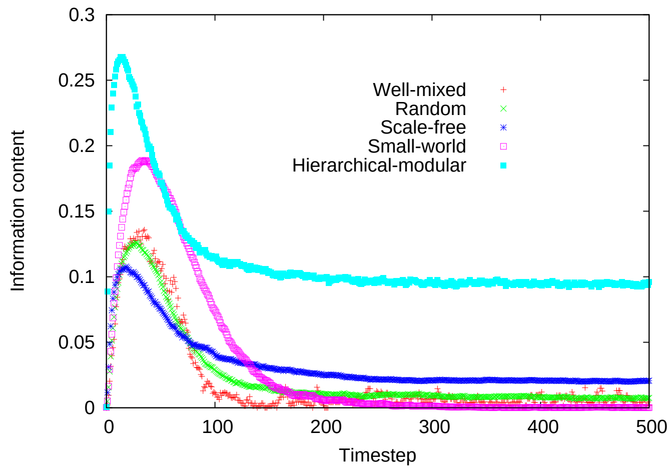


Figure 3.4: Information content in the network against the time-step for the five types of networks considered. The network size was $N = 1000$ nodes in all cases, and $T_e = 500$ time-steps are considered. $\beta = 1.8, \bar{k} = 4$.

all players within the network are coordinating.

It is important to note that there is not much difference in the time taken for the co-ordinators to dominate between the classes of networks that were studied. However, in the case of hierarchical-modular networks, non-coordinators are able to survive and hold a proportion of the network. In all other topologies, non-coordinators are 'wiped out' by evolution.

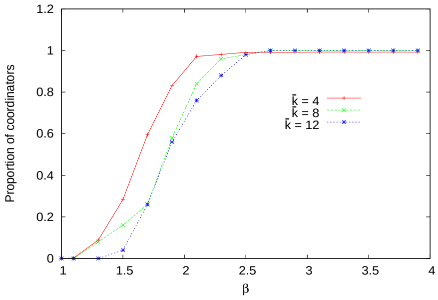
To complement this analysis, the information content of the node states is observed in each class of networks, and how they evolved as the simulation progressed over time. Shannon information is a generic measure of 'information content' in a system. In [240] an information content measure $I(q)$ was defined based on Shannon information for complex networks and on the remaining degree distribution of the network q_k, and [200] extended this definition so that node states are considered. The mutual information measure defined in [200] is,

$$I\left(q^t\right) = \sum_y \sum_z e^t_{y,z} \log \frac{e^t_{y,z}}{q^t_y q^t_z} \tag{3.4}$$

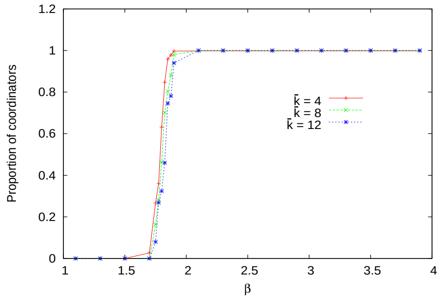
where $e^t_{y,z}$ is the proportion of links connecting, at time t, the nodes with states y, z respectively; q^t_y is the proportion of links, at time t, with a node (at one end) in the state y; and similarly, q^t_z is the proportion of links, at time t, with a node (at one end) in the state z. This measure was used to analyse how the mutual information in terms of node states (coordinator/non-coordinator) changes during simulation. The results are presented in Figure 3.4. Interestingly, the information content increases rapidly during the initial stages of evolution, and peaks at a point much earlier than when coordinators saturate the networks in the respective class. It appears that the time-step when the mutual information peaks is the time-step when the coordinators break - through, when they attain a critical number after which their eventual complete domination becomes inevitable. However, when coordinators completely dominate, the mutual information content is close to zero. This is not surprising since there is no 'information' left to be gained regarding the node status with respect to topology. Subsequently, it is evident that the steady state information content for hierarchical networks is greater than the other classes, since it was observed earlier that the domination of coordinators is not complete in this class and some non-coordinators manage not to convert.

Next, it is observed how the payoff parameter, β, influences the domination of coordinators. For this, let us consider each class of networks separately. In the case of scale-free networks, three different average degree values ($\bar{k} = 4, 8, 12$ respectively) were used. The populations were evolved over $T_e = 1500$ time-steps for each network, and the proportion of coordinators *after* the evolution was measured in each case. The obtained results against various β values are shown in Figure 3.5 a.

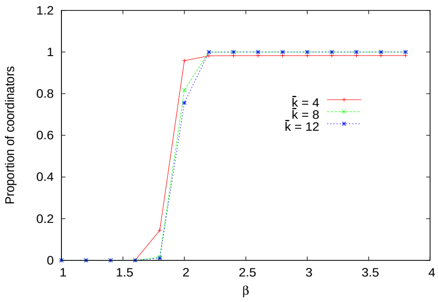
As evident from this figure, even though non-coordination is initially the better strategy, coordination emerges as the better strategy and is adopted by a majority of nodes after a period of time, for a range of β values less than 2.0. Moreover, there is a transition in terms of β, which occurs when $\beta \leq 2.0$ in most cases. For example, when $\bar{k} = 4.0$, it appears that coordination is a better strategy and is adopted by more nodes eventually, if $\beta \geq 1.6$. Therefore, it can be concluded that there is a range of β values (e.g. $2.0 \geq \beta \geq 1.6$ for networks with average degree 4.0), for which it is beneficial to adopt the non-coordination strategy if there is no information diffusion, however coordination is the evolutionarily winning strategy (and the evolutionarily stable strategy, ESS) if there is information diffusion about the cumulative payoff of the neighbours. However, if the relative payoff of coordination is sufficiently low (but higher than payoffs for non-coordination, e.g. $1.6 \geq \beta \geq 1.0$ for networks with average degree 4.0), it is evolutionarily better strategy to adopt non-coordination. It is also observed that for higher network density, it takes higher rewards of coordination for coordinators to become dominant.



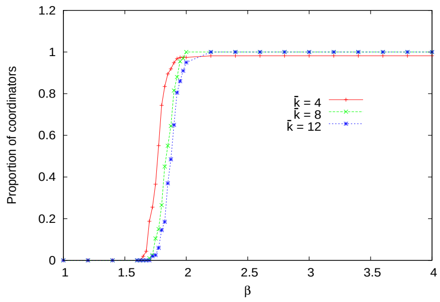
(a) Scale free



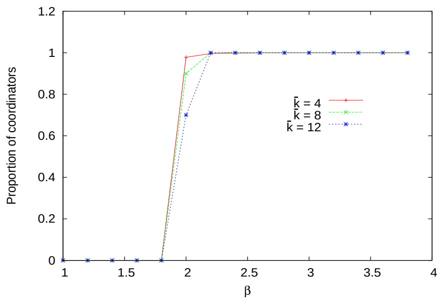
(b) Small world



(c) Hierarchical Modular



(d) Random



(e) Well mixed

Figure 3.5: Proportion of coordinators against the game parameter β for five classes of networks with different average degrees, after evolution. It can be seen that for each class of network there is a value β above which coordination dominates non-coordination. For low β values, non-coordinators dominate in all networks. The network size was $N = 1000$ nodes in all cases, and $T_e = 1500$ time-steps were used for evolution.

Similar analysis can be done in the case of small-world networks in order to ascertain the influence of payoff parameter β on the evolution of coordination. Small-world networks with size $N = 1000$ nodes and varying average degrees ($\bar{k} = 4, 8, 12$) were generated for this purpose. One hundred networks of populations were simulated in each case, for $T_e = 1500$ number of time-steps. Results that are similar to scale-free networks were found with small-world networks, as shown in Figure 3.5 b. That is, coordination emerges as the better strategy through evolution for higher values of β. However, there is one important difference. The phase transitions that were observed in terms of β are much sharper than scale-free networks. Therefore, a slight increase in the actual amount of payoff can very quickly change the evolutionary dynamics of coordination in small-world networks. The phase transition, for this particular set of parameters, seems to occur around $1.9 \geq \beta \geq 1.7$.

By comparing the results between various $\bar{k}$ values for both scale-free and small-world networks. It was also observed that the transition begins 'later' for higher average degrees, and coordinators begin to dominate only for higher values of β. For example, in the case of scale-free networks, when the average degree is 4.0, β is approximately 1.6 when the coordinators begin to dominate, while for average degree of 8.0, coordinators begin to dominate for $\beta = 1.8$ approximately. Even though the difference is small, it is possible to come to the important conclusion that when more links (relationships) are added to an existing network, increasing the link density and average degree, it takes higher relative rewards for coordination, for it to become the evolutionary dominant strategy. In a more densely connected society, the stag has to have relatively more meat for hunters to adopt coordination.

Let us now consider hierarchical-modular networks. As Figure 3.5 c shows, there is also a sharp transition. Does hierarchy in social systems favour the evolution of coordination? To answer this, further analysis was done using the hierarchical-modular networks generated by the method described in Sarkar and Dong [226]. Ensembles of 100 networks each were generated for various values of wiring parameter p, from 0.3 to 0.9. Sarkar and Dong explain that the higher this parameter, the higher the hierarchical nature of the networks. For each set of networks, the evolution of coordination was simulated as described before and measured the proportion of coordinators at the end of simulation (after $T_e = 1500$ time-steps) against the payoff parameter, β. Figure 3.6 shows our results. As the figure shows, hierarchy indeed aids the dominance of coordinators in a certain way. While coordinators do not dominate below a certain payoff parameter ($\beta = 1.6$), and the value of this cut-off is not influenced by the parameter p, the evolutionary behaviour for values higher than this β is influenced by the amount of hierarchy represented by p. For lower values of p, the coordinators do not dominate at all for any β. However, for higher values of p, the coordinators dominate and a phase transition is vaguely observable around $\beta = 1.6$. Therefore, it is possible to conclude that other topological features being similar, the presence of hierarchy encourages the emergence of coordinators (given sufficient relative payoff) in a social system.

Similarly, it may be possible to ask if small-worldness of small-world networks encourages or discourages the evolution of coordination. The small-world nature of a network is quantified by (i) relatively high clustering coefficient (ii) relatively low characteristic path lengths of a network [9, 73]. Therefore, it is possible to analyse how these two parameters influenced the evolution of coordination in small-world networks. Some typical results are shown in Figure 3.7. As shown in the figure, when clustering coefficient increases and network diameter decreases in a set of similar sized ($N = 1000, M = 2000$) small-world networks, again generated using the Watts- Strogatz algorithm, the phase transition in terms of relative coordinator payoff (β) becomes more pronounced. Thus, it can be argued that small-world nature encourages the rapid evolution of coordination, when the relative payoffs for coordinators increase.

Backing and intuitive explanations for these observations may be found in other recent studies, which looked at other games on graphs. For example, Masuda et al. [150] found that cooperative behaviour in spatial prisoner's dilemma (similar to coordination in stag-hunting) is optimised when the network is small-world. They arrived at this conclusion by comparing a range of graphs from regular lattices to random graphs, and the small-world characteristic was determined by the amount of randomness introduced into the lattice (as explained by Watts and Strogatz [260]). The topology was shown to be most conducive to cooperative behaviour when the randomness parameter was intermediate in value, which also maximises small-worldness. However, in a hawks and dove game, where coordination does not result in the highest payoff for an individual (the best scenario for an individual is to be the hawk themselves while the other player plays dove, whereas when both players playing hawk both players get a negative payoff), no such optimisation for coordination was observed in small-world networks [5]. The reason for these observations may be that, in games where mutual cooperation/coordination is not detrimental (in stag-hunting it results in best possible payoff, whereas in prisoner's dilemma it results in the second best possible payoff for an individual), the high clustering introduced by small-worldness helps sustain a group of coordinators /cooperators while the short-cuts available in topology help it spread. If the network is lattice-like, there are no short-cuts to further parts of the graph, hindering the spread of coordinators, while if it is totally random, the clustering is lost, making it harder for coordinators to sustain each other.

Similarly, in terms of modularity and hierarchy, it can be noted that in networks which are highly modular, most nodes belong to certain groups (modules), while there are other links which maintain the hierarchical structure. Recent work has shown that there is an optimal proportion of inter-group links, for which the spreading of cooperative behaviour for Prisoner's Dilemma is optimised [117]. Assuming a similar phenomena occurs in the case of the coordination game, it is possible that the increase in hierarchy as proposed by Sarkar and Dong [226] moves the fraction of intermodule links towards this optimal proportion, thus facilitating the spread of coordination. However, a detailed examination of this particular relationship is beyond the scope of this work.

Finally, the case with Erdős-Rènyi random networks is shown in Figure 3.5 d, and well-mixed populations in Figure 3.5 e. Here, too, it is possible to observe a sharp transition, around $1.9 \geq \beta \geq 1.7$. The average degree of network does not seem to influence much where this transition occurs. Therefore, we may argue that while all classes of networks display some degree of phase transition in terms of β, it is sharpest in small-world networks. This would imply that in a small-world network, with a scenario of increasing relative payoffs for coordinators, the decision to become coordinator has to be made quite swiftly in order not to lose out. However, in a society where social links are scale-free, it is possible to decide more slowly about becoming a coordinator(given that coordinator payoffs increase at a fixed temporal rate).

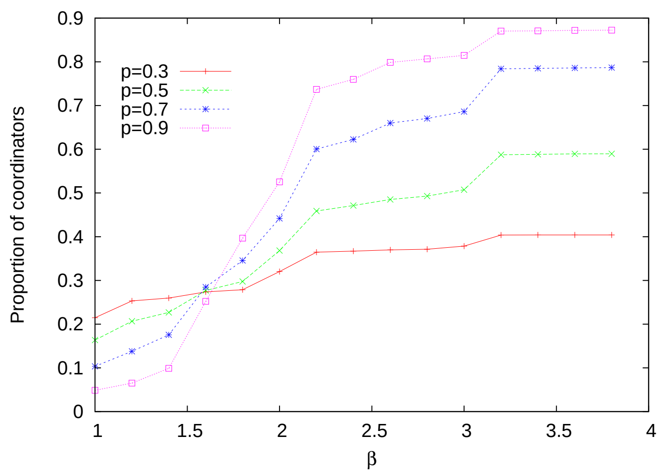


Figure 3.6: Proportion of coordinators against the game parameter β for hierarchical-modular networks with different rewiring p probabilities, after evolution. The network size was $N = 1000$ nodes in all cases, and $T_e = 1500$ time-steps were used for evolution.

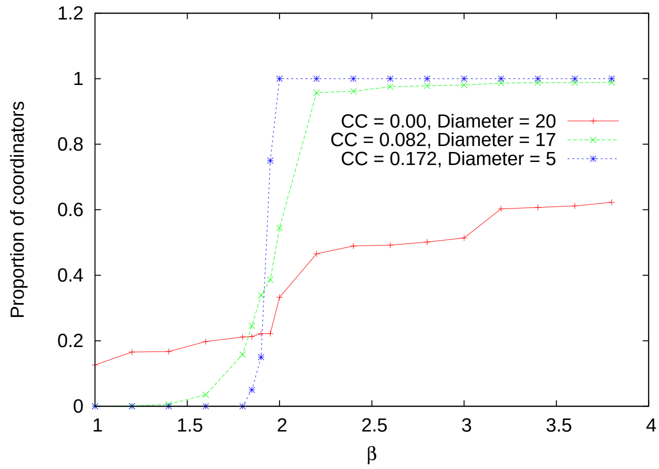
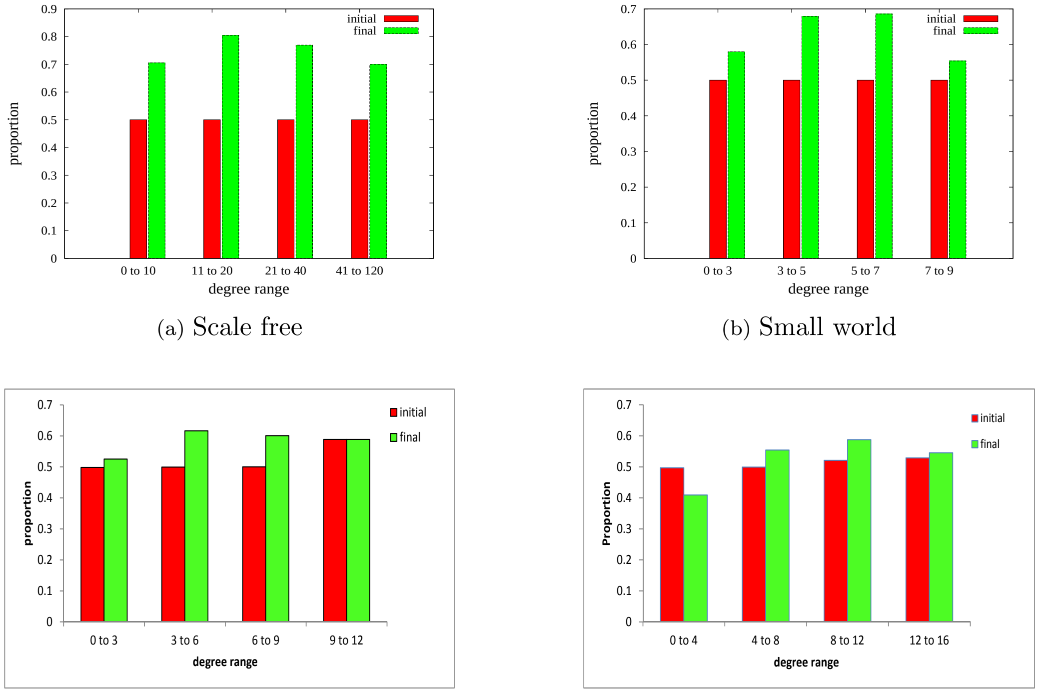


Figure 3.7: Proportion of coordinators against the game parameter β for small-world networks with different (a) network diameters (b) clustering coefficients after evolution. The network size was $N = 1000$ nodes in all cases, and $T_e = 1500$ time-steps were used for evolution.

3.5.3 Drivers of coordination and node degree

In order to gain an insight of how coordination ends up being the winning strategy, the degree distribution of coordinating and non-coordinating nodes during the process of evolution was observed. An example is shown in Figure 3.8 where the average of 100 networks with $\bar{k} = 4.0$, for a β of 2.1 are considered. Therefore, as shown in Figure 3.5, these are networks on which coordination dominates after T_e time-steps. An intermediate time-step was chosen deliberately, $T_i = 100$, with the view of understanding which degree range is first dominated by the coordinators. In case of scale-free networks, we observed that it is the provincial hubs, that first start to show a higher proportion of coordinators. It appears that main hubs resist the adaptation longer, and once they become predominantly coordinators, the evolution of coordination is almost complete. Some of the peripheral nodes also can remain non-coordinators for a long time. It is the provincial hubs that are the quickest to adapt.

In the case of small-world networks, the degree distribution of the network in terms of strategy at an intermediate time step ($T_i = 100$) was observed. The results, averaged over 100 networks of 1000 nodes each, are shown in Figure 3.8 b. Note that the small-world networks, by nature, have much smaller hubs than scale-free networks. Again, it is the provincial hubs that seem to be first adopting the coordination strategy completely. Similar results for hierarchical-modular and random networks were obtained, as shown in Figure 3.8 c and d. The regular lattices were not considered since node degrees are homogeneous in them and the analysis is unnecessary: that is, in all cases, it is the provincial hubs which are first fully converted into becoming coordinators.

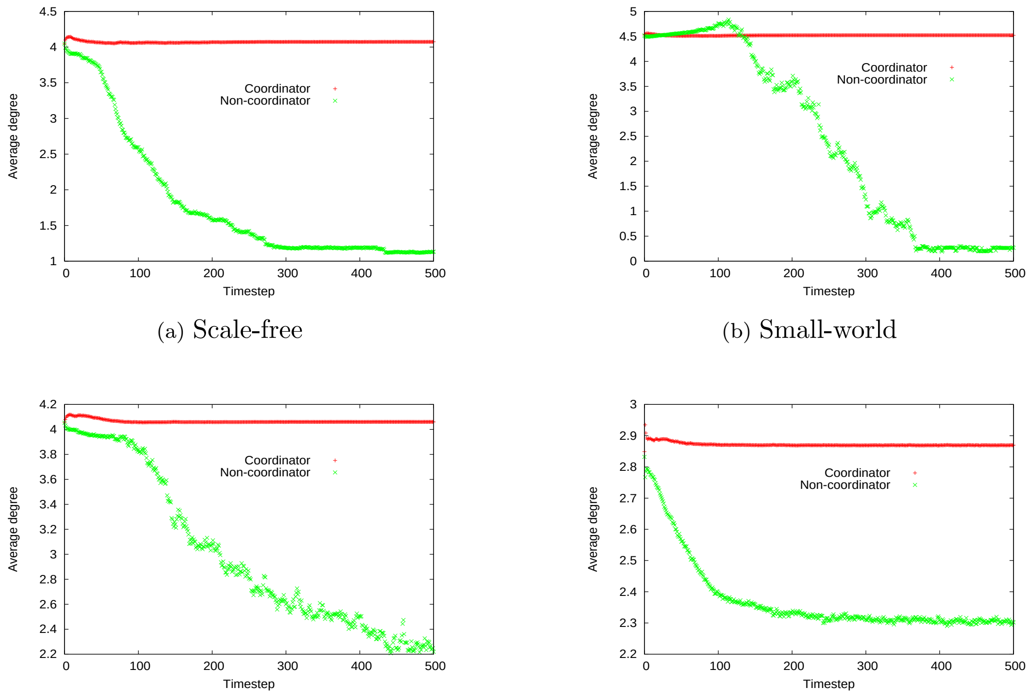


(a) Scale free (b) Small world

(c) Random (d) Hierarchical Modular

Figure 3.8: Evolution of coordinators by degree for four classes of networks (regular lattices are not considered since there is no variation of degree in them). The figure shows the fraction of coordinators for a number of degree intervals, averaged over 100 networks, at the beginning and at an intermediate time T_i evolution. Since strategies were initially randomly assigned, the proportion of coordinators is about half for each degree interval at the beginning. At the intermediate time however, the proportion of coordinators is much higher overall but highest among the provincial hubs. The network size was $N = 1000$ nodes in all cases, and $T_i = 100$ time-steps were used for evolution, $\bar{k} = 4$ and $\beta = 2.1$.

Now let us consider the question of which types of nodes convert first to coordination. To do this, the average degree of coordinators and non-coordinators throughout evolution for each class of networks (again averaged over 100 networks of $N = 1000$ each) were plotted in Figure 3.9. As seen in the figure, while the average degree for coordinators remain more or less the same, the average degree of non-coordinators declines steadily. This is not inconsistent with provincial hubs first adopting coordination, because it means that once provincial hubs (which have degrees higher than the network average) start becoming coordinators, the average degree of non-coordinators begins to decrease. However, there exists a special feature in terms of small-world networks. In this class, the average degree of non-coordinators briefly becomes higher than the average degree of coordinators before



(a) Scale-free

(b) Small-world

(c) Random

(d) Hierarchical-Modular

Figure 3.9: Average degrees of coordinators/non-coordinators against time-step. Four different network types are considered.

the average degree of non-coordinators decrease rapidly. This can only mean that in small-world networks, non-coordinators find refuge in main hubs (while provincial hubs are 'invaded' by coordinators) and resist the spread of coordinators for a while, before giving up and retreating to a few peripheral nodes. It is intriguing that players who play a larger number of games tend to adapt slower in small-world networks than in other topologies.

3.5.4 Influence of information diffusion

It is often unrealistic to expect that members of a community would know or can correctly predict, the strategies adopted by their neighbours or the payoffs they are receiving. Indeed, as observed earlier, a non-coordination strategy is transiently competitive and pays better dividends in the short term, so many members of the community may believe that it is not in their interests to share correct information about their strategies or their payoffs. If nodes only have partial information about payoffs of their neighbours, which is the evolutionarily competitive strategy in each of the network classes mentioned before? This is the primary question that is addressed in this section.

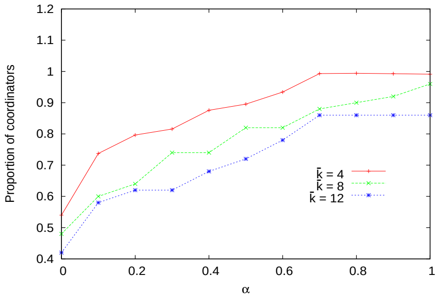
Eq. 3.2 could be used to introduce stochasticity in the adaptation. Nodes have 'noise' in the information which they have about the payoffs of their neighbours, and therefore have a level of randomness in changing decisions. The lower the parameter α, the higher the randomness. If $\alpha = 0$, no correct information diffuses about neighbours' payoffs and all decisions to change strategy are made randomly.

The evolution of the population under these conditions on the five classes of networks were simulated. Sale-free networks of size $N = 1000$ were used, and the evolution was done for $T_e = 1500$ time-steps, for various α values. A typical set of results (for $\beta = 2.3$) are shown in Figure 3.10 a. As shown previously, for this β, under complete information diffusion the coordinators will dominate after some time. Therefore, the proportion of coordinators were compared for a range of α values.

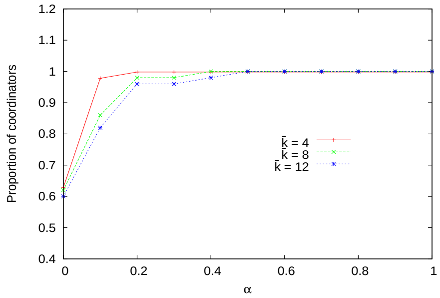
As expected, the proportion of coordinators increases with α. Further, there is a very small, but non-zero *alpha* value below which the non-coordinators dominate. Therefore, we can surmise that if the levels of information diffusion are sufficiently low then it is an evolutionarily winning strategy to be a non-coordinator. Similar experiments were conducted with other β values. While the starting points of plots vary with these β values (for smaller β, non-coordinators dominate for larger ranges of α), the results were qualitatively similar.

In the case of the small-world networks, it was found that the proportion of coordinators increased with information diffusion proportion α. However small-world networks were able to adopt coordination with smaller amounts of neighbour payoff information (smaller α), and the transition was sharper. Figure 3.10 b shows some of our results, again for $\beta = 2.3$, for hundred networks of size $N = 1000$ each after $T_e = 1500$ time-steps. It is found that for $\alpha \geq 0.2$, coordinators completely dominate. Therefore small-world networks seem even more resilient to noise in neighbour payoff information than scale-free networks.

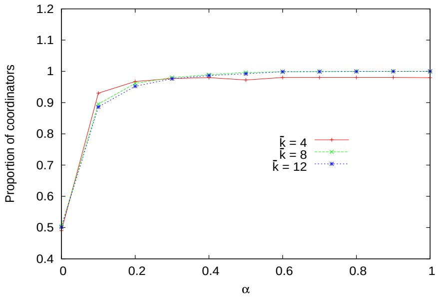
The experiments with E-R random networks, hierarchical-modular networks, and well-mixed lattices produce similar results, as shown in Figures 3.10 c, d and e.



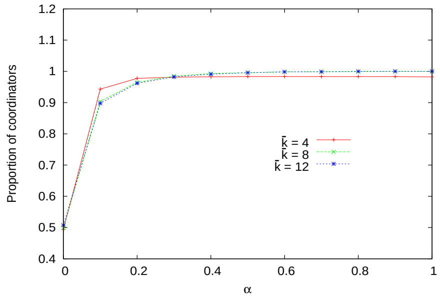
(a) Scale-free



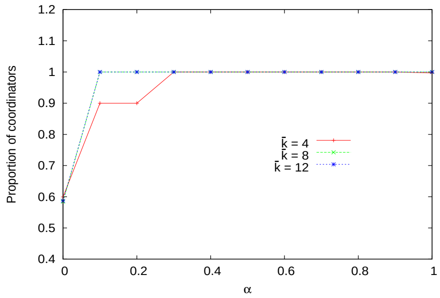
(b) Small-world



(c) Random



(d) Hierarchical-Modular



(e) Well-mixed

Figure 3.10: Influence of the information diffusion parameter, α, on the proportion of coordinators during evolution. The figure shows that the more information available about neighbour strategies, the higher the likelihood of coordinators dominating. If the levels of information available is relatively very low, non coordinators dominate even after evolution. The network size was $N = 1000$ nodes in all cases and $T_e = 1500$ time-steps were used for evolution, $\bar{k} = 4, 8, 12$ and $\beta = 2.3$.

3.5.5 Influence of timelag in information diffusion

In many real-world scenarios, information about the strategies adopted by other player, or the payoff received by them is not immediately available, since players tend to think that it is advantageous to hold such personal information secret. However, with the passage of time, the strategies adopted by players in the past and the payoffs received by them may become available, and this outdated information may partially help players to decide their current strategies. Even if players do not deliberately withhold information, it takes time for information to be transmitted and received. In other words, information about payoffs received by other players tends to have a time lag. In this section, the focus is on analysing how such a time lag influences the evolutionary patterns.

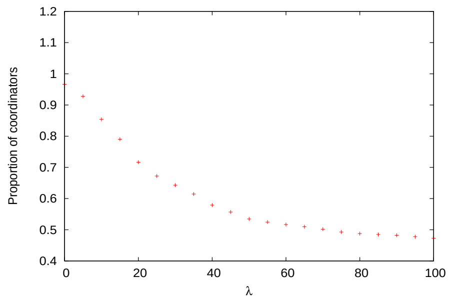
Eq. 3.2 was modified so that there is a lag in the payoffs known, giving us Eq. 3.3. For ease of reference, it is repeated here:

$$p = (1-\alpha)\rho + \alpha \max\left\{0, \frac{(P_y^\lambda - P_x^\lambda)}{k_{max}(R-T)\beta}\right\} \tag{3.5}$$

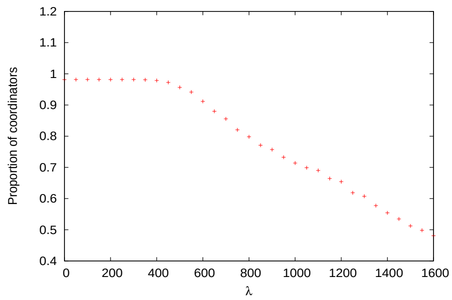
where P_y^λ, P_x^λ are the cumulative payoff values of players x and y, λ time-steps before the present time. Even though a particular player may know their current cumulative payoff, it might make more sense for cumulative payoffs from the same time-step to be compared, since the lag λ is used for both nodes. For $\alpha = 1.0$, our results for scale-free networks, small-world networks, hierarchical-modular networks, random networks and lattices are shown in Figure 3.11 a,b,c,d and e respectively. All these figures are for $\beta = 2.1$ and $\bar{k} = 4$. The results were obtained after 1500 time-steps.

As discussed previously, if there is no time lag, then coordinators dominate the network after a certain number of time-steps for this particular β. However, it is evident from the figures that if there is time lag, the dominance of coordinators is less pronounced. Beyond a certain amount of time lag, non coordinators become the dominant players. Importantly, the effect of time lag depends on the topology of the network. The evolutionary dynamics of scale-free networks change quickly, so that if the time lag is higher than 100 time-steps, then non-coordinators become the dominant players even after evolution. Whereas in the case of E-R random networks, hierarchical-modular networks and small-world networks, and indeed well-mixed populations, the time lag has to be much larger before the non-coordinators dominate evolved networks. The scale-free networks are the least resilient in coping with time lags in payoff information.

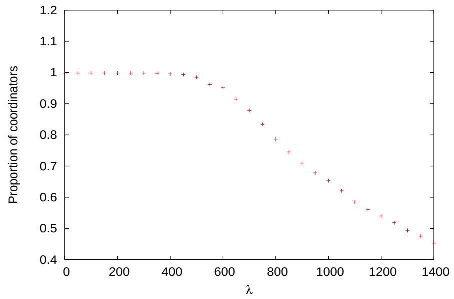
Since most real world networks are scale-free [9, 73, 198, 204, 201], it is a very important observation that scale-free networks are more sensitive to time-lags in payoff information than almost any other conceivable topology. The inherent heterogeneity of the degree distribution of scale-free networks could be a reason for this. This question could be further analysed by varying the amount of heterogeneity of synthesised scale-free networks



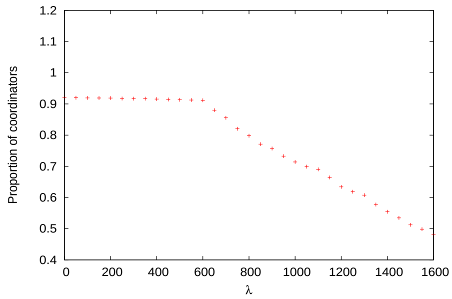
(a) Scale free



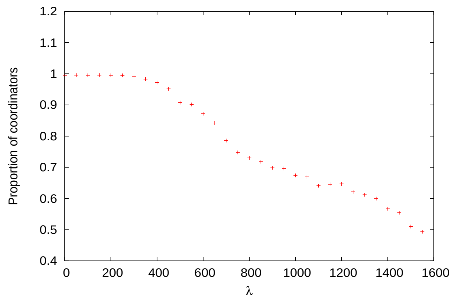
(b) Small world



(c) Random



(d) Hierarchical Modular



(e) Well mixed

Figure 3.11: Proportion of coordinators against the time delay parameter λ for five classes of networks after evolution. The network size was $N = 1000$ nodes in all cases, and $T_e = 1500$ time-steps were used for evolution. $\beta = 2.1, \bar{k} = 4$.

by, for example, changing the scale-free exponent γ, and analysing the value of threshold λ after which each network fails to evolve coordination. The $\lambda_{threshold}$ against γ plot would then give an indication about the relationship between time-lag sensitivity and scale-free nature. Such a detailed analysis, however, is beyond the scope of this preliminary study and subject to future research.

3.6 Discussion

In this chapter, the evolution of coordination in social systems was studied by simulating a coordination(stag-hunt) game on an ensemble of complex networks. A comparative study of topologies that are commonly found in social systems was conducted, by focusing on scale-free networks, small-world networks, hierarchical-modular networks and Erdős-Rènyi random networks. Well-mixed populations or lattices which approximate them were considered as a reference model. In all classes, it was observed that if nodes are unaware of the payoffs of their neighbours and cannot adapt, the relative payoff for coordination has to be quite high, for the average payoff of coordinators to be higher than the average payoff of non-coordinators. However, when nodes are aware of the payoffs of their neighbours are receiving and can evolutionarily adapt, coordination quickly emerges as the winning strategy, even for relatively lower levels of coordination payoff.

A number of general and topology-specific findings were observed. These can be summarised as follows:

General findings: (i) When there is no evolution, the relative coordinator payoff, β, has to be above two for coordinating nodes to have higher average payoff than non-coordinators. This result is independent of the topology of the system. However, after payoff information based evolution and adaptation, there emerges a range of β less than two for which coordinators are still a majority. (ii) In most topologies, the proportion of coordinators after sufficient evolution and adaptation goes through a phase transition when the relative coordinator payoff, β, is increased. (iii) It is the peripheral hubs, which first completely adopt coordination and drive the evolution of coordination. (iv) Noise and time-lags in payoff information adversely affect the evolution of coordination, though the level of this effect depends on topology.

Topology-specific findings: (i) The evolution of coordination is most pronounced, and the phase transition in terms of relative coordinator payoff, β, is sharpest, in small-world networks. However, the emergence of coordination after evolution is least rapid in scale-free networks. (ii) Scale-free networks are most sensitive to noise in payoff information, and the evolution of coordinators is most affected by such noise in them. Small-world networks are not so sensitive. (iii) Similarly, scale-free networks are most sensitive to time-lags in information about payoff. (iv) After the evolution of coordination, the average payoff of coordinators is higher than the initial stage in scale-free and hierarchical networks. On the other hand, it is lower than the payoff at initial stage for small-world and E-R random networks. Note however that the proportion of coordinators is higher than initial stage in all classes. (v) The 'amount' of hierarchy in hierarchical-modular networks, and the degree of 'small-worldness' in small-world networks, measured by appropriate parameters, both seem to aid the emergence of coordination.

In general, scale-free networks and small-world networks display contrasting characteristics in terms of the evolution of coordination. The hierarchical-modular class tends to display features similar to scale-free networks, while the E-R random networks display features similar to small-world networks. However, the overarching conclusion is that topological features, qualified here by the four classes of networks, influence the evolution of coordination in social systems in non-trivial ways.

There are several key implications from the results presented here. Both small-world and scale-free features are observed in real-world social systems up to various degrees. it was shown that while the emergence of coordination can be aided equally readily by both features, scale-freeness increases the sensitivity of the system to noise and time lags in information diffusion, while networks which are exclusively small-world are relatively unaffected by it. This would imply that systems that are small-world but *not* scale-free are likely to evolve into being dominant in coordination and sustain it under difficult information diffusion conditions. This contention is further validated by the fact that the 'small-worldness' itself, measured by the clustering coefficient and network diameter of the network, seems to aid the phase transition in terms of relative coordinator payoff. This has been corroborated by other studies in different game contexts. Experiments were conducted with several network densities (average degrees) in all classes of networks, and showed that the sparser the network is, the easier the emergence of coordination, other parameters being unchanged. Therefore, the smaller the number of games played within a network, the easier it seems for coordination to evolve as the winning strategy. These results are significant for understanding the behaviour of the spatially connected social system.

Finally, it would be useful to contextualise these results against some other recent advances made in networked game theory. A number of studies have looked at the evolution of cooperation (exemplified by cooperation in Prisoners Dilemma game), and how it is influenced by graph topology. For example, [225] considered a range of essentially scale-free networks and showed that the heterogeneity introduced by them helps in the spread of co-operation. The relationship between payoff aspirations and cooperation was highlighted by [52]. Other studies have looked at the role of assortativity in the emergence of coordination (for e.g., see [220]). The study by Jiang and Perc [117] highlighted, again within the context of Prisoners Dilemma, that there exists an optimal number of inter-modular links which aids the spread of cooperation between groups. Other studies have looked at the influence of topology in hawks and doves games [5]. However, most of these studies chose one or two arbitrary classes of network topologies, and did not consider the affects of information diffusion. Our contribution in this work lies in (a) comparing all well known classes of networks, as well as the well-mixed case, in a principled manner, and (b) explicitly studying the effects of information diffusion, in terms of noise as well as time-lags, in the evolution of coordination. This analysis focused on coordination rather than cooperation (i.e stag hunt rather than prisoner's dilemma), which is, while similar, less analysed in literature.

This study could be enhanced by looking at a broader set of parameters and network topologies. For example, only a limited range of average degrees were used, and mostly studied topological differences based on broad classifications rather than based on individual topological characteristics such as assortativity. The number of networks and initialisations used also could be increased. Despite these limitations, the results presented here are indicative of some general patterns with respect to different topologies and how coordination evolves in each of them.

While the simulations discussed in this chapter was done on relatively smaller networks, the results were consistent even when relatively large networks were used. In this chapter, we focused mainly on sparse networks and not dense networks, as social networks generally tend to be sparse in nature[162].

In this chapter, it was observed how the evolution of coordination is influenced by the topology of networks. While coordination is an important strategy in an evolving system of strategic players, studying the effect of network topology on the evolution of generic strategics is far more applicable in networked games. As such, the next chapter discusses how the evolutionary stability of strategies is influenced by the topology of complex networks.

Chapter 4

The influence of network topology on the evolutionary stability of strategies

Previous chapter focused on the effect of network topology on the evolution of coordination in network based games. While coordination is a significant strategy in evolving populations of strategic players, this chapter discusses the effect of topology on the evolution of generic strategies in evolutionary games.

4.1 Introduction

Evolutionary game theory is the branch of study that has resulted from the adoption of game theory into evolutionary biology [238]. It is used to study how a particular strategy or a group of strategies would evolve over time in a population of players. If a strategy is an evolutionarily stable strategy (ESS), once it is adopted by a population of players, any mutated strategy would not be able to invade it [237]. Evolutionary stability of a strategy could further be divided into two sub categories. strong ESS and weak ESS (also called asymptotic stable strategy and stable strategy [33]). If a strategy is in a weak evolutionarily stable state, the invading strategy does not completely die out but its population does not increase [33].

This chapter discusses how network topology affects the evolutionary stability of strategies. In order to do that, a class of strategies known as 'memory-one strategies' is used in prisoner's dilemma (PD) game to evaluate the effect of network topology on evolutionary stability. Iterated prisoner's dilemma game has widely been used to model the strategic decision making of self-interested opponents [224, 83, 4]. In memory-one strategies, each player would base his action on a probability derived based on the previous interaction with the same opponent. In this work, particular significance is placed on a sub-class of memory-one strategies known as zero-determinant(ZD) strategies, along with other well-known memory-one strategies. Zero-determinant strategies have been demonstrated to be extortionate strategies, meaning that they have the ability to unilaterally set the payoff of the opponent [210]. Intuitively, this would suggest that Zero-Determinant strategies have the potential to be evolutionarily stable against any competing strategy. However, Zero-determinant strategies have been shown to be evolutionarily unstable in a well-mixed population of players [4]. Extending from that basis, this chapter studies whether network topology affects the evolutionary stability of Zero-determinant strategies in a non-homogeneous network of players.

To test the effect of network topology, two well known network classes were used: scale-free networks [9] and well-mixed networks. Additionally, two different evolutionary processes were used to evolve the populations. First, the effect of topology on the evolutionary stability was tested using the death-birth Moran process [169], which is an evolutionary process used to model the evolution of players over time, particularly in biological systems. Then, a stochastic strategy adoption process that would update the strategy of a randomly selected node, by comparing it with a selected neighbour's strategy. The same process has been used as an evolutionary process by Santos et al. [223], with the pure-strategy PD game. For the rest of the chapter, this evolutionary process would be referred to as the 'strategy adoption process'. The evolutionary outcomes of these two processes when players are placed in both well-mixed networks and scale-free networks are compared. Based on the results of these observations, it is argued that the evolutionarily unstable strategies in a well-mixed population may survive and even dominate in a heterogeneous network of players. Further, it is shown that the topology of the interactions of the players, the evolutionary update process and the initial topological distribution of players are significant in determining the overall evolutionary stability of a strategy. When the players are distributed in a homogeneous network however, the evolutionary process used would not have a significant effect the evolutionary stability of a strategy. The effect of topology on the evolutionary stability is evaluated by varying network assortativity [181], which is a measure of the similarity of mixing of nodes in a network. Thus, it is possible to suggest that when the network becomes more heterogeneous, network topology would have a more significant effect on the evolutionary stability of strategies. We call this topologically influenced evolutionary stability of strategies as 'topological stability'[124, 130].

Understanding the topological effect on the evolutionary stability of a strategy would help us to make better predictions about the evolutionary stability of a strategy in a real-world environment. Even though a strategy may be theoretically stable or not, its actual evolutionary behaviour may depend on the topology of the interconnections in the population and the evolutionary update mechanism used. By studying these effects extensively, the modelling of evolutionary games may be improved, by increasing the accuracy of the predictions of the evolutionary stability of strategies.

The rest of the chapter is organised as follows. The next section provides a background on the theoretical aspects of evolutionary game theory and complex network science, within the scope of this work. Therein, the zero-determinant strategies in the iterated prisoner's dilemma (IPD) game is introduced. Further, the death-birth Moran process and its significance in determining the evolutionary stability of a strategy is compared with the strategy adoption process suggested by Santos et al. [223]. The following section describes the methodology applied in evaluating the evolutionary stability of strategies, under the two evolutionary update processes used. Next, the results obtained by simulating both the death-birth Moran process and the strategy adoption process in well-mixed and scale-free networks of players are presented. Further, the results obtained on how the variation of network heterogeneity, measured using the network assortativity, affects the evolutionary stability of strategies are outlined. The chapter concludes with a discussion on the results, presenting the conclusions.

4.2 Background

4.2.1 Evolutionary Game Theory

Evolutionary game theory is an outcome of the adaptation of game theory into the field of evolutionary biology [238, 237]. It studies how contending strategies evolve over time in a population of players. The equivalent concept to Nash Equilibrium in evolutionary game theory, is evolutionary stability [151]. If Nash Equilibrium can be considered as a static equilibrium, evolutionary stability represents a dynamic equilibrium of a strategy, over time. A strategy is called evolutionarily stable if it has the potential to dominate over any mutant strategy [238]. Evolutionary games are often modelled as iterative games where a population of players play the same game iteratively in a well-mixed or a spatially distributed environment. [140].

In iterated prisoner's dilemma(IPD), the prisoner's dilemma game is iterated over many time-steps, over a population of players [83]. Each player would play a single iteration of the game with its neighbours in each time-step. Iterated prisoner's dilemma game is widely used to model the autonomous decision making behaviour of self-interested players. It has been demonstrated that the topology of the network is significant in the evolution of cooperation of strategies in the IPD game [223]. For example, when the iterated prisoner's dilemma game is played among pure cooperation and pure defection strategies, cooperation evolves to be the dominant strategy in a population of players that are distributed in a scale-free topology.

4.2.2 Zero-determinant strategies

As opposed to pure strategies of cooperation and defection, mixed strategies of prisoner's dilemma game are based on the assumption that each player chooses a strategy based on a probability distribution. In fact, pure strategies can be regarded as a special case of mixed strategies where each strategy is chosen with the probability of one. Memory-one strategies [4, 137] are a special sub-class of mixed strategies, where the current mixed strategy of a game would depend on the immediate previous interaction between the two players in concern. Memory-one strategies are a specialisation of a more general class of strategies called finite-memory strategies [4, 137], where the current mixed strategy would be dependent on n number of historical states between the two players.

When considering the previous state between two players in a PD game, there could be four possible states. Namely CC, CD, DC and DD, where C represents cooperation and D represents defection, respectively. Memory-one strategies are represented by calculating the probabilities of cooperation by a player in the next move, given the type of the previous interaction of the player with the same opponent. For example, the strategy (1,1,1,1) would imply that the Player A would cooperate with player B, irrespective of the previous encounter between Player A and B. Thus, the pure strategy cooperation and defection can be thought of as a special case of memory-one or finite memory strategies. By varying the probabilities of cooperation under each of the previous encounters, it is possible to define any number of mixed strategies. Some of the well-known memory-one strategies include the Pavlov strategy (1,0,0,1) and the general cooperator (0.935, 0.229, 0.266, 0.42) strategy. General cooperator is the evolutionarily dominating strategy that evolved at low mutation rates as demonstrated by Iliopoulos et al. [112].

Zero-determinant strategies [210, 242] are a special sub-class of memory-one strategies that have recently gained much attention in the literature and media. ZD strategies denote a class of memory-one strategies that enable a player to unilaterally set the opponent's payoff. Due to this inherent property, ZD strategies have the ability to gain a higher expected payoff against an opposing strategy. However, for a strategy to be evolutionarily stable, it has to be stable against itself as well as the opponent strategies. It has been shown that ZD strategies do not perform well against itself. Due to this reason, ZD strategies have been demonstrated to be evolutionarily unstable [4], particularly against the Pavlov strategy.

ZD strategies are defined using a set of conditional probability equations [210]. Suppose p_1, p_2, p_3 and p_4 denote the set of probabilities that a player would cooperate given that the player's last interaction with the same opponent resulted in the outcomes CC (p_1), CD (p_2), DC (p_3) or DD (p_4). ZD strategies are defined by fixing p2 and p3 to be functions of p_1 and p_4, denoted by Eq. 4.1 and Eq. 4.2.

$$p_2 = \frac{p_1(T-P) - (1+p_4)(T-R)}{R-P} \tag{4.1}$$

$$p_3 = \frac{(1-p_1)(P-S) + p_4(R-S)}{R-P} \tag{4.2}$$

It was shown by Press and Dyson [210] that when playing against the ZD strategy, the expected utility of an opponent O can be defined using the probabilities p_1 and p_4, while p_2 and p_3 are defined as functions of p_1 and p_4. Eq. 4.3 gives the expected payoff of the opponent against the ZD strategy.

$$E(O, ZD) = \frac{(1-p_1)P + p_4 R)}{(1-p_1+p_4)} \tag{4.3}$$

Here, P and R represent the payoffs earned when both players defect and corporate, respectively.

Hence, ZD strategies allow a player to unilaterally set the opponent's payoff, effectively making them extortionate strategies. In the simulations performed here, the probabilities are set at p_1 and p_4 as 0.99 and 0.01 respectively, as in the study done by Adami and Hintze [4]. p_2 and p_3 are derived to be 0.97 and 0.02, using the ZD conditional probability equations Eq. 4.1 and Eq. 4.2.

4.3 Evolutionary processes

Two evolutionary processes are used in the evolution of populations. The first one is a well-known evolutionary process known as the death-birth Moran process. The second one is the stochastic strategy adoption process that was adopted from the work of Santos et al. [223].

4.3.1 The Death-birth Moran process [169]

As the name suggests, in the death-birth Moran process, a node is randomly selected for removal at each time-step. Its replacement node is then selected from its neighbours based on a probability proportional to the fitness of the neighbours. In the case of the iterated prisoner's dilemma game, the fitness is equivalent to the accumulated payoff of each node, averaged over its number of neighbours. Then, the selected neighbour is replicated to replace the node that is being removed. The new node would have zero payoff yet it will still have the same neighbours as the previous node that existed in the same topological space. This process is continued over n number of time-steps to evolve the entire population over time. The death-birth Moran process is commonly used to emulate the evolution of biological species where the strategies are hard-wired into the players. If the lifetime of a player is significantly less than the time-span of evolution, as with the case of biological evolution, the death-birth Moran process may effectively be used to simulate the evolution of strategies(players) over time.

4.3.2 Stochastic strategy adoption process

Since Moran process maybe be more applicable in the biological context where the players with hard-wired strategies get replaced, it does not take into account the individual payoff differences of the node being replaced and the replicating node. Thus, it maybe possible that the node being replaced would actually have a higher cumulative payoff (fitness) than the replicating node. On the other hand, in the social context, the time-span of evolution of strategies could be considerably less than the lifetime of a player. Hence, players would be more inclined to adopt the apparently successful strategy and survive without getting replaced from the population. In order to model this kind of social evolution, a stochastic strategy adoption process can be applied. Such a process has been used in Santos et al. [223] to demonstrate the evolution of cooperation in IPD games with pure strategies. This method is extended for mixed strategies in this work. When employing this particular strategy adoption process, a node going through evolution is not directly replaced. Instead, its strategy could be updated by comparing its cumulative payoff with that of a stochastically selected neighbour. As with the Moran process, in each time-step, a node is marked for update. A potential node to compare it with is selected from its neighbours, based on a probability proportional to the fitness (accumulated payoff) of the neighbours, then the probability of the marked node adopting the strategy of its selected neighbour is calculated using the following equation.

$$p = \max\{0,\ (P_y - P_x)/[k_> \ (E\,(ZD, Pav) - E\,(Pav, ZD)]\} \tag{4.4}$$

Where:
p - Probability that node X would adopt Y's strategy
P_x - Cumulative pay off of node X
P_y - Cumulative pay off of node Y
$k_>$ - Maximum degree of X's degree (k_x) and Y's degree (k_y)
$E\,(ZD, Pav) -$ The expected payoff of a ZD node against a Pavlov node
$E\,(Pav, ZD)-$ The expected payoff of a Pavlov node against a ZD node

As shown above, the population update probability depends on the payoff difference between the marked node and the selected neighbour node. The degree of those two nodes is also used to normalise the effect of degree differences. However, the cumulative payoff of the node with the higher degree would still be higher due to the fact that it will have more interactions with other players. Thus, this equation implicitly captures the network topology in calculating the adoption probability. Due to this, this particular strategy adoption process can be used to study the topological effect on the evolutionary stability of strategies.

4.4 Network analysis

Complex networks are self-organising networks that show non-trivial topological features [9]. Complex network analysis provides a network perspective in analysing complex systems. Different classes of complex networks have been defined to model real-world complex systems such as social and biological systems. This chapter mainly focuses on two such network classes: well-mixed networks and scale-free networks. These are widely discussed in the network analysis literature when evaluating the topological stability of strategies. Out of the network metrics discussed in the Background, *Assortativity* is used as a means of observing how the heterogeneity of mixing patterns affects the evolutionary stability of strategies.

4.5 Methodology

Initially, the experimental results obtained in the work by Adami and Hintze [4] were re-created, by mixing the Zero-determinant strategy with the Pavlov strategy in a well-mixed population. This further enabled the confirmation of the theoretical results presented in the same work using the replicator dynamics model, suggesting that the ZD strategy would be evolutionarily unstable against the Pavlov strategy. To do this, a population of 1000 nodes that are connected via a lattice where each node is connected to eight other random nodes were initialised. Initially, the two strategies were distributed in a random manner so that the ZD strategy would occupy different fractions of the population (0.6 and 0.4) in each simulation run. Then, using the death-birth Moran process, the population was updated over 150,000 time-steps to observe the evolution of the strategies.

Afterwards, the evolutionary process was changed to the stochastic strategy adoption process used by Santos et al. [223]. This tested whether it is the population update process or the network topology that affects the evolutionary stability of the strategies concerned.

Then, the well-mixed population was replaced with a non-homogeneous scale-free network with 1000 nodes. As with the case of the previous experiment, the ZD strategy and the Pavlov strategy were randomly assigned among the nodes. With the players spatially distributed in a scale-free network, both the death-birth Moran process and the strategy adoption process were applied separately to observe the effect that they have on the evolution of strategies.

Next, the initial distribution of the strategies were changed in such a manner that the ZD strategy occupied the majority of hubs. This was done by sorting the nodes according to their degree and assigning the top 60% of the nodes with the ZD strategy. In this configuration, the initial average degree of ZD nodes was measured to be 3.4 while the average degree of Pavlov nodes was 1.8. The evolution of strategies was observed under the death-birth Moran process as well as the strategy adoption process. The experiment was repeated with the Pavlov strategy occupying the majority of hubs.

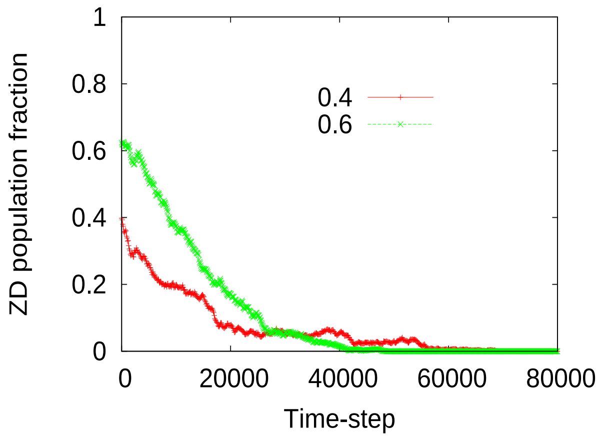
As the next step, the Pavlov strategy was mixed with the general cooperator strategy and the cooperator strategy in separate scale-free networks of players. The evolution of strategies was tested with a random initial distribution of strategies and a strategy distribution where the opponent strategy (GC or cooperator) was initially assigned mostly on hubs. This enabled us to determine whether any observed topological stability of ZD strategy is a unique and inherent property of the strategy itself or whether it is a more general behaviour that could occur with other strategies as well.

Finally, the evolution of the Pavlov and ZD strategies were observed in non-homogeneous networks while the network heterogeneity was gradually varied. To perform this test, a set of scale-free networks were generated with varying assortativity values by rewiring a scale-free network in a probabilistic manner. Then, both strategies were distributed randomly in each scale-free population in such a manner that the ZD strategy would occupy 60% of the nodes. Afterwards, the populations were allowed to evolve over 150,000 time-steps under the strategy adoption process and the remaining population fractions of ZD players were recorded. The results were averaged over 40 independent runs.

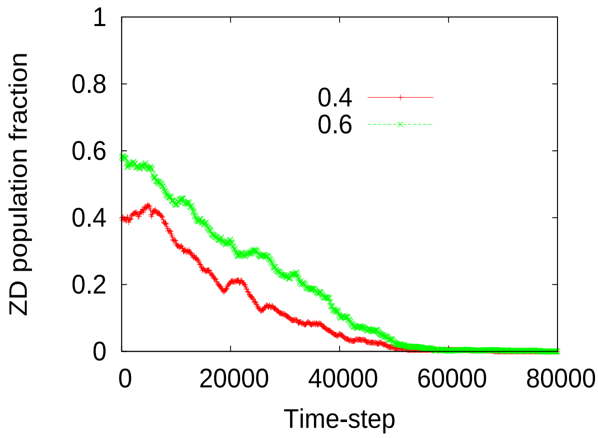
4.6 Results

In certain figures in this section, the number of time-steps shown is limited, when the strategy in concern becomes extinct reasonably quickly. Fig.4.1 shows the evolution of the fraction of ZD nodes when the ZD strategy is mixed with the Pavlov strategy in well-mixed and scale-free populations. The evolutionary process used is the death-birth Moran process. As expected, ZD strategy gradually becomes extinct in a well-mixed population. This confirms that in a homogeneous network, ZD strategy is evolutionarily unstable against the Pavlov strategy that operates as a strong evolutionarily stable strategy, as suggested by Adami and Hintze [4]. Moreover, ZD does not survive even in a non-homogeneous population distributed in a scale-free network, when the same evolutionary process is applied.

Next, Fig.4.2 depicts the evolution of the ZD and Pavlov strategies in a scale-free non-homogeneous network of players, under different initial configurations. The figure shows the evolution of the ZD fraction when the strategies are initially distributed randomly as well as when more hubs are assigned with the ZD strategy initially. As the figure depicts, Pavlov clearly dominates and eradicates the ZD strategy, suggesting that regardless of the initial distribution of strategies, ZD cannot survive when the population is allowed to evolve under the death-birth Moran process.



(a) Well-mixed



(b) Scale-free

Figure 4.1: The evolution of ZD population fraction against the Pavlov strategy, in well-mixed and scale-free populations. The population is allowed to evolve under the death-birth Moran process with 0.1% replacement rate. The strategies are initially distributed randomly, with the fraction of ZD nodes being 0.4 and 0.6, respectively.

Fig.4.3 depicts the scenario where a well-mixed population of ZD and Pavlov strategies are allowed to interact with each other over time, according to the strategy adoption process instead of the Moran process. Here too, ZD is gradually eradicated from the population. However, when the same evolutionary process is applied in a scale-free population of players, ZD strategy manages to survive, as shown in Fig4.4[a]. As shown in the figure, Pavlov strategy shows weak evolutionary stability, failing to eradicate the ZD strategy completely, when the two strategies are initially assigned randomly. On the other hand, when the ZD is initially assigned to the majority of hubs as depicted in Fig.4.4[b], ZD manages to become the weak evolutionarily stable strategy over the Pavlov strategy, becoming the dominant strategy in the network. However, as the same figure shows, when

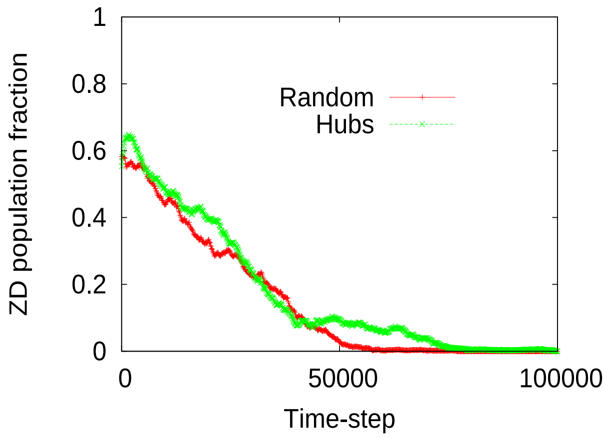


Figure 4.2: The evolution of ZD population fraction against the Pavlov strategy, in scale-free populations of varying initial configurations. The population is allowed to evolve under the death-birth Moran process with 0.1% replacement rate. In the two initial configurations, ZD strategy is either assigned randomly or assigned more on hubs (in hubs initialisation, the initial average degrees of ZD and Pavlov nodes are 3.4 and 1.8, respectively).

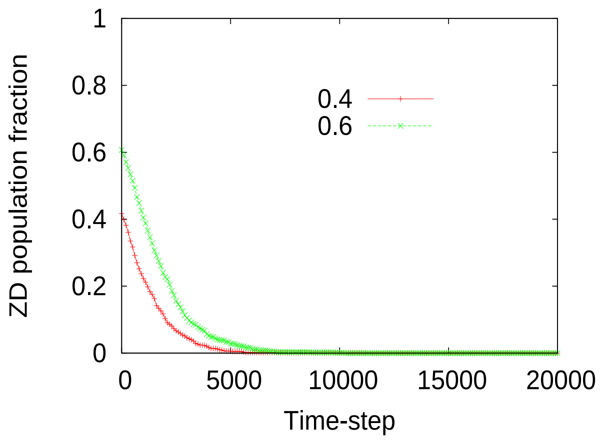
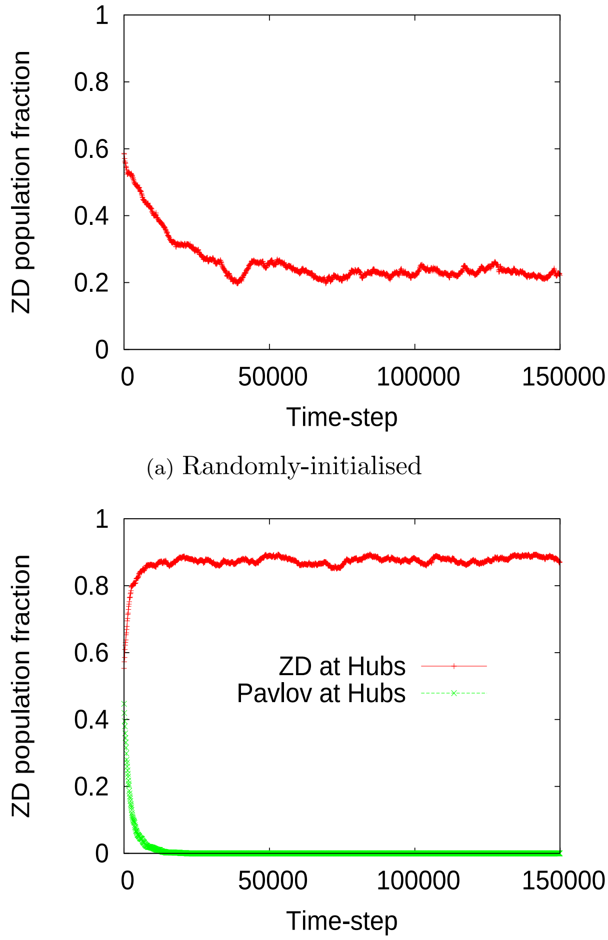


Figure 4.3: The evolution of ZD population fraction against the Pavlov strategy, in a well-mixed population. The population is allowed to evolve under the strategy adoption process. The strategies are initially distributed randomly, with the fraction of ZD nodes being 0.4 and 0.6, respectively.

the Pavlov strategy is initially assigned to the majority of hubs, it behaves as a strong evolutionarily stable strategy, wiping out the ZD population. This suggests that under the strategy adoption evolutionary process, the evolutionarily unstable ZD strategy may not only survive, but may even become the more prominent strategy in a non-homogeneous population of players.

The Pavlov strategy was then mixed with the GC and the cooperator strategies in a scale-free population. Fig.4.5 shows the evolution of the GC and cooperator strategy fractions



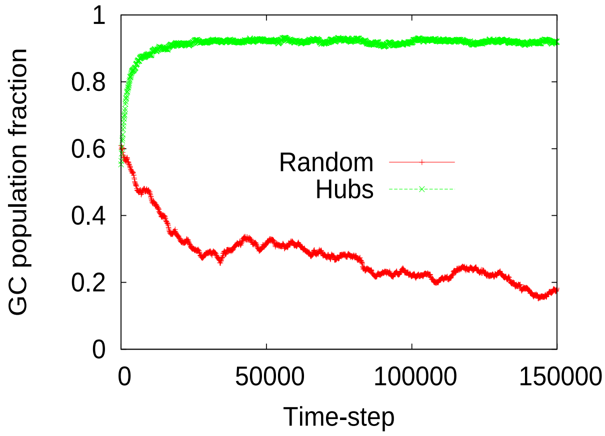
(a) Randomly-initialised

(b) Hubs-initialised

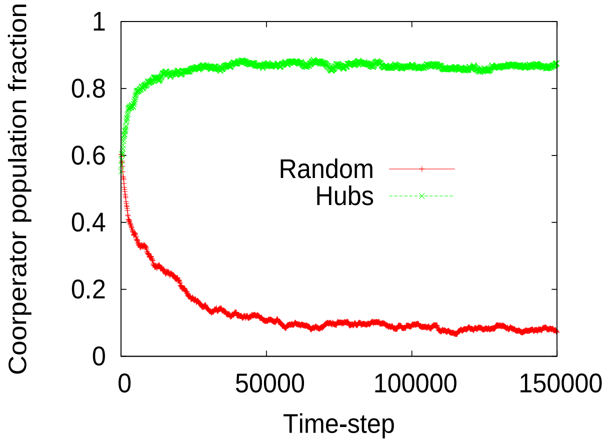
Figure 4.4: The evolution of ZD population fraction against the Pavlov strategy, in scale-free populations of varying initial configurations. The population is allowed to evolve under the strategy adoption process. (a) ZD and Pavlov strategies initially distributed randomly (b) Majority of hubs are either assigned with ZD or Pavlov (the initial average degrees of hub and non-hub strategies are 3.4 and 1.8, respectively).

over time, when those strategies are initially placed randomly or mostly on hubs. As the figures depict, GC and cooperator too may survive or dominate the population based on the initial distribution of the strategies, when the population is updated using the strategy adoption evolutionary process. This suggests that topological influence on the evolutionary stability is not limited to the ZD strategy, but may apply to other strategies as well.

Finally, the evolution of the ZD population against the Pavlov strategy was tested while the heterogeneity of the networks are gradually changed. Fig.4.6 shows the variation of the remaining ZD fraction after 150,000 time-steps under the strategy adoption process.



(a) GC-vs-Pavlov



(b) Cooperator-vs-Pavlov

Figure 4.5: The evolution of GC and cooperator strategies competing against the Pavlov strategy, in scale-free populations of varying initial configurations. The population is allowed to evolve under the strategy adoption process. In the two initial configurations, the competing strategy is either assigned randomly or assigned more on hubs (in hubs initialisation, the initial average degrees of competing nodes and Pavlov nodes are 3.4 and 1.8, respectively).

As the figure shows, there exits a negative correlation between the network assortativity and the remaining ZD fraction. The actual Pearson correlation value of the two series is -0.85, suggesting a strong negative correlation. Network assortativity is a measure of the similarity or the homogeneity of the mixing patterns of the nodes. Therefore, this result suggests that the effect of network topology on the evolutionary stability of a strategy increases as the network becomes more heterogeneous.

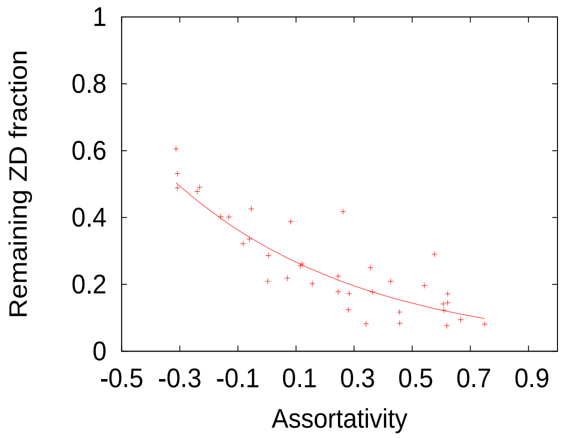


Figure 4.6: The evolution of ZD population fraction against the Pavlov strategy, in scale-free populations with varying network assortativity values. The population is allowed to evolve under the strategy adoption process. The results are averaged over 40 independent runs.

4.7 Discussion

This chapter attempted to evaluate how the network topology of a population of players affects the evolutionary stability of a strategy. In particular, focus in this work was given to a class of strategies known as zero-determinant strategies, which have been demonstrated to be evolutionarily unstable against the Pavlov strategy.

Based on the results gathered from the experiments conducted in this chapter, it is possible to argue that network topology has an effect on whether a particular strategy is evolutionarily stable or not. However, the topologically influenced evolutionary stability is a weak evolutionary stability and not a strong evolutionary stability. In other words, the stable strategy would not be able to completely eradicate the competing strategy and the competing strategy would be able to survive within the confines of the network.

Further, it was identified that the topological effect of evolutionary stability is determined by the evolutionary process used. When using the death-birth Moran process to evolve the population, topology does not seem have a significant effect on the evolutionary stability of strategies. However, when the strategy adoption process suggested by Santos et al. [223] is applied, topology does have significant effect on evolutionary stability of a strategy within a population. Strategy adoption process takes into account the cumulative payoff of each node in determining whether a strategy should be replaced or not. Therefore, this result suggests that an evolutionarily unstable strategy could survive when they occupy the hubs surrounded by leaf nodes assigned with the evolutionarily stable strategy. In a heterogeneous network of players, hubs tend to have more strategic interactions with their opponents compared to leaf nodes. Thus, a hub with an evolutionarily unstable strategy would continue to be irreplaceable by the neighbouring nodes' strategies, as it would continue to have a higher payoff than its immediate neighbours.

The significance of the evolutionary process may have implications in the real-world networks of strategic players. Moran process would be more appropriate in the biological context where the lifetime of a player is significantly less than the evolutionary time-span. It could be effectively used to model the evolution of species where the strategies are hard-wired to the players and the evolution happens through the replacement of players with the replicas of better performing players. However, in the social context, the evolution of strategies may be driven by the adoption of strategies by the players based on the performance of their neighbouring players. In other words, a stochastic strategy adoption process could be used to model the evolution of strategies when the lifetime of a player maybe considerably larger than the time-span of evolution. Examples of such situations involve the interactions that happen in corporate sector and financial markets. There, it is often observable that the players continually adopt the strategies of other players, in their struggle to survive. Thus, the strategy adoption evolutionary update process may be more relevant when the evolution of strategies is applied in the social context. Accordingly, topological effect on the evolutionary stability of strategies may be more prevalent in the social context, than the biological context.

Further, it is important to note that it is not only the topology, but also the initial distribution of the strategies within the network too plays a significant role in shaping the evolution of the strategies. For instance, when an evolutionarily unstable strategy occupies hubs as opposed to the leaf nodes at the initiation of the evolution, it even manages to become the more prominent strategy within the network over time, resembling a weak evolutionarily stable strategy.

Even though the main focus was on the ZD and Pavlov strategies in this work, it was possible to replicate similar observations with other well-known strategies such as the general cooperator and cooperator strategies, competing against the Pavlov strategy. This could mean that the variation of evolutionary stability due to topological stability of strategies is a more general phenomena that may be applicable to most strategies that are competing with each other.

The simulations discussed in this chapter was done on relatively smaller networks. However, the results were consistent even when relatively large networks were used. Further, we mainly focused on sparse networks and not dense networks in this work, as social networks generally tend to be sparse in nature[162].

In conclusion, it is possible to identify three key factors that determine the topological stability of strategies in a non-homogeneous network: network topology, evolutionary process and the initial distribution of the strategies. By varying these three factors, an evolutionarily unstable strategy may be able to survive and may even operate as a weak evolutionarily stable strategy, in a population of players connected in a non-homogeneous topology. Based on these observations, the topological stability of strategies may be more prevalent in the social context of the evolution of strategies in comparison to the biological context.

In this chapter, we studied the effect of network topology on the evolutionary stability of strategies was examined. While the evolution that is discussed here is based on the classic notion of individual fitness, the optimisation of the global value of an socio-economic system too is of particular importance, especially in a social context. Thus, in the next chapter, the placement of contending strategies is evolved in order to maximise the collective utility of a population of strategic players.

Chapter 5

The influence of network topology on the optimisation of public good in complex networks

In the previous chapter, we discussed the effect of network topology on the evolution of strategies. Accordingly, it is observed that the initial strategy placement is critical in the evolution of strategies. While the work on evolutionary stability focuses on individual fitness, in real-world socio-economic systems, improving the collective utility of a population is also of vital importance. Thus, this chapter focuses on the evolution of the placement of contending strategies in order to maximise the collective utility of a population of players.

5.1 Introduction

As the next step in observing how the network topology affects strategic interactions, this work examines how to optimise strategy placement in populations of players based on network topology. Game theory has long been used to model cognitive decision-making in societies. While traditional game theoretic modelling has focused on well-mixed populations, recent research has suggested that the topological structure of social networks plays an important part in the dynamic behaviour of social systems. Any agent or person playing a game employs a strategy (pure or mixed) to optimise pay-off. So far, we focused on how selfish agents can optimise their payoffs by choosing particular strategies within a social network model. In this chapter we pose the question that, if agents were to work towards the common goal of increasing the public good (that is, the total network utility), what strategies they should adapt within the context of a heterogeneous network. We consider a number of classical and recently demonstrated game theoretic strategies, including cooperation, defection, general cooperation, Pavlov, and zero-determinant strategies, and compare them pairwise. The iterative prisoner's dilemma game is simulated on scale-free networks, and use a genetic-algorithmic approach to investigate what optimal placement patterns evolve in terms of strategy. In particular, we ask the question that, given a pair of strategies are present in a network, which strategy should be adopted by the hubs (highly connected people), for the overall betterment of society (high network utility).

The social structures of people have often been modelled as complex networks. While the well-mixed or random models have been used earlier in this research to characterise social interactions, the heterogeneous nature of some interactions, whereby some individuals have more links than others, is nowadays taken into account. It has been found that most social networks are, in fact, the so-called scale-free networks, with power law degree distributions. As such, networked game theory has come into prominence, to analyse the payoff of individuals in such scenario. At the same time, public goods games have begun to be studied as a branch of games where the individual pay-offs for agents are less important than the overall payoff (utility) for the community. Not many studies have been done on networked public good games.

The evolutionary stability of a game refers to the ability of a particular strategy to dominate over any mutant strategy [151, 33]. There may be situations, however, where weaker strategies are allowed to sustain within a network due to the factors external to the game itself. A good real-world example of this is the welfare systems that are in place in many financial environments to safeguard the financially weaker individuals or organisations.

Normally, evolution within the context of game theory or networked game theory is taken to mean that individual agents adopt or evolve strategies with the view of maximising their individual payoff. This is indeed often the case; for example each deer in the forest adapts strategies to maximize its lifetime and food intake, and such strategies are passed on to the next generation, either by observation or as some kind of genetic memory. However, environmental pressures may also dictate collective evolution, whereby each individual tries to adapt the best strategy for the collective gain of the society, as opposed to its individual gain. For example, a herd of deers may be forced to evolve collective strategies to better survive against a pride of lions. The strategy adapted by each deer, then, is dictated not so much by its individual gain but the collective gain of the society. It is easy to find similar examples in the human society as well.

In this work, we observe how the evolutionarily stable and evolutionarily unstable strategies should be distributed within a network in order to maximize the cumulative payoff of the entire network. That is, how best to assign the strategies over the nodes of a network to maximize the cumulative payoff of all players. In order to do that, first we try to determine whether the spatial distribution of players have an effect on the cumulative payoff of the network. Next, we observe the variation of average degree of players with each strategy is observed to see which strategies tend to occupy the hubs and which occupy the peripheral nodes when the cumulative payoff increases. Since the ratios of different strategies and their distributions need to be kept fixed, we assume that there is no evolution of strategies among the players when a game is played iteratively. Instead, merely the initial configurations of players are varied to observe which configuration would provide the best overall utility of the network over time.

A genetic algorithm-based approach is used where a population of networks that are structurally identical but employ different placement of strategies, evolve to maximise the network utility. The evolving networks answer the question of how best to distribute strategies in order to maximise payoff for the society. The iterative prisoner's dilemma game is used as the game of choice. A number of well known strategies, including cooperation, defection, general cooperation, Pavlov, and the recently introduced zero-determinant strategies are employed. We compare strategies pairwise, and our particular goal is to identify which strategy must occupy the hubs (highly connected nodes) against the other for maximum network utility.

Potential applications of such optimisation may be found in organisational structures. Quite often, even the weaker strategies are allowed to survive due to the external environmental conditions (for example, welfare, legal or political pressures). Thus, the optimisation technique suggested in this work may help to determine the optimum distribution of strategies to maximise the overall utility of the network, while the strategies are not allowed to freely evolve and the ratios of players with each strategy remains fixed.

This chapter is organised as follows. The next section elaborates on the game theoretical and genetic algorithm based background used in this work. Then, we describe how genetic optimisation was used to optimise for the cumulative network payoff. Next, we present the results obtained, followed by the discussion and conclusion.

5.2 Background

Memory-one strategies [4, 137] are a special sub class of strategies in prisoner's dilemma games, where the current mixed strategy of a game would depend on the previous interaction between the two players in concern. In a mixed strategy scenario, there is a probability distribution that defines the potential strategies that could be adopted by a particular player against an opponent strategy. In memory-one strategies, this distribution is conditional to the immediate previous state of the two players in concern. In fact, Memory-one strategies are a specialisation of a more general class of strategies called finite-memory strategies[4, 137], where the current strategy is dependent on n number of historical states between the two players.

When considering the previous state between two players, in a prisoner's dilemma game, there could be four possible states: CC, CD, DC and DD, where C represents cooperation and D represents defection, respectively. Memory-one strategies are represented by stipulating the probabilities of cooperation by a player in the next move, given each type of interaction of the player with the same opponent. For example, a strategy (1,1,1,1) would imply that the Player A would cooperate with player B, regardless of the previous encounter between players A and B. Thus, the pure strategy cooperation and defection can be thought of as a special case of memory-one or finite memory strategies. By varying the probabilities of cooperation under each of the previous encounters, it is possible to define an infinite amount of mixed strategies. Well-known memory-one strategies include the Pavlov (1,0,0,1) and general cooperator (0.935, 0.229, 0.266, 0.42) strategies. Both these strategies are considered in this study.

Zero-determinant (ZD) strategies [210, 242] are a special sub class of memory-one strategies that has recently gained much attention in the literature. As the name suggests, ZD strategies denote a class of memory-one strategies that enable a player to unilaterally set the opponent's payoff. Due to this inherent property, ZD strategies have the ability to gain higher expected payoff against an opposing strategy. However, it has been shown that ZD strategies do not perform well against themselves. Due to this reason, ZD strategies have been demonstrated to be evolutionary unstable [4], particularly against the Pavlov strategy. In order to observe how the distribution of evolutionary stable and unstable strategies affect the overall utility optimisation in a network of players, the scenarios where ZD strategy is mixed with Pavlov and GC strategies were simulated.

5.2.1 Genetic algorithms

Genetic algorithms [92] are widely used as an optimisation technique. Genetic algorithms adopt the established concepts in biology to optimize a population of candidate solutions based on a particular fitness function. Each potential solution is identified as a genome. Recombination and mutation are the genetic operators that are used to evolve a population, until a certain boundary condition is met. In recombination, two most fit solutions in the population are selected for reproduction and they are randomly recombined to produce a new offspring solution. When each offspring is born, it would go through a mutation process with a relatively small probability of adding new genetic information to the population. When generating each population set, the weaker solutions are allowed to die out, keeping the overall population size fixed. In this context, genetic optimisation may be a suitable candidate for optimising the payoff of a network game as there isn't a deterministic and computationally efficient algorithm to perform that task.

5.3 Research Method

The genetic optimisation technique was used to test the hypothesis that the strategy distribution in a heterogeneous network affects the cumulative payoff of all nodes. Here, the assignment the strategies to players was evolved within the network to maximise the overall payoff. The algorithm 4 describes the steps followed to apply genetic optimisation to evolve the network to optimise for the cumulative payoff.

In a heterogeneous network of players, the players at hubs may play a larger number of games than the players at the peripheral nodes. Therefore, depending on the spatial distribution of strategies, the cumulative payoffs of a particular game would be different.

Thus, given a particular set of strategies and a spatial distribution, finding the optimum distribution of strategies over the network in order to maximise the total cumulative payoff could be regarded as an optimisation problem. Such optimisation may have applications where the overall benefit of a particular strategic decision making environment has to be maximised instead of maximising a particular player's payoff.

Initially, we wanted to test whether there's a correlation between the cumulative payoff and the initial distribution of strategies. To do that, we distributed the cooperator and the defector strategies were distributed in 100 different initial configurations, with the underlying network topology being a scale-free topology. The configurations were made to remain static without any evolution. For each configuration, we repeated the game over 1000 iterations and compared the accumulated payoff compared with the average degrees of nodes with each strategy.

When using the genetic optimisation, a scale-free network was chosen for observation. Scale-free networks make good candidates as heterogeneous networks as they are commonly observed in social networks. A genome is represented as a binary string to represent the collection of nodes, with 1 and 0s being used to denote the two strategies assigned to the nodes. Initially, n number of different initial distributions of players were placed randomly, ensuring that exactly 50% of the players follow each strategy. Afterwards, the game is played iteratively for t number of time-steps among the players within the network. In the prisoner's dilemma game, the parameter b was set to 1.8 while for the memory-one strategies, the variables were assigned to constants as T = 5, R = 3, P = 1 and S = 0. Note that the strategies of the nodes remain fixed during these iterations, thus the game is not simulated as an evolutionary game in the strict sense of the word. This is necessary as this work is interested in observing the effect of topological arrangement of strategies of players on the cumulative payoff of the network, for which the topological distribution of strategies should remain unchanged. The fitness function of the network game is the total cumulative payoff of all the players after the iterative game is played. In each generation of candidate player distributions, the fittest 10% of networks are chosen for recombination. Upon recombining, the positions of players are randomly mutated with a very small probability. Following each recombination and mutation, the strategies of players are adjusted to keep the ratio of two strategies the same. This is done to ensure that the changing ratios of players' strategies do not affect the cumulative payoffs and it is just the arrangement of the strategies that affect the cumulative payoff of the network. Following this process, it was observed that genetic optimisation of player positions does improve the overall payoff of the network. Hence, the cumulative payoff of a network of a players are affected by the spatial distribution of players with heterogeneous strategies. This also suggests that GA could be effectively used to identify the optimum distribution of players/strategies within a network.

Algorithm 4: Genetic optimisation to optimise for the cumulative payoff of the network

```
Start with an initial N number of networks of players with coordinators and
defectors;
forall the Network net in the Network collection N do
    forall the node n in Network net do
        Randomly assign a strategy
while The termination condition is not met do
    Play an iteration of the prisoner's dilemma game over the network;
    Assign payoffs to each node;
    Calculate the collective payoff of the network;
    Select the 10% of networks that have the highest collective payoffs;
    Randomly recombine the selected networks;
    After recombining, 0.05% of the networks are mutated% Replace the networks
    with least collective payoff with the new population;
```

5.4 Results

First, we simulated the classical prisoner's dilemma game for the optimisation of strategy placement. It has been shown that the cooperation strategy is the evolutionary stable strategy in a scale-free network. Fig. 5.1 depicts the variation of the cumulative payoff of players in a collection of randomly distributed strategy distributions in a scale-free topology. The strategy distributions are sorted based on their resulting cumulative payoff. Fig. 5.2 shows the variation of the average degrees of cooperators and defectors in the same set of networks. As shown in the figures, there exists a clear correlation between the cumulative payoffs of strategy distributions and the average degrees of each strategy. Fig. 5.3 shows the average cumulative payoffs of network populations do increase over time when the initial configuration of the strategies is optimised using a genetic algorithm, suggesting that it is the networks with cooperators occupying the hubs that generate higher cumulative payoffs. While the cumulative payoff of the network is increasing, we can observe that the average degree of the cooperators of network populations do increase over time, while the average defector degree decreases, as shown in Fig. 5.4. This suggests that in order to maximise the cumulative payoff of a network, the cooperators should be placed as hubs.

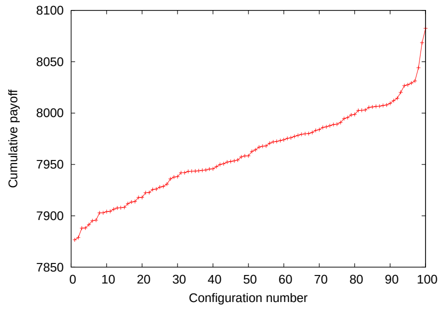


Figure 5.1: The variation of cumulative network payoff of a collection of random strategy distributions on a set of players playing the prisoner's dilemma game. The variable b was set to 1.8. The underlying network has a scale-free topology, consisting of 1000 nodes. The game was iterated for 10,000 time-steps. The strategy distributions are sorted based on the cumulative payoff.

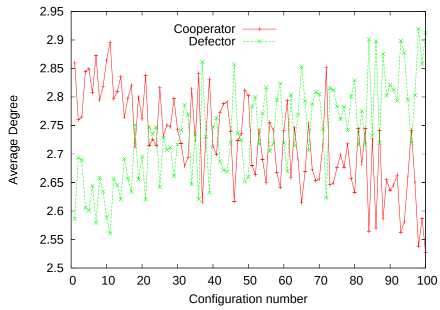


Figure 5.2: The variation of the average degrees of the cooperators and defectors of a collection of random strategy distributions on a set of players playing the prisoner's dilemma game. The variable b was set to 1.8. The underlying network has a scale-free topology, consisting of 1000 nodes. The game was iterated for 10,000 time-steps. The strategy distributions are sorted based on the cumulative payoff.

Next we performed a similar optimisation was performed on memory-one strategies of the prisoner's dilemma game. Memory-one strategies are a branch of strategies in prisoner's dilemma games where each the cooperation of each node depends on the previous move

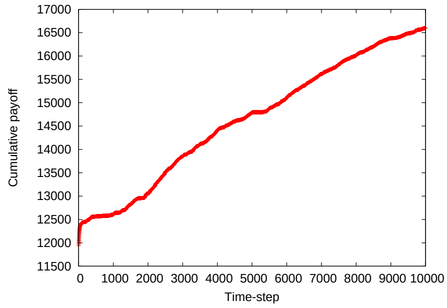


Figure 5.3: The variation of cumulative network payoff of the network of players playing the prisoner's dilemma game. The variable b was set to 1.8. The network is a scale-free network consisting of 1000 nodes. The game was iterated for 10,000 time-steps.

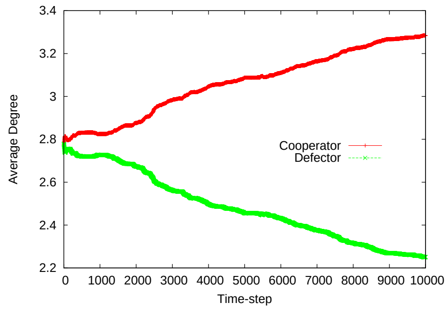
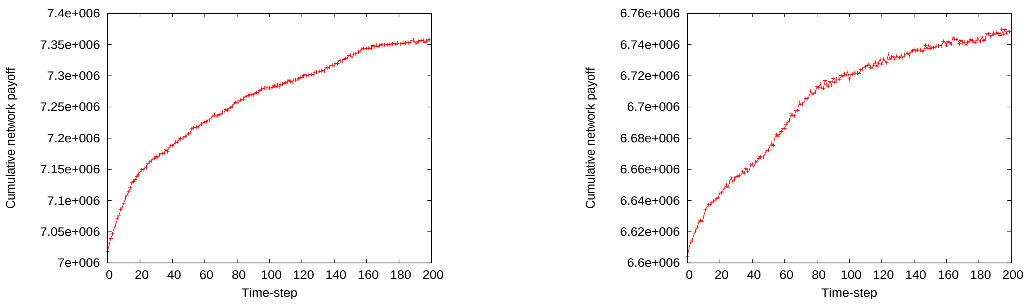


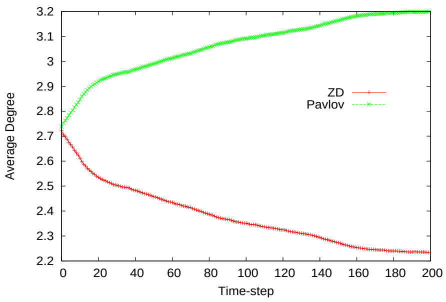
Figure 5.4: The variation of the average degrees of the cooperators and defectors of a network of players playing the prisoner's dilemma game. The variable b was set to 1.8. The network is a scale-free network consisting of 1000 nodes. The game was iterated for 10,000 time-steps.

of each node. By varying the probabilities of cooperation based on each of the previous combinations (CC, CD, DC, DD), it is possible to derive different strategies. Some of the well-known strategies include General Cooperator, Pavlov and zero-determinant strategies. ZD strategies have been recently shown to be evolutionary unstable against

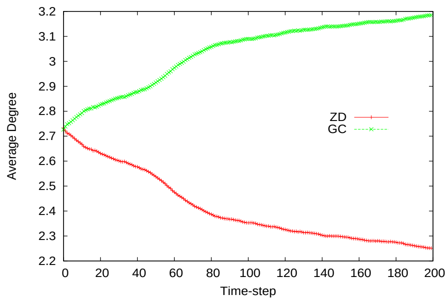


(a) $ZD - Pavlov$ (b) $ZD - GC$

Figure 5.5: The variation of cumulative network payoff of the network of players consisting of ZD-Pavlov and ZD-GC strategies. The network is a scale-free network consisting of 1000 nodes. The game was iterated for 200 time-steps.



(a) $ZD - Pavlov$



(b) $ZD - GC$

Figure 5.6: The variation of the average degrees of the network of players consisting of ZD-Pavlov and ZD-GC strategies. The network is a scale-free network consisting of 1000 nodes. The game was iterated for 200 time-steps.

Pavlov strategy. Thus, we use the Genetic optimisation technique is used to identify the optimum positioning of the ZD and Pavlov strategies. Fig. 5.5[a] depicts the increase of the cumulative payoff of the players when the networks are being evolved, suggesting that in memory-one strategies too, strategy placement does contribute to the optimisation of cumulative network payoff. Fig. 5.6[a] shows the evolution of player configuration using genetic optimisation. As the figure shows, there is an apparent increase in the average degree of the Pavlov strategy compared to the ZD strategy within the network population as the average cumulative payoffs of the networks are optimised. Even though the payoff of a ZD node would be higher against a Pavlov node, Pavlov performs well against itself compared to ZD strategy, making it evolutionary stable against ZD. This suggests that when Pavlov and ZD strategies are mixed in a population of players, the cumulative payoff of the entire network could be maximised by assigning the hubs with the Pavlov strategy.

Similarly, when the ZD strategy mixed with the general cooperator strategy, the GC strategy tended to occupy the hubs as the networks are evolved over time. As with the ZD versus Pavlov strategies, the cumulative payoffs of the networks continued to increase when the initial configuration of the strategies are changed, as shown in Fig. 5.5[b]. Fig. 5.6[b] shows the evolution of the average degree of the nodes occupying the two strategies over time.

Next, we mixed the Pavlov and general cooperator strategies were mixed in order to observe which strategy tends to occupy the hubs as the cumulative payoff of networks evolve as in Fig. 5.7. Again, it is when the Pavlov strategy is placed on the hubs that the cumulative payoff of the network tends to increase, as shown in Fig. 5.8.

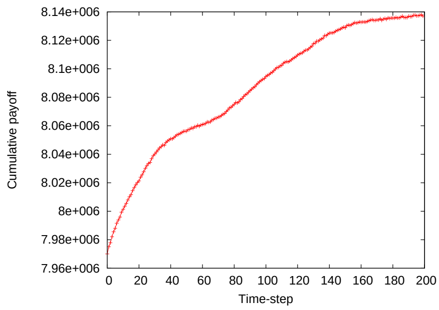


Figure 5.7: The variation of cumulative network payoff of the network of players consisting of GC and Pavlov strategies. The network is a scale-free network consisting of 1000 nodes. The game was iterated for 200 time steps.

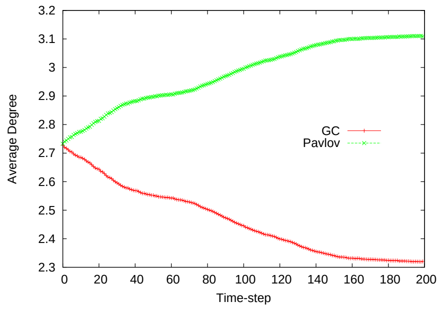


Figure 5.8: The variation of the average degrees of the network of players consisting of GC and Pavlov strategies. The network is a scale-free network consisting of 1000 nodes. The game was iterated for 200 time steps.

The observations made above can be summarised as follows:

Given a society which can be modelled as a scale-free network (bearing in mind the fact that most social networks have been proven to be scale-free) and considering a scenario where nodes in that social network can choose from one of two strategies and the overall balance of strategies across the network should be maintained such that at any given time, the number of agents playing either strategy must be the same, certain strategies evolutionarily win the competition against certain other strategies in occupying the hubs (highest connected nodes) in the network.

- Cooperation occupies the hubs against defection
- Pavlov occupies the hubs against general cooperation
- Pavlov occupies the hubs against zero-determinant strategies

- General cooperation occupies the hubs against zero-determinant strategies

These results are arrived at by comparing the average degrees of nodes implementing each strategy after evolution. Noting here that nodes evolve (switch strategies) not to maximise their own pay-off, but to maximise the cumulative network payoff. Thus, the environmental pressure is for maximisation of the public good.

These results are significant for the following reasons. The constraint of the network having to have an equal number of nodes implementing a pair of strategies might seem artificial at first. However, if we consider a scenario where nodes are merely place-holders for individuals who roam in the network, while the strategies for these individuals is actually fixed, it is conceivable that such a scenario may indeed occur in real world. Therefore, nodes do not actually change strategies, but swap individuals who themselves always use a certain strategy. Thus, all individuals 'coordinate' by swapping positions for the 'public good'. For example, consider a soccer team, which has 11 fixed positions (left extreme, right extreme, centre back, goal keeper etc). The positions can be thought of as a complex network (the goal keeper position is connected to the three backs, and so forth). There would be certain players who are better at offence and others who are better at defence. The coach could rotate players around the positions in order to maximize the 'public good', which, in this case, is to increase the net number of goals (goals scored by the side minus goals scored by the opposition). Similar scenarios can be described in the case of an army comprised of many strategic units advancing, or a business magnate placing his subordinates in various parts of his business empire to derive maximum benefit to his business. Therefore, understanding which classical strategies must be used by the hubs as opposed to peripheral nodes for maximum overall utility is of vital importance.

5.5 Discussion

Game theory can be successfully applied to understand the dynamics of a society. The concept of public goods games has recently gained prominence where the emphasis is not on the individual gains of agents but the overall payoff for the society. In this paper, a novel approach is taken by utilising the classical iterative prisoner's dilemma game as a public goods game. That is, agents play prisoner's dilemma repeatedly and adapt their strategies with the goal of increasing the total network utility. Evolution was simulated by implementing a version of genetic algorithm optimisation, where each member of the population is a network with a particular distribution of strategies. Thus, the evolution of networks (social structures), rather than evolution of individuals is considered.

It was found that for high network utility, networks evolve which prefer a certain type of strategy to be at their hub over another type[122]. As such, the evolved networks preferred cooperation over defection, general cooperation over zero-determinant, Pavlov over general cooperation, and Pavlov over zero determinant at their hubs. This indicates that when societies compete, societies that can efficiently order individuals within those societies according to their strategies to have a better chance of gaining a high overall payoff. This is a significant result in understanding cooperation for public good.

The genetic algorithm is only one form of optimisation. Similar experiments can be performed with another optimisation techniques, including simulated annealing and ant-colony optimisation. Also, genetic optimisation does not guarantee a global optimum and it could be the optimum that is arrived at is a local optimum. To avoid this limitation, a more formal optimisation technique on networks, such as the convex function may be used, as potential future work. Broader range of memory-one and other strategies can be considered (tit-for-tat, for example). Furthermore, experiments could be performed on particular application domains, such as defence and project management, to better demonstrate the utility of our results. The findings presented in this chapter demonstrate that the network topology or form is key in determining the optimum global wealth of a population based on the placement of strategies.

While the results presented in this chapter were mainly for relatively smaller networks, the results were consistent even when relatively large networks were used. Further, we focused mainly on sparse networks and not dense networks in this chapter, as social networks generally tend to be sparse[162].

It has to be acknowledged that genetic optimisation doesn't always guarantee a globally optimum solution. For the optimisation conducted in this chapter, many experiments were performed to select the GA parameters such that a global optimum is reached. It has to be noted that there is a limitation in applying formal optimization methods to find the optimum solution evolutionary game theory as the global search space (all possible networks and individual/collective payoffs) is extremely large and heterogeneous and demonstrates non-linear behavior.

One crucial detail that has to be noted here is that the collective payoff that we consider is based on the repeated prisoner's dilemma game, which is a pairwise-interaction, while the traditional public goods game is a multi-point game. It is to be seen whether the traditional public goods game would produced the same results as the repeated prisoner's dilemma game.

This chapter concludes the first segment of the thesis which studied the influence of topology on networked games from numerous aspects such as the evolution of coordination, evolutionary stability of strategies and the optimisation of collective utility of a population. The next segment of the thesis focuses on the concept of non-optimal or bounded rationality and how it may be defined by the network topology and information diffusion constraints. Since the bounded rationality of players implicitly affects the outcome of strategic interactions, it allows network topology and information diffusion to be an integral part of networked game dynamics.

Chapter 6

Topological distribution of bounded rationality in network-based games

The previous three chapters primarily focused on the direct impact of network topology on different aspects of network based games, such as the evolution of coordination and the evolutionary stability. Chapters 6,7 and 8 of this thesis investigate the concept of non-optimal or bounded rationality, which provides an implicit mechanism for capturing the influence of network topology and information diffusion constraints on networked game dynamics.

6.1 Introduction

Socio ecological systems are increasingly being modelled by games played on complex networks. While the concept of Nash equilibrium assumes perfect rationality, in reality players in such systems display heterogeneous bounded rationality. This chapter presents a topological model of bounded rationality in socio-ecological systems, using the rationality parameter of the quantal response equilibrium. It is argued that system rationality could be measured by the average Jensen-Shannon divergence between Nash and quantal response equilibria, and that the convergence towards Nash equilibria on average corresponds to increased system rationality. Using this model, it is shown that when a randomly connected socio-ecological system is topologically optimised in order to converge towards Nash equilibria, scale-free and small world features emerge. Therefore, optimising system rationality is an evolutionary reason for the emergence of scale-free and small-world features in socio-ecological systems. Further, it is demonstrated that in games where multiple equilibria are possible, the correlation between the scale-freeness of the system and the fraction of links with multiple equilibria goes through a phase transition when the average system rationality increases. The results presented explain the influence of the topological structure of socio ecological systems in shaping their collective cognitive behaviour and provide an explanation for the prevalence of scale-free and small-world characteristics in such systems[125].

Game theory is widely used to study and model strategic decision making scenarios, ranging from politics and market economics to ecosystems and information routing [217, 28, 237, 270]. Network based games are increasingly used to understand critical phenomena in socio-ecological systems [186, 185, 52, 220]. The concept of Nash equilibrium has been an important cornerstone in understanding the dynamics of such systems [174]. While Nash equilibrium assumes that all players in a system are fully rational, most real-world strategic decision making scenarios involve players with non-optimal or bounded rationality, resulting in their strategies and behaviour deviating from those predicted by the Nash equilibrium [91]. The possible limitations, such as the amount of information at hand, cognitive capacity and the computational time available, may force a self-interested autonomous player or agent to have bounded rationality and therefore to make non-optimal decisions [88].

Numerous theories have been presented to model the non-optimal rationality of players in strategic games, including the concepts of the near-rationality equilibrium and the quantal response equilibrium [55, 90, 264, 219]. However, these models do not attempt to quantify and predict the levels of rationality prevalent in individual players based on their observable characteristics. Meanwhile, studies in psychology and cognitive science have conjectured that the rationality of individuals is correlated to the level of their social interactions [21, 75, 46]. In this chapter, therefore, a topological model of bounded rationality in socio ecological systems is proposed, based on this conjecture. Using this model, it is investigated how such systems could topologically evolve to have higher system rationality, given a heterogeneous bounded rationality distribution. Since the calculation of Nash equilibrium assumes the perfect rationality of all players, the average Jensen-Shannon divergence between the Nash and quantal response equilibria of each game played within the system is applied as an indicator of overall system rationality. It is important to note that distinguish this system rationality from the average rationality, which is simply the average of the heterogeneous rationality distribution of a system and therefore not influenced by its topology.

This work shows that among different topological classes of complex networks modelling socio ecological systems, the scale-free class minimises this divergence and maximises the system rationality. Further, it is shown that when a network is grown under the Barabàsi-Albert model [25], its system rationality increases, suggesting that it is this drive towards increasing the system rationality that makes the networks grow under the Barabàsi-Albert model. Based on this assumption, a rationality based interpretation of the preferential attachment model is provided, observing the evolution of the topological features while comparing and contrasting them with the standard, degree-based Barabàsi-Albert model. Conversely, when a socio ecological system with a random topology is optimised towards higher system rationality (the system on average is driven towards Nash equilibrium), scale-free and small world features emerge. This result is true for games with single or multiple equilibria. In the case of games with multiple equilibria, the fraction of links in a network where multiple equilibria are actually prevalent is topologically dependent. It is shown that when average rationality is lower, the scale-freeness of the socio-ecological network aids in increasing the fraction of links with multiple equilibria. However, when the average rationality is higher, the scale-freeness actually aids in decreasing this fraction. In fact, with this topological interpretation of bounded rationality, it is possible to demonstrate that the correlation between the 'scale-freeness' and the fraction of links with multiple equilibria goes through a phase-transition when average network rationality is increased. The results presented here provide a possible explanation for the prevalence of scale-free features in the topologies of real world socio-ecological systems [9], and explore how the scale-freeness in turn affects the cognitive decision making behaviour of such systems.

The rest of the chapter is organised as follows. The section 6.2 discusses the background relevant to this work, including the quantal response equilibrium model for players with bounded rationality. Section 6.3 introduces the topological model of bounded rationality, on which this work is based. Section 6.4 discusses the methodology followed in measuring the divergence of Nash equilibrium from quantal response equilibrium and devising the methods applied in performing the other experiments. The next section presents the analysis conducted based on the assumption of topologically distributed bounded rationality (TDBR), and the results obtained. Within that, each sub-section elaborates on the simulation method used in each experiment. Finally, an overall discussion of the topological rationality model is presented.

6.2 Background

6.2.1 Games among players with bounded rationality

Game theory [28, 172, 230, 45] is an effective tool for prisoner's complex socio ecological systems that involve multiple self-interested entities and decision making scenarios [62, 32, 83, 140]. The concept of Nash Equilibrium [174, 101] states that in a strategic decision making environment there exists an equilibrium which no player would benefit deviating from. However, it has been observed that in experimental settings, the equilibrium states of players deviate substantially from those predicted by the Nash equilibrium [97]. One key reason for this deviation is the non-perfect, or bounded rationality of players.

Nash equilibrium assumes that players always adopt the strategy that maximises their utility, with rationality defined as the tendency to maximise one's own utility under uncertainty [91]. However, in the real world, the players may not be perfectly rational due to the limitations mentioned before [88]. Since these limitations vary from player to player, it is to be expected that the players would have heterogeneous bounded rationality, and would make some sub-optimal or apparently random decisions. The qusantal response equilibrium (QRE) [91, 153, 154] presents an analogous way to model games with 'noisy' strategies, by using probabilistic choice model functions such as logit and probit [91]. These functions map the vector of expected payoffs from available choices into a vector of choice probabilities that is monotone with the expected payoffs.

Let us consider the payoff matrix of a generic normal form game (an example is given in Fig.6.1 for two-player games). The quantal response logit function, shown in Eq.6.1, can be used to derive the Quantal Response Equilibrium of a player with a particular level of non-perfect rationality, as shown in Methods.

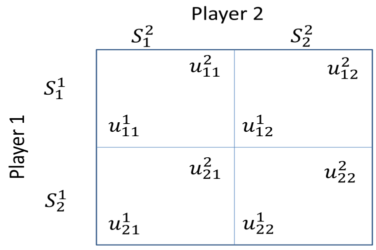


Figure 6.1: The payoff matrix of a generic normal-form game which involves two players. S_j^i denotes strategy j adapted by player i, while u_{jk}^i denotes the payoff to player i, when the first player adopts the strategy j and the second player adapts strategy k.

$$P_j^i = \frac{e^{\lambda_i E^i(s_j^i, \mathbf{P})}}{\sum_{k=1}^{\kappa} e^{\lambda_i E^i(s_k^i, \mathbf{P})}} \tag{6.1}$$

Here, P_j^i is the probability of player i selecting the strategy j. $E^i(s_j^i, \mathbf{P})$ is the expected utility to player i in choosing strategy j, given that other players play according to the probability distribution $\mathbf{P}$. The total number of strategies that player i can choose from is given by κ. The parameter λ_i is known as the rationality parameter of player i, and denotes the level of relative rationality the player i possesses, and can vary from zero to infinity. The average of λ_i over all players, $\bar{\lambda}$, can therefore be an indicator of the average levels of rationality prevalent in the system. It can be shown that as $\lambda_i \to \infty$, the equilibrium probabilities tend towards those given by the Nash equilibrium, and as $\lambda_i \to 0$, the player would operate in a totally random (irrational) fashion [91]. Thus, the rationality parameter λ_i provides a convenient way to quantify the bounded rationality of a particular player and the resulting probability distribution denotes the quantal response equilibrium for that player at that particular bounded rationality level.

6.2.2 Relationship between rationality and social interaction

In this section, we present an argument that there exists an implicit relationship between the amount of social interaction of a particular player and their bounded rationality. This argument is critical in topologically quantifying the bounded rationality of players. Indeed, a number of theories and models have already articulated this view.

Social cognitive theory

Social cognitive theory[21] is a theory used in psychology. It is also applied in areas such as education and mass media. The core argument in social cognitive theory is that 'learning' or cognitive capacity is a social function. In other words, the cognitive capacity of an individual may depend on his ability to observe others within the context of social interactions.

Social learning theory[22] identifies four key factors in learning new behaviour. Those are identified as drives, cues, responses and rewards. These four factors are abundant in strategic decision making environments, making them relevant in determining the cognitive capacities of the players involved.

Developing on the ideas formulated in social learning theory, social cognitive theory[21] identifies five core concepts in modelling the social perspective of cognition. Those are observational learning/modelling, outcome expectations, self-efficacy, goal setting and self regulation. The first aspect, which is observational learning suggests that knowledge acquisition is directly correlated to the observation of models. All these aspects can be related to strategic decision making environments, where the players may learn from each other. One example where social cognitive theory is applied in a strategic decision making environment is organisational modelling, where individuals with conflicting interests may interact with each other.

The key assumption that is made based on social cognitive theory is that in a game theoretic setting, a player's cognitive capacity is proportional to the observational capacity of that player. Thus, a player with a relatively high amount of social interactions may have higher cognitive capacity than a player with a relatively low amount of social interactions.

Social brain hypothesis

Social brain hypothesis [75, 76] provides another interesting avenue to theorise how the social influence, or the social group size, of an individual may reflect upon the cognitive capacity of that individual. Conventionally, it was assumed that the relatively large brain size of humans was due to the evolutionary advantage that it provided in collecting and processing information. However, the social brain hypothesis suggests that the larger brain sizes of primates and humans in general are correlated to the complex and large social groups that they form.

Extending on this evolutionary brain hypothesis, brain studies have been conducted to study the correlation between the human brain cortex size and the social cognitive capacity of humans. Indeed, it has been observed that there exists such a correlation. Recent studies, however suggest that there is a neuro-anatomical correlation with that of cognitive competencies, where the increased brain volumes observed in more intelligent human beings may be accounted for by selectively enlarged brain volumes especially relevant for higher cognitive function [209]. Therefore, these findings suggest that there may be a strong correlation between the social group size and complexity and the cognitive capacity of individuals.

Cognitive hierarchical model

Modelling the heterogeneity of the rationality of players has been previously attempted in game theory. One of the key models that captures this heterogeneity is known as the cognitive hierarchical model[46]. Cognitive hierarchical model was developed to account for the variation of empirical results from the predictions of the Nash equilibrium. The basic assumption in the cognitive hierarchical (CH) model is that each player 'believes' that he or she is the most sophisticated player in the population. It also makes use of iterated decision rules that reflect the iterated process of strategic thinking. The CH models predicts that the players in a population are distributed in k levels of rationality or cognitive capacity. A step k player would assume that all other players are distributed in cognitive levels 0 to $k-1$. The frequency distribution $f(k)$ of other players is assumed to be a Poisson distribution.

Cognitive hierarchical model is based on a 'subjective' hierarchy, where each player assumes that the rest of the population is distributed in a Poisson distribution in their ability to make 'rational' decisions. However this work considering a more objective hierarchy, where the rationality of players are distributed based on the topological characteristics they encompass. The rationale behind this is that network topology reflects the access to information and the cognitive capacity of players.

Based on these theoretical foundations, it could also be argued that the cognitive capacity of a person (player) is an inherent property of a person, and the amount of interactions they engage in is a reflection of that cognitive capacity. Either way, based on the above mentioned studies, it is reasonable to argue that the cognitive capacity, i.e rationality, of a player is positively correlated to the amount of social interactions they undertake.

6.3 Modelling bounded rationality as a topological attribute

Although there have been attempts to model the rationality of players, they have mostly been concerned with proposing a rationality model that identifies the rationality as a constant for all players under a particular strategic decision making context. For example, Wolpert [264] proposes a model to derive the rationality of an abstract player by solving the Maxent (maximum entropy) Lagrangians that model the probability distribution of a human player as a Boltzmann distribution. However, as the cognitive hierarchical model [46] and related empirical observations suggest the rationality of players in a population is typically distributed in a heterogeneous distribution instead of all players having the same level of rationality for a particular strategic decision-making environment. A social-interaction based modelling of bounded rationality would account for this heterogeneity.

Indeed, capturing the heterogeneity of the rationality of players using the quantal response equilibrium has been studied before [219, 93], particularly with models such as the heterogeneous QRE and truncated QRE models [219]. It has been demonstrated that the cognitive hierarchical model [46] is a special case of truncated QRE model. However, these models too limit themselves to varying the heterogeneous rationality parameter λ_i to fit the empirical results, modelling or applying it as an arbitrary parameter without any physical interpretation, while acknowledging that the rationality would be heterogeneous in a population of players. While this approach increases the versatility of the rationality parameter, it limits the predictive capacity of the QRE model. Therefore, in this work a model is proposed with more predictive power, at least in relative terms for players within a population, as long as the assumption that the rationality of a player could be mapped (by a linear or non-linear non-decreasing function) to their amount of social interaction is justified. The model that is proposed defines the rationality parameter λ_i for each player (node) as a function of social interactions.

At a very basic level, the number of social ties a player has (i.e, the degree of a node) could be an indicator of the amount of social interaction a player engages in. However, the amount of interaction would also depend on the tie strength attributes, such as the amount of time spent, the volume of information exchanged, between each pair of players. Furthermore, the correlation between the amount of interaction of a player with other players and the rationality of a player could be linear or non-linear. To model such a dependency, therefore, a generic function f is used, to which the weighted degree (on simply the degree, if tie strengths are considered equal) of a node is an input, as shown in Eq. 7.1.

$$\lambda_i = r.f(\sum_{j=1}^{n} w_{ij}) \tag{6.2}$$

Here λ_i is the rationality of node i; r denotes a network rationality parameter that would be a property of the network and represent the general level of rationality in the system. It should be noted that the average rationality of the system, $\bar{\lambda}$, is proportional to this parameter, as any change in r will result in a corresponding proportional change in every λ_i. The weight w_{ij} denotes the weight of the link connecting node i with each neighbour j, while n is the number of neighbours that node i has. In this work, the function f is modelled as simple linear, convex or concave functions, though in future studies empirical data could be used to fit a more accurate function for a given decision making context. The linear, convex and concave functions that are used are $f(x) = x$, $f(x) = x^2$ and $f(x) = \sqrt{x}$ respectively, due to the simplicity and the computational efficiency of those functions and also due to the fact that they facilitate the $[0 : \infty]$ range of possible rationality values of the rationality parameter. Under this model, a node may behave completely randomly if the network rationality parameter is set to $r = 0$ or when the node is completely disconnected (ie. degree is zero). On the other hand, a node may make choices as predicted by the Nash equilibrium as the network rationality parameter $r \to \infty$, or when the degree of the node is extremely large.

6.4 Methodology

6.4.1 Calculating QRE equilibrium of a strategic interaction

The logit function given in Eq. 8.2 is used for computing the quantal response equilibrium, as often done in literature [91, 268].

$$P_j^i = \frac{e^{\lambda_i E^i(s_j^i, \mathbf{P})}}{\sum_{k=1}^{\kappa} e^{\lambda_i E^i(s_k^i, \mathbf{P})}} \tag{6.3}$$

Here, P_j^i is the probability of player i selecting the strategy j. $E^i(s_j^i, \mathbf{P})$ is the expected utility to player i in choosing strategy j, given that other players play according to the probability distribution $\mathbf{P}$ (which is also denoted $\mathbf{P}^{-i}$ in some literature to highlight the fact that entries 'belonging' to player i should be discounted when the other players are considered collectively). The total number of strategies that player i can choose from is given by κ. The rationality parameter λ_i can vary from zero to infinity.

For a two-player prisoner's dilemma game, it is possible to derive Eq. A.4 and Eq. A.5 from Eq. 8.2 to represent the probabilities that the two players would cooperate.

$$p_c^1 = \frac{e^{\lambda_1(p_c^2 u_{11}^1 + (1-p_c^2)u_{12}^1)}}{e^{\lambda_1(p_c^2 u_{11}^1 + (1-p_c^2)u_{12}^1)} + e^{\lambda_1(p_c^2 u_{21}^1 + (1-p_c^2)u_{22}^1)}} \tag{6.4}$$

$$p_c^2 = \frac{e^{\lambda_2(p_c^1 u_{11}^2 + (1-p_c^1)u_{21}^2)}}{e^{\lambda_2(p_c^1 u_{11}^2 + (1-p_c^1)u_{21}^2)} + e^{\lambda_2(p_c^1 u_{12}^2 + (1-p_c^1)u_{22}^2)}} \tag{6.5}$$

Using the utility values set as ($u_{11}^1 = 3, u_{11}^2 = 3, u_{12}^1 = 0, u_{12}^2 = 5, u_{21}^1 = 5, u_{21}^2 = 0, u_{22}^1 = 1, u_{22}^2 = 1$), these can be simplified to:

$$p_c^1 = \frac{e^{\lambda_1(3p_c^2)}}{e^{\lambda_1(3p_c^2)} + e^{\lambda_1(4p_c^2+1)}} \tag{6.6}$$

$$p_c^2 = \frac{e^{\lambda_2(3p_c^1)}}{e^{\lambda_2(3p_c^1)} + e^{\lambda_2(4p_c^1+1)}} \tag{6.7}$$

Here, p_c^1 and p_c^2 are the probabilities of player 1 and 2 cooperating, respectively, and λ_1, λ_2 denote the rationality parameters of player 1 and 2, which are derived using their respective degrees, the rationality function and the network rationality parameter used, as prescribed in Eq.7.1, with identical link weights set to unity. The two equations have two unknowns p_c^1 and p_c^2. Thus, these two equations can be solved and the probability of cooperation and defection can be calculated for a particular pair of players with varying rationality parameters. The resulting probability distributions provides the QRE for the particular pair of players.

It is already known that the only Nash equilibrium for this game would occur when both players defect (that is, $p_c^1 = 0$, $p_d^1 = 1$ and $p_c^2 = 0$, $p_d^2 = 1$). The payoffs of the prisoner's dilemma game were set to the static values, $u_{11}^1 = 3, u_{11}^2 = 3, u_{12}^1 = 0, u_{12}^2 = 5, u_{21}^1 = 5, u_{21}^2 = 0, u_{22}^1 = 1, u_{22}^2 = 1$, in the simulations conducted in this section, unless otherwise specified. Fig. 6.2 shows the variation of the cooperation probability when the rationality parameter λ is gradually increased, under the QRE model, with the above payoffs set in the payoff matrix.

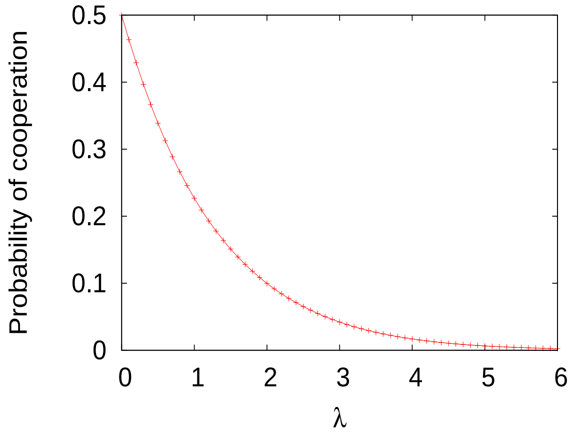


Figure 6.2: The variation of the cooperation probability of a player in the PD game, when the rationality parameter λ is increased under the QRE model.

A similar procedure was followed to calculate the QRE for other games used in this study, using the respective payoff matrices as appropriate.

6.4.2 Measuring the divergence of QRE from Nash equilibrium

In order to measure the divergence of each strategic interaction of the Prisoner's dilemma game from the Nash equilibrium, the Kullback-Liebler (KL) divergence[63] is used. In probability theory and information theory, the KL divergence is used to measure the distance between two probability distributions. In this work, the two probability distributions concerned are the strategic distribution at the Nash equilibrium and the strategy distribution at the quantal response equilibrium. By applying the Kullback-leibler divergence, the asymmetric distance between these two distributions can be measured. This measure can be used to quantify the divergence of each interaction from that of Nash equilibrium. However, this measure is an asymmetric measure as it does not follow the triangular inequality. In order to come up with a symmetric measure divergence, the average divergence of NE and QRE from the average of those two distributions could be utilised. In order to use the KL divergence as a metric, it can be used as a symmetric measure by measuring the KL divergence of the respective QRE strategy distribution from the averaged probability distribution of NE and QRE. Next the KL divergence from the NE to the averaged probability distribution of NE and QRE is also calculated. The average of these two KL divergence values is considered as the divergence metric of a particular QRE from NE.

Following equations depicts the KL divergence of the QRE from NE.

$$D_{KL}(P||Q) = \sum_i P(i) ln \frac{P(i)}{Q(i)} \tag{6.8}$$

Based on this asymmetric measure, the Jensen-Shannon divergence [84] is adopted as a symmetric metric to measure the divergence of QRE from NE.

$$JS(P||Q) = \frac{\left(\sum_i P(i) ln \frac{P(i)}{M(i)} + \sum_i Q(i) ln \frac{Q(i)}{M(i)}\right)}{2} \tag{6.9}$$

Here P and Q are the probability distributions of Nash equilibrium and QRE respectively. M is the average of the Nash and QRE probability distributions. The average Jensen-Shannon divergence between the Quantal Response and Nash equilibria as an indicator of the system rationality, based on the assumption that the more 'selfishly rational' the players are on average, the less this divergence will be. Therefore, system rationality ρ is given by Eq.6.10. The negative sign indicates that the lower this divergence is, the higher the average system rationality. Note that in Eq.6.10, a pair of players are represented by a link k in the corresponding social network and there are M links in the social network in total.

$$\rho = -\frac{1}{M}\sum_{k=1}^{M} JS(Nash_k||QRE_k) \tag{6.10}$$

Elaborating on this further, for the prisoner's dilemma game considered, the Nash equilibrium would be the state where both players defect, thus the probability distribution at Nash equilibrium would be $P1 = 0{,}1$ and $P2{=}0{,}1$ where $P1$ and $P2$ are the probability distributions of the two players respectively. The first probability is the cooperation probability and the second one is the defection probability. Thus, these two distributions would denote the probabilities when both players defect. Suppose the quantal response equilibrium for a particular rationality parameter is $P1_{QRE} = 0.3, 0.7$ and $P2_{QRE} = 0.4, 0.6$. Thus, the average probability distribution for the two players would be $M1 = 0.15, 0.85$ and $M2 = 0.2, 0.8$. Therefore, in order to use the Jensen-Shannon divergence to measure the average divergence of the two distributions, the divergence of NE and QRE to the averaged distribution can be calculated and then the average of those two divergence values could be considered.

Thus, the resulting Jensen-Shannon divergence metric value is:

$$KL(NE||M) = 0 + 1.log\frac{1}{0.85} + 0 + 1.log\frac{1}{0.8} = 0.3857 \tag{6.11}$$

$$KL(QRE||M) = 0.3.log\frac{0.3}{0.15} + 0.7.log\frac{0.7}{0.85} + 0.4.log\frac{0.4}{0.2} + 0.6.log\frac{0.6}{0.8} = 0.1767 \tag{6.12}$$

Thus, the average divergence metric for that particular interaction would be $= (0.3857 + 0.1767)/2.0 = 0.2812$. The average divergence of all such interactions in the networks can be regarded as the 'system divergence' of the network. The negative value of the divergence could be regarded the system rationality within the context of this work. The average system divergence is used as the main metric of quantitative analysis in this work. It would give an indication of how 'far' the network has diverged from Nash equilibria in its constituting strategic interactions.

6.4.3 Measuring scale-free correlation

Throughout this chapter, the proximity of a network to the appropriate scale-free degree distribution is used to denote whether a network is converging to a scale-free topology. There are several well-known methods used to fit a curve into a power-law[59]. The least square method is used, particularly since the degree distributions involve non-negative integers. The coefficient of determination (R), also known as the R-squared value, given in Eq. 6.13, can be used to measure the correlation of given points the corresponding points in the fitted line.

$$R = \sum_i \frac{(f(X_i) - \overline{Y})^2}{(Y_i - \overline{Y})^2} \tag{6.13}$$

where $f(X_i)$ denotes the corresponding fitted power-law curve for each degree X_i, Y_i denotes the actual corresponding histogram value for each degree in the degree distribution and $\overline{Y}$ denotes the average of all input histogram values. Usually, the degree and histogram values are converted to the logarithmic scale in performing this calculation. In addition to the coefficient of determination, we also used the degree distribution and the respective power-law curve was also used to measure the accuracy of the power-law curve fitted using least square method.

6.5 Analysis and Results

Using the topological model for bounded rationality presented in Eq.7.1, the following questions are addressed: (i) Which topological features in a socio ecological system facilitate the highest system rationality, given a particular heterogeneous rationality distribution among players? (ii) Is there a connection between the emergence of scale-free features in socio-ecological systems, and the need to optimise for better system rationality? (iii) What would be the evolution of the average system divergence from Nash equilibrium, when a population is evolved under the Barabasi-Albert model? (iv) Provided that convergence towards Nash Equilibrium is used as a driver for preferential attachment, how would the topological properties of a network evolve? (v) Is there a connection between the emergence of multiple equilibria in systems with heterogeneous rationality, and the topological structure of such systems? In answering these questions, it is important to first define system rationality. Of course, the average of rationality parameters of all players in a system, $\bar{\lambda}$, is one indicator for system rationality, however this definition disregards the topological effects. Therefore, this work will use an independent measurement of system rationality ρ, which is defined as the average Jensen-Shannon divergence of Nash and QRE equilibria over all pairs of players, with a minus sign to account for the fact that the higher this divergence, the lower the system rationality. Details of the computation of ρ are given in the Methods. It is obvious that systems which have the same $\bar{\lambda}$ may have different ρ, since the later is topologically dependent.

6.5.1 Comparing network topologies based on their average divergence from Nash equilibria

To answer the first question mentioned above, three different network models are chosen for comparison: the scale-free network model, Erdős-Rènyi random network model and a regular (well-mixed) network model. The networks that was analysed contained 500 or 1000 nodes with average degrees of $4, 6$ or 8. The Prisoner's Dilemma game was applied in this analysis, since the focus was on games with a single equilibrium first. The rationality of each node was calculated using the linear, convex and concave functions separately. Note that only the equilibria are required, and it was not necessary to actually simulate the games. Based on the Eq.6.1, the QRE was derived for each pair of players (each link). The network rationality parameter r was set to 0.2, 0.002 and 0.5 for the linear, convex and concave functions, respectively. The single Nash equilibrium for prisoner's dilemma occurs when both players defect. Therefore, once the QRE for each pair of players is obtained, the average Jensen-Shannon divergence between Quantal Response and Nash equilibria for the network was computed.

Following the parametrised Prisoner's dilemma game suggested by Santos et. al[223], the payoffs of the Prisoner's dilemma game were set to $u^1_{11} = 1, u^2_{11} = 1, u^1_{12} = 0, u^2_{12} = b, u^1_{21} = b, u^2_{21} = 0, u^1_{22} = 0, u^2_{22} = 0$, where $2 > b > 1$, which is the only parameter of the game, represents the cheating advantage of the defectors over cooperators.

Table A.1 depicts a typical set of results. In this particular experiment, the number of nodes is $N = 1000$ and the number of links is $M = 2000$. The results are averaged over 100

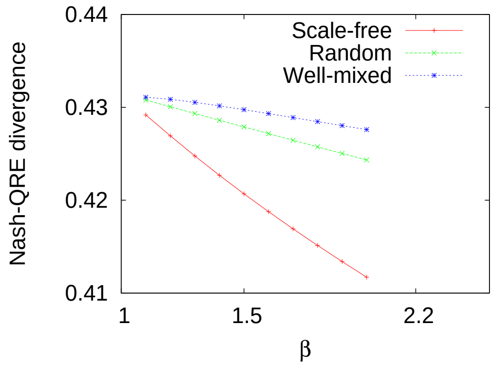
Table 6.1: The average Nash-QRE divergence ($-\rho$) of network topologies for different rationality functions. The rows represent the rationality functions while the columns contain the topology of the population. The network rationality parameter r was set to $0.2, 0.002$ and 0.5 for the linear, convex and concave functions, respectively. The average rationality parameter of nodes, $\bar{\lambda}$ is also shown. Note that the 'average' divergence and 'average' rationality parameter were averaged separately: once over all nodes ($N = 1000$) and again over 100 different instances of the same topological class. The parameter b was set to 2.

	Scale-free	Random	Well-mixed	$\bar{\lambda}$
Linear	0.298	0.334	0.352	0.8
Convex	0.412	0.424	0.428	0.008
Concave	0.306	0.325	0.337	2.0

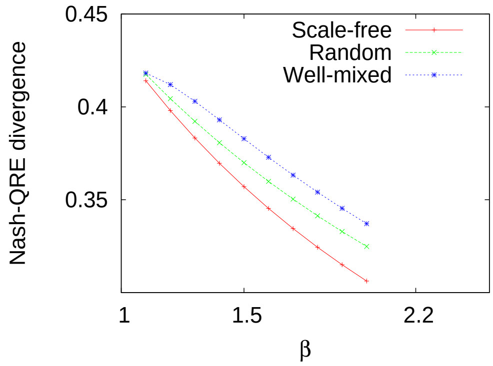
instances, and in each instant, a different topology belonging to the same network class was used, while the number of nodes and links was kept constant. A degree-preserving re-wiring technique was used to create different instances of the same topological class. All scale-free networks had a scale-free exponent of 2.0 with a 90% R-squared correlation. Fig. 6.3 depicts the variation of the average Nash-QRE when the parameter b is varied, under the three types of the topological rationality functions considered. According to the results given in Table A.1 and the Fig. 6.3, it is evident that the Nash-QRE divergence is minimum for the scale-free topology class under all three types of rationality functions. As expected, the convex rationality function gives the highest variations of average Nash-QRE divergence among different topology classes. The divergence is highest for the well-mixed topological class. Thus, it is possible to conjecture that one reason for the prevalence of scale-free topology [9] in real-world socio-ecological systems in which strategic decision making takes place is that this topology facilitates the highest system rationality for a given heterogeneous rationality distribution among players.

6.5.2 Evolution of system divergence under the Barabàsi-Albert model

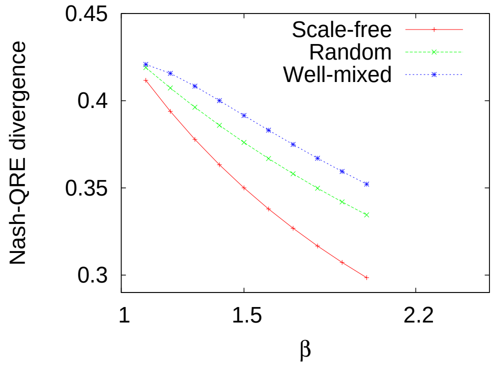
The results obtained in the previous section suggest that scale-free network topologies facilitate strategic interactions that are closer to Nash equilibrium, in comparison to Erdős-Rènyi random and well-mixed networks. This could imply that the real-world networks evolve to be scale-free networks in order to operate closer to Nash equilibrium in strategic interactions. Since the preferential-attachment and growth based model proposed by Barabàsi-Albert is often used to generate scale-free networks, this model could be used to further test this hypothesis by observing the variation of QRE in a network being grown based on the Barabàsi-Albert model, along with a topologically distributed bounded rationality.



(a) Convex



(b) Concave



(c) Linear

Figure 6.3: The Nash-QRE divergence with varying β under (a) Convex. (b) Concave and (c) Linear topological rationality functions. The Prisoner's Dilemma game was used in simulations.

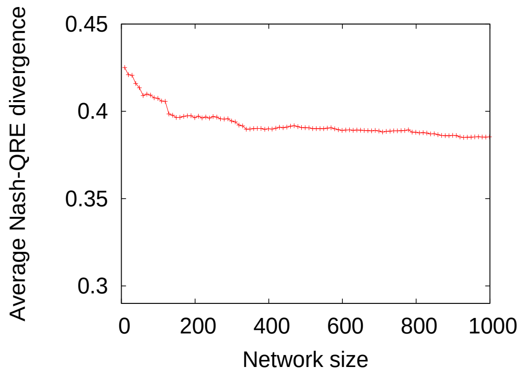
Methodology

In order to test this hypothesis, a network was generated using the Barabàsi-Albert model, where growth and preferential-attachment are the two main factors making a network evolve. For every 10 nodes added to the network, the average Nash-QRE divergence or the system divergence of each intermediate network that is generated after playing a single iteration of the Prisoner's dielmma game. This was done by averaging the Jensen-Shannon divergences of the QRE probability distributions of strategies resulting from the interactions of the network and the probability distribution of strategies at Nash equilibrium. Next, the evolution of the system divergence against the network size. This experiment was repeated for linear, convex and concave rationality functions. As with the previous experiment, the network rationality parameter was adjusted under each rationality function to distribute the rationality values over the network.

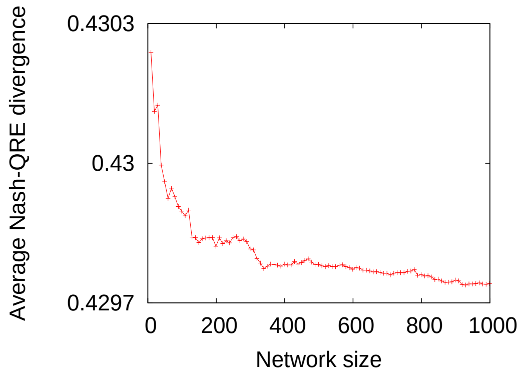
Results and Discussion

Fig.6.4 depicts the evolution of the average Nash-QRE divergence of a scale-free network that is grown by applying the Barabàsi-Albert model, under different types of topologically derived rationality functions.

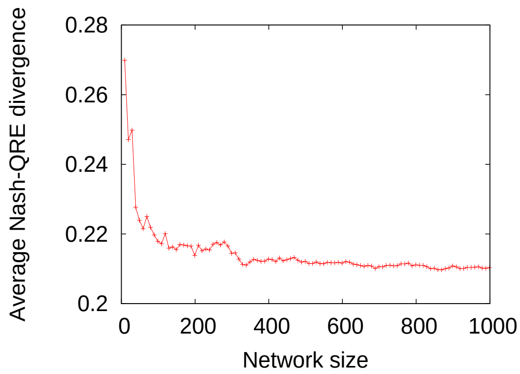
As shown in Fig. 6.4, it is evident that when a network is grown using preferential attachment, the average Nash-QRE divergence of the entire network shows a downward trend, suggesting that the strategic interactions among the players approach Nash equilibrium. Figures 6.5 and 6.6 depict the evolution of the scale-free exponent and the scale-free correlation of the network under growth. According to these figures, the scale-freeness of the network increases and stabilises as it grows. These results confirm the earlier observation presented in Table 6.1 and the hypothesis that the scale-free nature of a network growing under preferential attachment model has a correlation with the tendency of nodes to optimise the outcome of their strategic interactions by converging towards Nash equilibrium. Thus, based on these observations, it can be argued that the growth in Barabàsi-Albert model could be driven by the population's tendency to increase its rationality and thereby optimise the strategic interactions.



(a) Convex



(b) Concave



(c) Linear

Figure 6.4: The variation of average Nash-QRE divergence of the network over the number of nodes in the network. The network is grown till 1000 nodes, based on the Barabàsi-Albert model. Convex and concave and linear topological rationality functions used. The network rationality parameters 0.002,0.5 and 0.2 were used respectively.

6.5.3 Network growth model based on bounded rationality and convergence towards Nash Equilibrium

Based on the results discussed so far in this chapter, it is possible to identify an apparent correlation between the scale-free topology generated using the preferential attachment

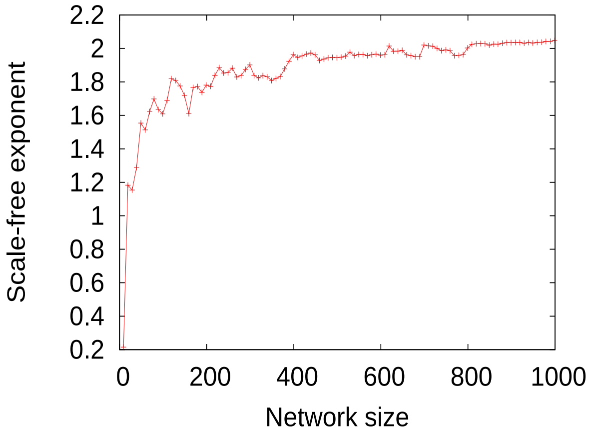


Figure 6.5: The variation of the scale-free exponent of the network over the number of nodes in the network. The network is grown till 1000 nodes, based on the Barabàsi-Albert model. Linear topological rationality function used with the network rationality parameter being set to 0.2.

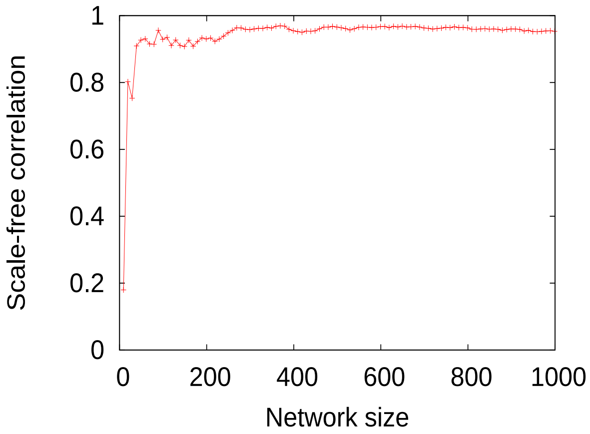


Figure 6.6: The variation of the scale-free R-squared correlation of the network over the number of nodes in the network. The network is grown till 1000 nodes, based on the Barabàsi-Albert model. Linear topological rationality function used with the network rationality parameter being set to 0.2.

based Barabàsi-Albert model[25] and the tendency of the nodes in a network to converge towards Nash equilibrium. Inspired by this outcome, this sub-section is dedicated to the proposition that a network growth model takes the tendency to operate towards Nash equilibrium as a form of preferential attachment. In other words, the hypothesis suggested is that when a new node joins a network, it is more likely to establish a link with an existing node that would produce an interaction, which could optimise the outcome of the interaction. Since self-interested players would move towards Nash equilibrium when they try to optimise their outcomes, this means that a new node is more likely to establish a connection with an existing node, where the interaction would have less divergence from Nash equilibrium compared to other nodes. Such a growth model could be relevant for networks formed in order to perform strategic interactions, such as collaboration networks.

Methodology

Algorithm 9 details the steps that were used to grow a network using preferential attachment interpreted as the tendency to optimise the payoff of a node in its strategic interactions. The prisoner's dilemma game was used to compute the QRE and the Jensen-Shannon divergence from NE in each interaction. The network was grown till 1000 nodes. The evolution of the scale-free exponent, the R-squared correlation of the scale-free exponent and the network assortativity [176] measured over time to observe the characteristics of the resulting network. The network rationality parameter r was set at 0.1, while a linear function of degree was used to determine node rationality.

Algorithm 5: Preferential attachment model based on the convergence to Nash equilibrium in strategic interactions.

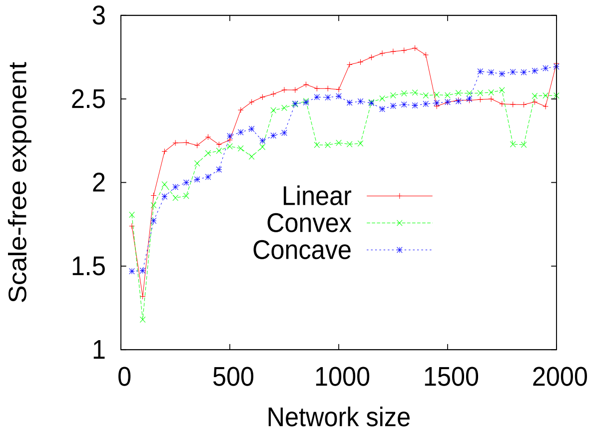
Data: Network size-n

Result: Network generated from a strategic interaction based preferential attachment growth model.

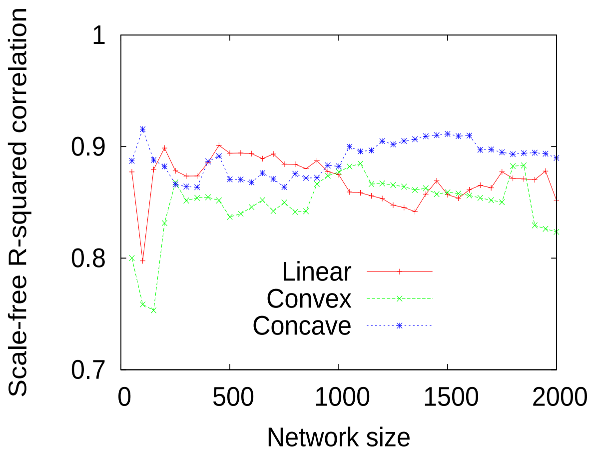
create an empty network;
create new node;
add the node to the network;
while *The size of the network is less than* n **do**
    create new node i;
    measure the Nash-QRE divergence of interacting with each node within the network;
    calculate the probability p of connecting to each node j in the network (negatively proportional to the Nash-QRE divergence);
    foreach *node* j *in network* n **do**
        create a link $i - j$ based on the probability p;

Results and Discussion

Fig. 6.7 and Fig. 6.8 depict the evolution of the scale-free exponent, R-squared correlation of the respective power-law curve and the assortativity values of the intermediate networks, over the network size. The network rationality parameter was set to 0.2, 0.002 and 0.5 in linear, convex and concave rationality functions were used, respectively.



(a) Scale-free-exponent



(b) R-squared-correlation

Figure 6.7: The variation of scale-free exponent and the R-squared correlation of the respective power-law curve, over network size, when the network is grown using the convergence to Nash equilibrium as an indication of preferential attachment.

As depicted by figures 6.7 and 6.8, the growth model defined by interpreting preferential attachment as the tendency to converge towards Nash equilibrium generates networks that evolving into scale-free networks. This is evident from the evolution of the scale-free exponent and the R-squared correlation of the respective power-law curve. In most real-

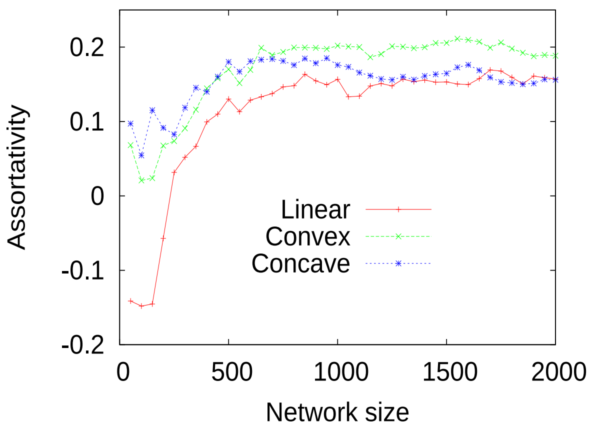


Figure 6.8: The variation of network assortativity over network size, when the network is grown using the convergence to Nash equilibrium as an indication of preferential attachment.

world scale-free networks, the scale-free exponent is between 2 and 3 and there exists a significant positive R-squared correlation between the actual degree distribution and the power-law fitted curve, reminiscent to the characteristics of the network generated from this model. This may be suggested as further evidence of the validity of a topological interpretation of the bounded rationality of networked population players.

Additionally, the non-negative assortativity values of the networks resulting of this particular growth model is significant since the Barabàsi-albert model does not produce assortative or disassortative networks. However, most real-world social, collaboration and biological networks are observed to be either assortative or disassortative [175]. In particular, collaborative networks that operate on strategic interactions show strong assortative tendencies [175]. Thus, a topologically distributed rationality based interpretation of the preferential attachment model may be useful in generating networks with real-world characteristics of scale-free nature and assortative mixing.

6.5.4 Optimising network topology for maximum system rationality

In the results obtained so far, it an be observed that the scale-free topological class aids the convergence towards Nash equilibria on average, when compared to other topological classes. Conversely, it is possible to test whether it is the evolutionary pressure on any given system that forces it to move towards Nash equilibrium on average, while allowing the system to rewire itself, that results in the system evolve into a scale-free topology.

Therefore, this subsection elaborates on the topological optimisation that was performed using the Erdős-Rènyi random network class as the null model, and the resulting topological evolution that was observed. In order to perform the optimisation based on the convergence towards Nash equilibrium, a variant of the simulated annealing technique [3] was applied.

Methodology

In this particular set of experiments, bounded rationality was measured using a convex function of degree (because the convex function $f(x) = x^2$ facilitated rapid topological evolution compared to the other functions) with the network rationality parameter being set to $r = 0.1$. The scale-free R-squared correlation of each intermediate network was measured in order to observe the emergence of scale-free characteristics. Moreover, the clustering coefficient and the average path length of the intermediate networks were also measured, since these are the parameters that could be utilised to identify the small-world nature of the networks [9]. Relatively higher clustering coefficients and lower average path length are indications of small-world characteristics emerging [9].

The process begins with an Erdős-Rènyi random network of size $N = 1000$ and $M = 2000$. In each iteration, 0.03% (i.e $M/300$) randomly selected links were rewired so that the average Jensen-Shannon divergence from Nash equilibrium decreases. The iterations were continued until the network had been rewired M times (i.e 300 iterations). Algorithm 6 explains the optimisation technique that was employed.

Algorithm 6: Network optimisation using the convergence towards Nash Equilibrium and simulated annealing.

Data: Random network

Result: Network optimised to minimise the average divergence between Nash and QRE equilibria

```
while counter is less than 300 do
    increase counter; randomly select a link l1;
    measure the Nash-QRE Jensen-Shannon divergence of the interaction denoted
    by the link l1;
    randomly select a node j to connect with node i of link l1;
    measure the Nash-QRE Jensen-Shannon divergence of a potential link l2
    consisting of nodes i and j;
    if the Nash-QRE KL divergence of the link l1 is lower than that of l2 then
        drop the link l1;
        create the link l2;
        save the current network;
    Repeat until (M/300) link replacements have been made
```

For each intermediate network, the scale-free R-squared correlation, the average clustering coefficient and the average path length were recorded. The definition and computation of these metrics is well-understood and therefore they are not explained here [9]. If there are multiple equilibria, the lowest divergence for each pair of players was used.

Results and Discussion

The results obtained from the simulated annealing optimisation are shown in Fig. 6.9 and Fig. 6.10. As depicted by Fig. 6.9, the scale-free R-squared correlation shows an upward trend when the network evolves. Meanwhile as Fig.6.10 shows, the clustering coefficient clearly increases while the average path-length decreases over time, indicating that the small-worldness also increases over time. Even though these results are for networks with size $N = 1000$ and $M = 2000$, similar results were obtained for networks with average degrees $4, 6$ and 8, attempting all possible permutations. From these results, it is clear that when network topology is optimised towards maximum system rationality (i.e converegence

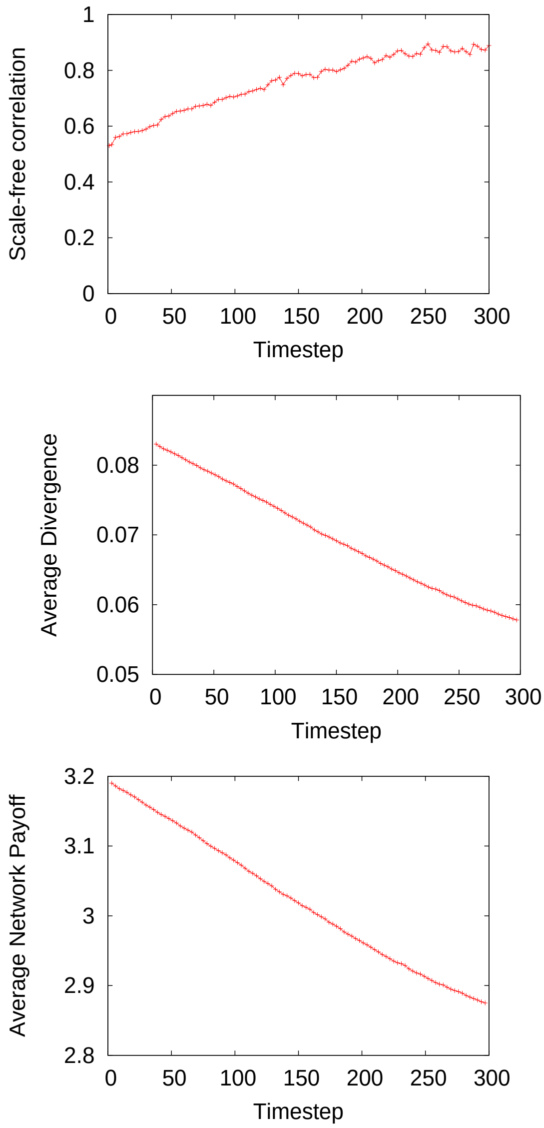


Figure 6.9: (a) The evolution of R-squared scale-free correlation of a network over time, when it is optimised to minimise the average Jensen-Shannon divergence between Nash-QRE equilibria, using simulated annealing. (b) The evolution of $-\rho$, average Jensen-Shannon divergence between Nash-QRE equilibria, of the network over time. (c) The evolution of the average pay-off over time. Network size is $N = 1000, M = 2000$. The prisoner's dilemma game was used in simulations.

towards Nash equilibria on average is favoured), scale-free and small-world features emerge in systems with random topology.

Interestingly, it was also found that the average trend towards system rationality does not imply that the average pay-off for the players will also increase. In fact, for Prisoner's Dilemma at least, the opposite is true, as Fig.6.9 indicates. However, it is important to bear in mind that the premise behind Nash equilibrium in prisoner's dilemma is that

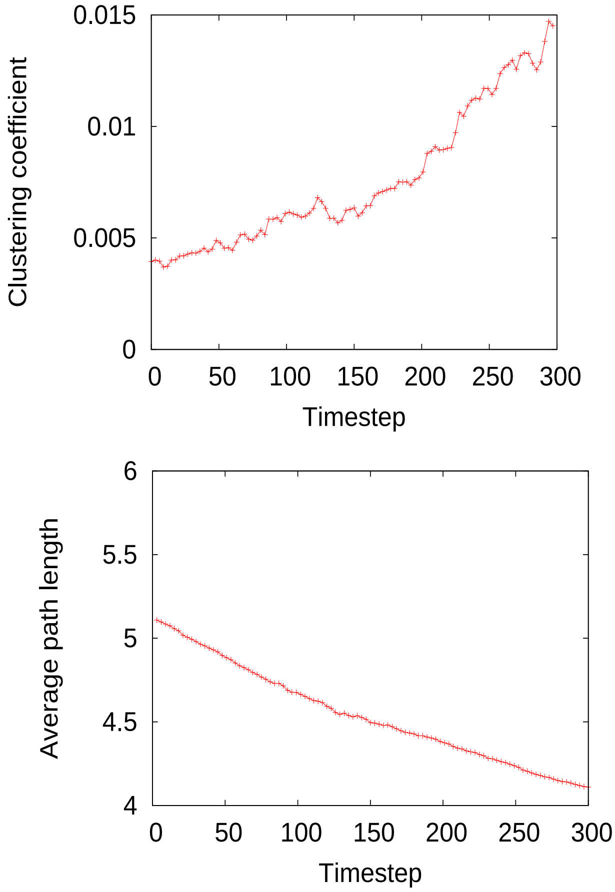


Figure 6.10: The evolution of the clustering coefficient and average path length of a network over time, when it is optimised to minimise the average Jensen-Shannon divergence between Nash-QRE equilibria, using simulated annealing. The Prisoner's Dilemma game was used in simulations. Network size is $N = 1000, M = 2000$.

given the uncertainty about the other player's decision, each player will make a selfish decision which would ensure that they are not worse off than the other player, even though the expected utility of that decision is lower than both cooperating. That is indeed the 'dilemma'. The drive towards Nash equilibria in this case does not imply an increase in the 'common wealth' of the system [122]. This would be explained further in the discussion presented at the end of this chapter.

Emergence of scale-freeness and small-worldness in games with multiple equilibria

So far, the simulations have been performed on games with single pure Nash equilibria. However, most normal-form games consist of multiple pure and mixed Nash equilibria. When the assumption is made that the bounded rationality of a population of players is heterogeneous, the existence of multiple equilibria adds an extra layer of complexity. In order to observe how the rationality parameter would affect the quantal response equilibria in a game where there exists multiple equilibria, a set of coordination games that have two pure Nash equilibria were used. These included (i) the stag-hunt game, where the two pure Nash equilibria occur when either both players coordinate or both players defect (ii) the meeting game, where the Nash equilibria occurs when both choose one location or when both players choose the other location to meet (iii) The matching-pennies game [168], where the Nash equilibria occurs when the symbol on the penny each player comes up with (head/tail) does not match.

The stag-hunt game [237, 127, 223] was simulated with the payoffs set to $u^1_{11} = 5, u^2_{11} = 5, u^1_{12} = 0, u^2_{12} = 3, u^1_{21} = 3, u^2_{21} = 0, u^1_{22} = 3, u^2_{22} = 3$. The stag-hunt games contains two pure strategy Nash equilibria. The same optimisation process described in the algorithm 6 was followed. Note that since there are multiple equilibria, the *lowest* divergence for each pair of players was used to compute the average Jensen-Shannon divergence. Again, the scale-free R-squared correlation of each intermediate network was recorded in order to observe the emergence of scale-free characteristics.

As depicted by Fig. 6.11[A], the scale-free correlation shows an upward trend when the network evolves. The corresponding average divergence $-\rho$ is shown in Fig. 6.11[B], which, as expected, is minimised during the process. Interestingly, the average-payoff of players, shown in Fig. 6.11[C], increases as well. This is in contrast to the Prisoner's Dilemma game, where the increase in the selfish-rationality of the system, represented by ρ, does not increase the average pay-off. As noted before, depending on the nature of the game, the average selfish rationality corresponds to the common good in some games, and does not in others. Regardless, the main observation that the topological optimisation towards high system rationality results in the emergence of scale-free characteristics is true also for games with multiple equilibria. This was further confirmed by conducting experiments with other two-player games, such as battle of the sexes and matching-pennies. The payoffs of the meeting game was set to $u^1_{11} = 5, u^2_{11} = 3, u^1_{12} = 0, u^2_{12} = 0, u^1_{21} = 0, u^2_{21} = 0, u^1_{22} = 3, u^2_{22} = 5$. For the matching-pennies game, an asymmetric payoff matrix was used with the payoffs $u^1_{11} = 0, u^2_{11} = 2, u^1_{12} = 5, u^2_{12} = 5, u^1_{21} = 4, u^2_{21} = 4, u^1_{22} = 2, u^2_{22} = 0$. The results obtained for these two games are not shown here, while they were quantitatively similar.

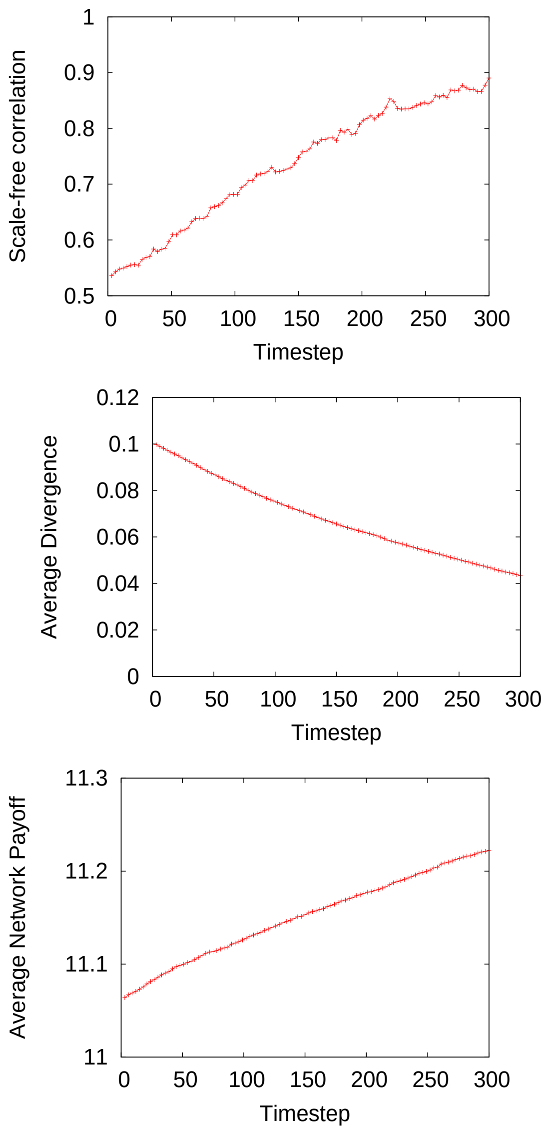


Figure 6.11: (A) The evolution of the R-squared correlation to power-law degree distribution (scale-freeness) of the network over time, when it is optimised using Nash-QRE divergence and simulated annealing. (B) The evolution of the average Nash-QRE divergence of the network ($-\rho$) over time. (C) The evolution of the average pay-off over time. Network size is $N = 1000, M = 2000$. The stag-hunt game was used in simulations.

Further, for all the above mentioned games, the average clustering coefficient and the average path length during this optimisation process was observed. Similar to the re-

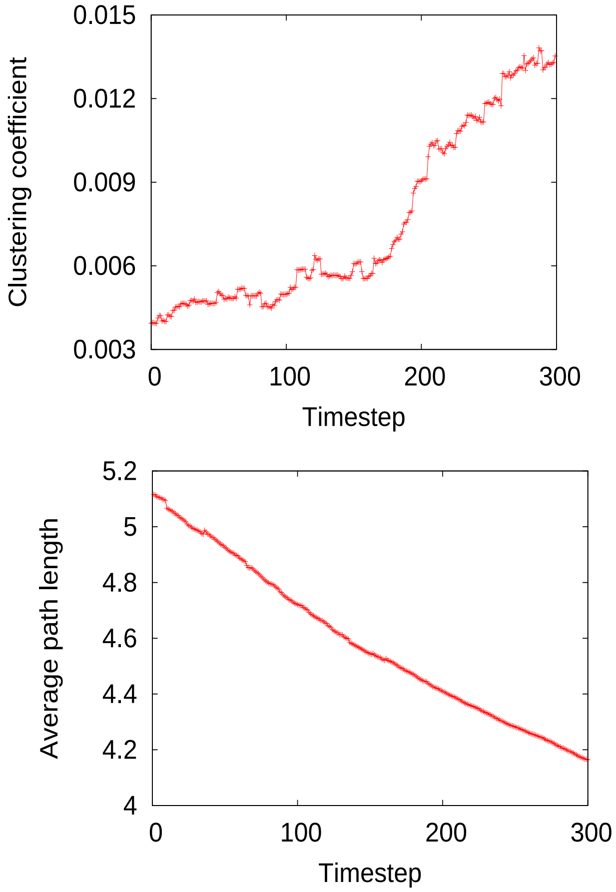


Figure 6.12: The evolution of the clustering coefficient and average path length of a network over time, when it is optimised to minimise the average Jensen-Shannon divergence between Nash-QRE equilibria, using simulated annealing. The stag-hunt game was used in simulations. Network size is $N = 1000, M = 2000$.

sults obtained for Prisoner's Dilemma, the average path length decreases and, the average clustering coefficient increases, when the network is optimised towards higher system rationality. Fig. 6.12 shows the evolution of the average path length and the clustering coefficient for the stag-hunt game based simulations. In summary, all of these results confirm that when a network is topologically optimised towards increased system rationality, scale-free and small-world features emerge for a range of single and multiple equilibria games. Conversely, it was also verified that among the three topological classes that were considered (scale-free, random and regular), it is the scale-free class which showed the highest system rationality ρ for all the above-mentioned multiple-equilibria games.

6.5.5 Network topology and fraction of links with multiple equilibria

Moving further on the analysis of games with multiple equilibria, this sub-section discusses on the actual prevalence of multiple equilibria in games in which multiple equilibria are possible, and how the interplay between heterogeneous rationality and network topology influences this prevalence. In particular, the fraction of links with multiple equilibria in the landscape defined by varying scale-freeness and varying average rationality is computed, indicated by the network rationality parameter r. It has been previously shown that in two-player games, the players go through phase-transitions of knowledge of opponents when the rationality parameter increases [98]. Initially, a single pair of players playing stag-hunt was considered to verify that the rationality of both players would influence the number of multiple equilibria in the system. The quantal response equilibria equations were solved for a range of λ_i values for both players, as described in Methods. Fig.6.13 depicts the results observed. For a given player, when the opponent's rationality is relatively high ($\lambda_2 = 1.0$ or $\lambda_2 = 2.0$), multiple equilibria can exist and the probability of coordination goes through a phase transition, as predicted by Harrè et al. [98]. If the rationality of the opponent is relatively low ($\lambda_2 = 0.1$), a phase transition does not occur. Therefore, it can be verified that for a single pair of players, multiple equilibria does not always occur and that the rationality levels of both players influence whether there could be multiple equilibria. Hence, it is clear that in a socio-ecological system (represented by a complex network) with a heterogeneous rationality distribution, on which a multiple-equilibria game is played, only a fraction of links would actually support multiple equilibria.

Next, a socio-ecological system of players were considered with a heterogeneous rationality distribution who engage in such a game with multiple equilibria and analyse how the system topology (particularly the level of scale-freeness) would influence the number of multiple equilibria. Therefore, a range of scale-free networks were generated with varying scale-freeness and identical size $N = 1000$ and $M = 2000$. In order to do this, perfect scale-free networks were generated using the Barabàsi-Albert model [9], and introduced a measure of 'randomness' in each network by randomly rewiring $m < M$ number of links. While varying m, the scale-freeness of each resulting network was measured by fitting a power-law degree distribution and measuring the fitness, as mentioned before. For each of these networks, Eq. 7.1 was used to generate a heterogeneous rationality distribution, and then for each pair of nodes, computed the QRE equilibria as shown in Methods. Then,

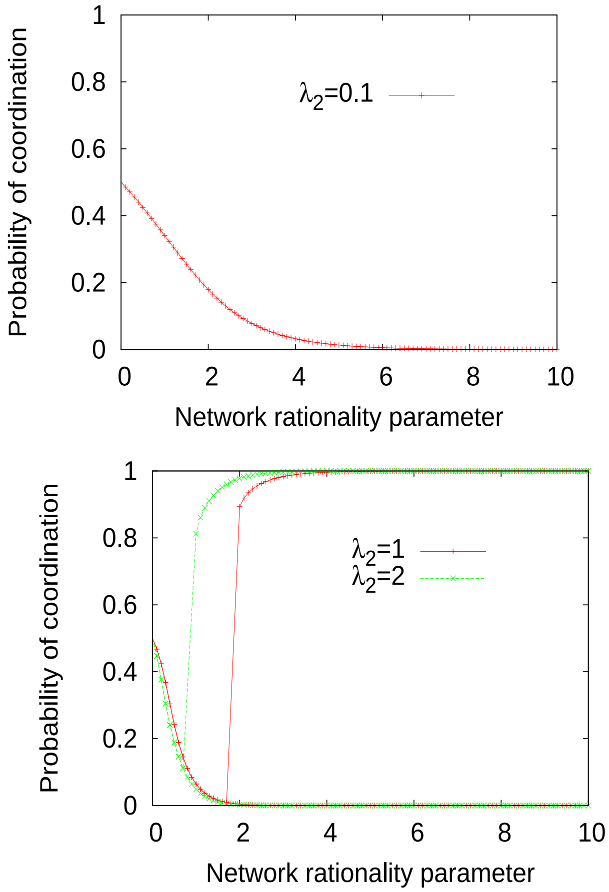
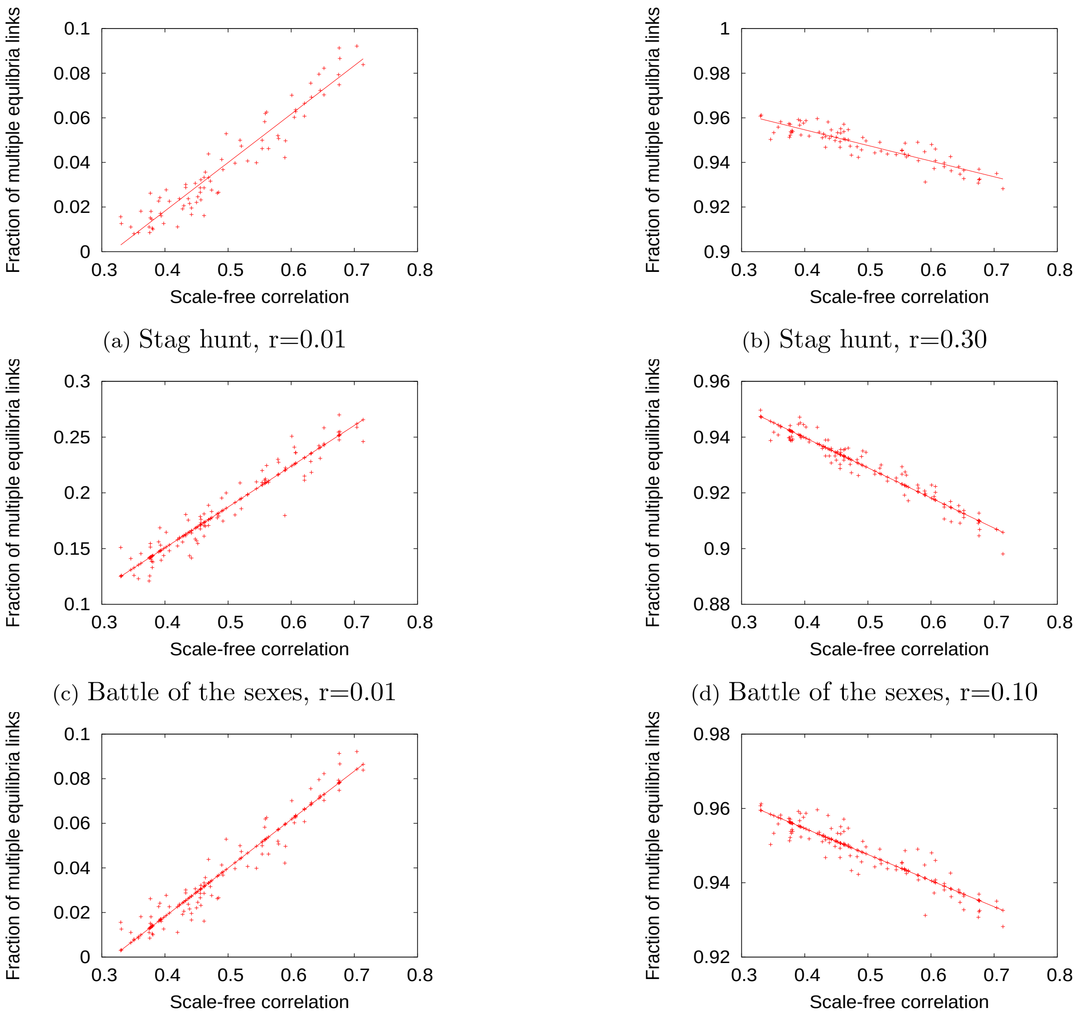


Figure 6.13: The variation of player 1's coordination probability when the rationality parameter of player 2 is fixed (at either $0.1, 1.0$ or 2.0) and the rationality parameter of player 1 is varied. When the rationality of the opponent is small, there is only one equilibrium and otherwise, there are multiple equilibria. The stag-hunt game was used.

the number of links which would have multiple were counted (in this case, two) equilibria, and finally thus computed the proportion of links in the entire network which had multiple equilibria. For this experiment, we used a convex rationality function. We repeated the whole process was repeated for different network rationality parameter values, beginning from $r = 0.01$ (low average rationality) to $r = 0.3$ (high average rationality).

The simulation results were obtained for two particular values of network rationality, $r = 0.01$ and $r = 0.3$, in Fig. 6.14[a] and Fig. 6.14[b], for the stag-hunt game. In these figures, the fraction of links where multiple equilibria is possible were plotted against the 'scale-freeness' of the network, represented by the scale-free R-squared correlation. Eighty different networks of increasing scale-freeness were shown in each plot. It was observed that for relatively lower network rationality ($r = 0.01$), the scale-freeness of networks has clear



(a) Stag hunt, r=0.01
(b) Stag hunt, r=0.30
(c) Battle of the sexes, r=0.01
(d) Battle of the sexes, r=0.10
(e) Matching pennies, r=0.01
(f) Matching pennies, r=0.30

Figure 6.14: The variation of the fraction of links with multiple equilibria of networks with varying scale-freeness, under smaller (0.01) and larger (0.30 or 0.10) network rationality parameter values. The results for the stag-hunt game, battle of the sexes and matching pennies games are shown.

positive correlation with the fraction of links with multiple quantal response equilibria. That is, when the scale-free nature of the network increases, it becomes easier for links to attain multiple equilibria. By contrast, when the network rationality is relatively high ($r = 0.3$), the fraction of links with multiple equilibria has a negative correlation with the 'scale-freeness' of the network. That is, the emergence of scale-freeness encourages single equilibria among the pairs of players in the population. In fact, to generalise this result, the correlation between the fraction of links with multiple equilibria and the 'scale-freeness' (R-squared correlation) of the network could be computed for several network rationality parameter (r) values. A change of the sign of this correlation can expected to be seen for r between $r = 0.01$ and $r = 0.3$. Fig. 6.15[A] depicts the results of such an experiment, where 16 different values of r from $r = 0.01$ to $r = 0.3$ are used. From this figure, it is evident that indeed such a change of sign occurs when $r \approx 0.1$. Moreover, this change of sign is not gradual but appears to be, again, a sudden transition. Note that the same set of 80 scale-free networks were used to generate each data point in this plot, with differing values of r. Thus, the correlation of scale-freeness with the fraction of links with multiple quantal equilibria goes through a phase-transition when the overall network rationality, represented by the network rationality parameter r, is increased. On the other hand, as Fig.6.15[B] shows, for the same level of 'scale-freeness' the fraction of links with multiple equilibria increases with rationality parameter r. This is also confirmed by Fig.6.14 which shows that the range of fraction values is much higher when $r = 0.3$ compared to $r = 0.01$. These results are summarised in a 3D plot (Fig.6.16), which shows the fraction of links with multiple equilibria in a stag hunting game for a range of scale-free networks with differing 'scale-freeness' and rationality parameter r. The dominant trend shows the fraction increasing then stabilising against rationality; however, it is important to note that the positive correlation with scale-freeness for lower r values and the negative correlation with scale-freeness for higher r values. These correlations are relatively less visible however, and it is for this reason, they have showed separately in Fig. 6.14 which clearly identifies the correlation tendencies.

These results are vital in understanding the relationship between network topology and cognitive decision making in systems with bounded rationality. They suggest that when the socio-ecological system as a whole has less average rationality (i.e players are more likely to make random decisions), the scale-free structure of the system helps players to have a higher number of rational choices. Yet, if the system becomes more rational on average, the same scale-freeness becomes a hindrance to players having a higher number of rational choices. If a society is increasingly becoming 'selfishly wise' (i.e, rational in the game theoretic sense), there will come a time where a slight change in the average rationality will have a huge bearing in the number of rational choices the players may have. In this set of experiments, when $r \approx 0.1$, slight changes in rationality will hugely impact the fraction of multiple equilibria. Finally, the observations suggest that even though a particular strategic decision making scenario may potentially encompass multiple equilibria, the actual prevalence of multiple equilibria in a population is topologically

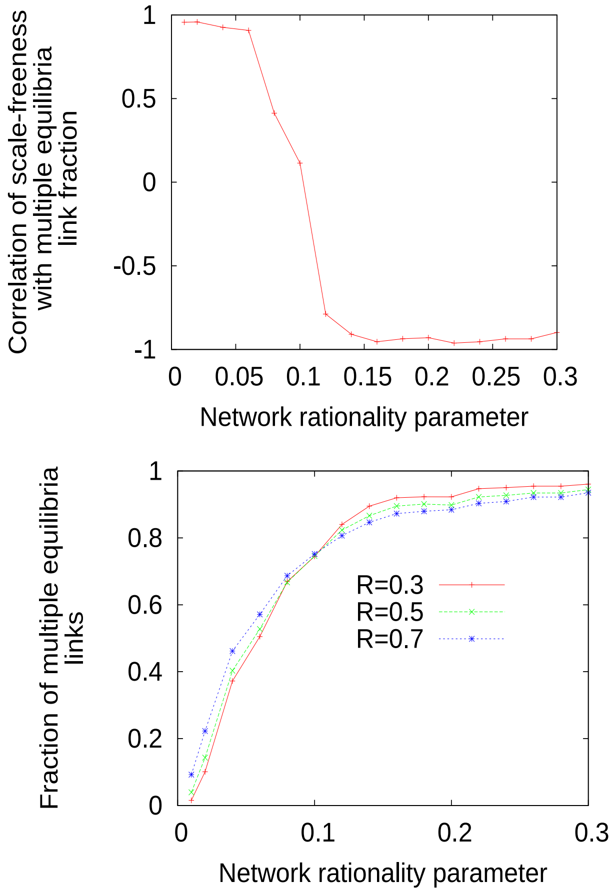


Figure 6.15: (a) Correlation between the scale-freeness of networks and the fraction of links with multiple equilibria, against the network rationality parameter, for the stag-hunt game. (b) The variation of the fraction of links with multiple equilibria of networks with varying network rationality parameter using the stag hung game. Three scale-free networks with differing R-squared correlations of 0.3, 0.5 and 0.7 were used.

dependent and connected to the average rationality of that population.

To verify whether the results observed were specific to the stag-hunt game or could be generalised to other games with multiple equilibria, similar experiments were conducted with the meeting game (battle of the sexes) and the matching-pennies game described earlier. The same set of scale-free and partially scale-free networks which were used in the experiments with stag-hunt game were used. The typical results are shown in Fig. 6.14 [Parts C,D,E,and F]. These figures indicate that these observations made earlier are generic and not specific to the stag-hunt game alone, and that in any game with multiple equilibria, the scale-free features facilitate the prevalence of multiple equilibria when average network rationality is lower, and hinder the prevalence of multiple equilibria when average network rationality is higher.

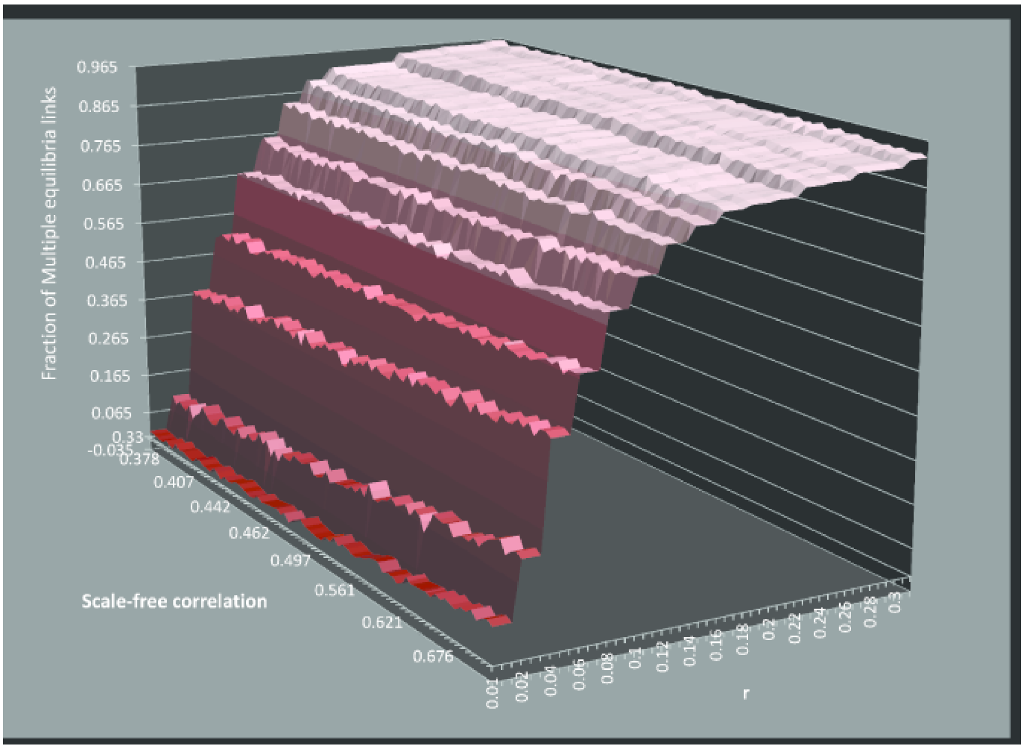


Figure 6.16: A three-dimentional plot showing the fraction of links with multiple equilibria against 'scale-freeness' (R-squared correlation) and network rationality parameter r, for the stag-hunt game. Please note that the apparent stratification is simply a result of the limited number of r values used. To increase clarity of the figure, we only show a section of the scale-freeness range used are shown.

6.6 Discussion

Real world socio ecological systems consist of players whose rationality is bounded. A range of existing theories and hypotheses such as social cognitive theory, the social brain hypothesis and the cognitive hierarchical model have implied that the rationality of a player might be correlated to the amount of social interactions they undertake. Based on this assumption, this chapter proposed a topological model of bounded rationality. This model was then used to understand the relationship between the topology of socio-ecological systems and their dynamics. In particular, attention was paid to the scale-free characteristic and its influence on social network dynamics. Given a particular heterogeneous distribution of rationality among players, it was considered how the topological characteristics encourage system rationality. Since the Nash equilibrium predicts the strategies that players with perfect rationality would choose, the Jensen-Shannnon divergence of Nash and the quantal response equilibrium states was computed for each pair of players in a given social system, and the average of these divergences was used as a measure to quantify the system rationality. A number of well known games were considered, including the Prisoner's Dilemma, the stag-hunt, the meeting game and the matching pennies game to simulate scenarios where cognitive decisions must be made.

The topological analysis of bounded rationality resulted in some significant findings. Firstly, a number of network classes were compared, including scale-free, Erdős-Rènyi random, and lattice networks (representing well mixed populations). Based on this comparison, it was shown that among these classes, it is the scale-free networks which facilitate the best convergence towards Nash equilibrium (highest system rationality), on average. It may be argued that this might be one reason why many real-world social systems are scale-free.

Exploring the possible relationship with the scale-free networks and bounded rationality further, the variation of the average Jensen-Shannon divergence was measured while a network is grown according to the Barabàsi-Albert[25] model. The resulting observations suggest that network growth has a negative correlation with the Jensen-Shannon divergence, suggesting that the networks converge towards Nash equilibrium as they grow. This observation is used to indicate that there could be a game-theoretical explanation on why there is preferential attachment in networks. Exploring further on the same argument, a network growth model is proposed that interprets 'preference' in preferential attachment as the tendency to optimise a strategic interaction by converging towards Nash equilibrium. The resulting network of this growth model displays both scale-free and assortative tendencies. However, it has to be noted that this the convergence towards Nash Equilibrium may not be the sole reason for the abundance of the scale-free and small-world topologies. There may be other more complex explanations for this behaviour. However, the convergence towards Nash equlibrium could be one possible reason for the abundance of scale-free and small-world topologies.

Seeking further evidence for this conjecture, the topological evolution of social systems was simulated using the simulated annealing technique, beginning from a random network topology. It became apparent that when evolutionary pressure is applied on social systems to converge, on average, towards Nash equilibria, scale-free and small world features emerge. This finding could have significant implications, since it provides an alternative explanation for the prevalence of scale-free networks in many real world systems and societies.

Following this, the topological analysis of bounded rationality was extended to games with multiple equilibria. Again, it was demonstrated that when evolutionary pressure is applied on systems to converge, on average, towards Nash-equilibria (regardless of which equilibrium state a particular pair of players converge towards), scale-free and small world features emerge. Further, the likelihood of the existence of multiple equilibria among the players of a system with a bounded heterogeneous rationality distribution was considered. It was observed that a delicate balance exists: when the average rationality (this must be distinguished from what the system rationality, which is computed from the Jensen-Shannon divergence between QRE and Nash equilibria) is low, the scale-free nature of the system encourages the emergence of multiple equilibria, while when the average rationality is high, the scale-free character in fact hinders the existence of multiple equilibria. Therefore, the number of rational choices available to players, from which they cannot deviate without loss, depends on the social network topology as well as the level of rationality prevalent in the system.

It is important to stress that rationality of players and that of a system have been defined in a very specific way in this work. It could be argued that rational players are those who try to maximise their average individual pay-offs. If players attempted to do this within a heterogeneous system, they may well make choices that are contrary to Nash equilibrium. Therefore, a system which converges towards Nash equilibrium will not necessarily have increasing average pay-offs. Indeed, in the case of prisoner's dilemma game, the convergence towards Nash equilibrium results in decreasing average pay-offs. Thus, it could be argued that such a system is, on average, not becoming more rational. However, in an environment where there is a lot of mistrust and/or competition, the priority of the players will be to make sure that their average pay-offs are better than other players with whom they compete - that is, they would want to ensure that they are not cheated by others. The self-interest, and the relative well-being in the system, therefore gains prominence over the absolute well being, represented by the cumulative pay-off. In such systems, the convergence towards Nash equilibria, on average, means the players are getting better at preserving their relative self-interest, and thus becoming more rational in a selfish sense. The findings that are present here are related to this sense of rationality, and not the 'common good' of the system [122]. However, in games other than prisoner's dilemma (for example, in the stag-hunt game), it is found that the average pay-off indeed could increase as the system converges towards the (multiple) Nash equilibria, depending on the actual values of pay-offs for each scenario. Thus, the public good of the system matches with the selfish rationality of players. Therefore, it is important to realise that the results that were obtained are applicable in terms of average selfish rationality of players, which sometimes matches with the common good of the system and sometimes does not. In any case, it is quite conceivable that players would put their relative well being over their absolute well being, since human beings perceive their level of well being primarily by comparing themselves with their local neighbours. In summary, it remains a vital research question of great scientific and practical significance to understand how the cognitive decision making of players and the resultant dynamics in socio ecological systems are shaped by both the topology of such systems and the bounded rationality of actors in such systems.

It is widely known that network topology affects the strategic decision making scenarios of self-interested players [224, 223]. However, game theoretic models have been proposed to model network growth[207], suggesting that network formation can be regarded as a strategic decision that a node makes. This topological rationality model suggests that there could be a simultaneous and cyclic inter-dependency between network topology and strategic decision making of self interested agents, based on heterogeneous bounded rationality. Thus, strategic games maybe more sensitive to the local context than generally assumed, when they operate over a spatially distributed network of players. This is highlighted by identifying such games as network-based games, in comparison to network game models where a two-player game is simply iterated over a network [223]. Thus, understanding how node rationality is affected by network topology may be significant in contextualising an abstract gaming model. There may also be other centrality measures and topological characteristics that may serve as better indications of the rationality of a node in its social context. Further, quantifying the information flow along a link with an opponent may provide a better indication of a node's rationality of that particular opponent, instead of merely depending on the physical topology. More empirical studies are required to confirm the possible correlation between the bounded rationality of players and their network topological placement and properties.

In this work, a topological model for bounded rationality in networked games was proposed. Bounded rationality provides an implicit mechanism for capturing the effect of network topology and information diffusion on networked game dynamics. The next chapter discusses several potential applications of such a bounded rationality model in the domain of computer networks, presented as case studies.

Chapter 7

Applications of topologically distributed bounded rationality in network-based games

The previous chapter introduced a topological model of bounded rationality, which implicitly captured the effect of network topology and information diffusion on networked game dynamics. In this chapter, the topologically distributed bounded rationality (TDBR) model is applied to three different strategic decision-making scenarios in computer networks as case studies.

7.1 Introduction

In strategic decision-making scenarios, different agents or players may demonstrate heterogeneous non-optimal rationality. The rationality of an autonomous agent may depend on the amount of information at hand, cognitive capacity and the computational time available. When a strategic game is played over a spatially distributed network of players, the cognitive capacity and information availability may be reflected in the topological arrangement of the players. Based on this assumption, in a previous chapter, a topological model for quantifying the bounded rationality of strategic players was developed. This chapter discusses the potential applicability of this model using several real-world scenarios in computer science. By analysing the results obtained from these case studies, it is shown that the suggested topologically distributed bounded rationality (TDBR) can be effectively used to improve the accuracy and predictive capacity of Nash equilibrium based game theoretic models[131].

This chapter is organised as follows. The next section, Section 7.2, gives a background of the relevant areas of study, including the proposed topologically distributed bounded rationality model. Then, the following discuss three case studies where a topological model of bounded rationality may be applicable. These case studies include peer-to-peer network formation, network security and network routing-based applications where topologically distributed bounded rationality may be applicable. Further, how the existing game theoretic models on these domains can be extended using a topologically distributed bounded rationality model is examined. Finally, the conclusions based on these case studies are presented.

7.2 Background

Game theory is widely used to study and model strategic decision-making scenarios [28], ranging from politics and market economics to ecosystems and information routing. Nash equilibrium is considered to be the most important theoretical cornerstone of game theory, which predicts the existence one or more equilibrium states in a strategic game from which no player has an incentive to deviate [174]. One of the key underlying assumptions of the Nash equilibrium is that players are fully rational, meaning they are fully aware of opponents' strategies and the respective payoffs [91], and there are not any cognitive or temporal limitations to their decision-making ability. However, most real-world strategic decision making scenarios involve players with non-optimal or bounded rationality, prompting them to deviate their behaviour from that of the Nash equilibrium [97]. The possible limitations, such as the amount of information at hand, cognitive capacity and the computational time available, may force a self-interested autonomous player or agent to have bounded rationality and therefore to make non-optimal decisions [88].

At the same time, studies in psychology and cognitive science suggest that the rationality of individuals may be correlated to the level of their social interactions [21, 75, 46]. In particular, the level of information available about the environment and their cognitive capacity may affect the rationality of an autonomous agent, which may be reflected in the social structure of that agent. There have been numerous attempts to model the non-optimal rationality in strategic games, based on models such as near-rationality and quantal response equilibrium [55, 90, 264, 219]. However, they do not attempt to quantify the heterogeneity of rationality that is prevalent in the populations of real-world players in a predictive manner, based on the physical and observable characteristics of the players. Based on this premise, the previous chapter argued for a topologically distributed bounded rationality model, that attempts to quantify the distribution of rationality of players placed in a heterogeneous network.

One of the criticisms that can be made against the quantal response equilibrium is the usage of the rationality parameter as an arbitrarily set parameter [268]. Quite often, it is used as a model to fit empirical results by varying the rationality parameter [53].

Social cognitive theories [21] and the social brain hypothesis [75, 76] suggest that there is a strong correlation between the cognitive capacity of a player and the amount of social interaction that the player may have. Based on this argument, a topological model of bounded rationality has been proposed [125]. In this topological rationality model, the rationality parameter λ of QRE model is defined as a function of social interactions [125], in order to incorporate the topology of a node as a measure of its bounded rationality, as shown in Eq. 7.1. In a more general context, the cumulative weights of a node's connections to its neighbours quantify the social interactions it may have in the context of the social network. If the weights of the links are all identical and unified, the degree of the node can be used as a relative measure of its social interactions.

$$\lambda_i = r.f(\sum_{j=1}^{n} w_{ij}) \tag{7.1}$$

Here, λ_i is the rationality of node i. r denotes a network rationality parameter that is a property of the network. The higher the network rationality parameter, the greater the sensitivity of the nodes' rationality to the amount of social interactions that they would have. Thus, it is possible to argue that the network rationality parameter provides a measure of the rationality level or the overall network rationality of the entire network of players. The weight w_{ij} denotes the weight of the link connecting node i with each neighbour j, while n is the number of neighbours that node i has. Under this model, a node may behave completely randomly if the network rationality parameter is 0 or when the node is disconnected (i.e. the degree is 0). However, a node may behave according to the Nash equilibrium as the network rationality parameter $r \to \infty$, or when the degree of the node is extremely large. Thus, this model captures the two extremes of rationality and the discrete levels of rationality in between those two, given by the aggregate of link weights of each node. In the case studies considered, unweighted networks (or networks with an identical link weight of unity) are used. This makes it possible to directly capture the topological implications of the strategic interactions of a population of players where the bounded rationality levels are topologically influenced.

This chapter attempts to discuss the potential applications of this model using real-world applications used as case studies. Such case studies are useful in identifying the real-world implications and potential applications of this interpretation, enabling the quantal response equilibrium model to be used as a predictive model, in addition to being used for fitting empirical results with an arbitrarily set rationality parameter. The case studies considered are all related to computer network-based applications. There are abundant real-world network data relating to such applications, making it possible to validate the simulated results. Further, all these applications involve the decision-making of humans or human-computer interaction, making them susceptible to the inherent bounded rationality of humans. In addition, the computer agents that are engaged in these applications too may not have the necessary information, computer power or computing time to make fully rational decisions, thus making them behave as bounded rational players.

7.3 Applications of topologically interpreted bounded rationality

7.3.1 Peer-to-peer network formation

The first case study (application) is the network formation on peer-to-peer overlay routing networks. These networks are formed by self-interested players who want to share resources amongst themselves. These players may be human or software agents and they have limited visibility of the network based on their topological positioning. Peer-to-peer overlay networks make a good candidate to study heterogeneous bounded rationality as it has been observed that different players operate with different levels of rationality. For instance, free-running or free loading behaviour is prevalent in such networks where some players act in an extortionate manner exploiting others while other players contribute towards the common good [213, 37]. In a non-cooperative game theoretic perspective, free-runners could be thought as rational players while contributors could be regarded as irrational players, under the assumption that rationality is based on the tendency to maximise one's own utility. Such heterogeneity of rationality is more prevalent when the network is formed by human players and could even be relevant to autonomous agents due to the limitation of information that they might have about the overall network [269]. This heterogeneity of rationality may be captured by a topological distribution of bounded rationality, similar to the one discussed in the previous chapter.

Methodology

In order to test how a topologically distributed bounded rationality would affect the formation of a peer-to-peer overlay network, an existing game-theoretic overlay network construction model [57] that defines a network construction game is used. In this particular network construction model, each node is assumed to be running the link state (LS) protocol, where each node periodically performs link addition and link dropping. The total cost of a node being part of the network is a function of cost paid for maintaining links and the distances from the node in concern to the other nodes in the overlay network. The strategy adopted by a node is the subset of other nodes in the network that the node chooses to connect to. The Eq.A.6 calculates the cost incurred by a particular node by being part of the overlay network.

$$C_i(s) = \alpha \sum_{j \epsilon NB_i} t_j + \sum_{j=0}^{n-1} d_{G[s]}(i,j) \tag{7.2}$$

Here, NB_i is the set of neighbours of node i, t_j is the cost incurred to connect to node j and $d_{G[s]}(i,j)$ is the distance from node i to node j in the overlay network $G[s]$. The distance between two nodes is calculated by measuring the shortest path between them in the overlay and then adding the distances of the intermediate links along the underlying base network. Here, α can be regarded as the relation between the cost of establishing a link and the change in distance to other nodes caused by the addition of that link. Also, the cost can be regarded as the inverse of the pay-off obtained by a node. The higher the cost, the lower the pay-off. The algorithms that define the link addition and dropping protocols [57] under perfect rationality are described under Algorithms 7 and 8.

Algorithm 7: Link Addition for node i

```
Randomly select node j not in the neighborhood of i;
Compute Cost_new with j included;
if Cost_old − Cost_new > 0 then
    Add the link;
```

Algorithm 8: Link Dropping for node i

```
NodeToDrop = -1;
Mincost = Cost_old;
forall the node j in the neighbourhood of i do
    Compute Cost_new without j;
    if MinCost − Cost_new > 0 then
        MinCost = Cost_new;
        NodeToDrop = j;
    if MinCost − Cost_new > 0 then
        Drop the link between i and NodeToDrop.
```

The resulting network that evolves based on the assumption of perfect rationality could be regarded as the Nash equilibrium solution [57] under the network construction game. The decision to create a link or not is a pure strategy (with probability 1 or 0) and these algorithms assume that all nodes operate with perfect rationality, suggesting that each node possesses the cognitive capacity, information and the computational time to calculate all the costs (and therefore payoffs) under each strategy. However, in a real-world population of players that form a peer-to-peer overlay network, this may not be the case. The players would have varying bounded rationality, possibly making them operate in a non-optimal manner. Thus, the above algorithms could be modified by introducing a topologically distributed bounded rationality in making the decision to create a link or to drop a link.

Here, the opponent of each node can be regarded as the overlay network itself, as the creation and dropping of links affects the network and the respective costs and payoffs would be derived from the network that is formed. The modified link addition and link dropping protocols, based on a topologically distributed bounded rationality parameter in a quantal response equilibrium is given in Algorithms 9 and 10.

Algorithm 9: Link addition for node i under bounded rationality and QRE.

Randomly select node j not in the neighborhood of i;
Compute $Cost_{new}$ with j included;
Payoff = $Cost_{old} - Cost_{new}$;
Compute the rationality parameter λ_i based on node degree;
Compute the probability of creating the link p_a using λ and QRE;
Generate a random probability p;
if $p < pa$ **then**
    Add the link;

Algorithm 10: Link Dropping for node i under bounded rationality and QRE.

$NodeToDrop$ = -1;
$Mincost = Cost_{old}$;
forall the *node* j *in the neighbourhood of* i **do**
    Compute $Cost_{new}$ without j;
    Payoff = $Mincost - Cost_{new}$;
    Compute the rationality parameter λ_i based on node degree;
    Compute the probability of dropping the link p_d using λ and QRE;
    Generate a random probability p;
    if $p < p_d$ **then**
        $MinCost = Cost_{new}$;
        $NodeToDrop = j$;
    if $MinCost - Cost_{new} > 0$ **then**
        Drop the link between i and $NodeToDrop$.

To add a link in the modified algorithm 9, the probability p_a is calculated by using the equation Eq.A.7. The probability of dropping a link used in Algorithm 10 is calculated using the Equation A.8. These probabilities depend on the rationality of each node, thus the hubs have made more rational decisions in adding and dropping links in comparison to leaf nodes. Note that the opponent of each node, that is the node at the receiving end is assumed be in an always connected state, and the decision to create or drop a link is only made by the active node of the link in that particular instance.

$$p_a = \frac{e^{\lambda_i . P}}{e^{\lambda_i . P} + e^{\lambda_i . 0}} \tag{7.3}$$

Here, p_a is the probability of adding a new link. λ_i is the bounded rationality of node i, which is measured using a convex function of the node degree with the network rationality parameter set to 0.01. P is the payoff of creating the link, which is the difference between $Cost_{new}$ and $Cost_{old}$. The node would thus obtain a payoff of P if the link is created and a payoff of 0 if not. This simple subgame with an incorporation of a rationality parameter could be used to model a node with non-optimal bounded rationality.

$$p_d = \frac{e^{\lambda_i . P}}{e^{\lambda_i . P} + e^{\lambda_i . 0}} \tag{7.4}$$

In Eq.7.4, p_d is the probability of dropping a new link. λ_i is the bounded rationality of node i, which is measured based on a convex function of the node degree with the network rationality parameter set to 0.01. P is the payoff of dropping the link, which is the difference between $Cost_{new}$ and $MinCost$. Thus, P could be regarded as the expected payoff of dropping the link in concern, in comparison to the payoff 0 of keeping the link.

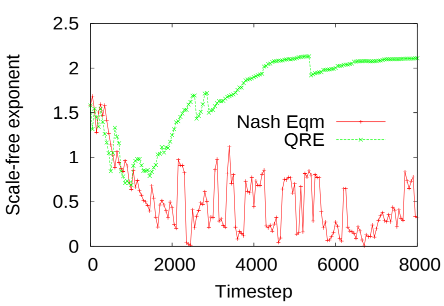
Using these two sets of algorithms and corresponding equations, peer-to-peer overlay networks were generated using perfectly rational nodes (Nash equilibrium) and bounded rational nodes whose rationality levels are topologically distributed. Next, the topological properties in each class of networks were compared, particularly the scale-free exponent and the R-squared correlation to the power-law degree distribution. Real-world peer-to-peer overlay network topologies such as Gnutella were observed to show scale-free topology [227]. By comparing the topologies of each of the generated networks, it is possible to compare each network construction game in its ability to generate networks that map closer to the real-world overlay networks.

In the experiments conducted, scale free networks of 500 nodes served as the underlying base networks. Since the Internet has been observed to be scale-free in nature [9], using scale-free networks as the underlying networks is justifiable. Next, an overlay network was generated randomly over the physical base network. Then, the link-state algorithms given above were iterated over time under both the Nash equilibrium and QRE algorithms. The resulting networks were compared on their scale-freeness. As previously mentioned, the parameter α used in the cost function Eq.A.6 is an indication of the relationship between the cost of creating a link and the change in distance to other nodes that occurs due to the creation of that link. For instance, if $\alpha \leq 1$, that means that it is always more beneficial to create a link than having to traverse at least two nodes to reach a non-neighbouring node. If α is significantly large, a link addition would only happen if it substantially reduces the distances to the other nodes in the overlay network. By varying α, it is possible to construct different overlay topologies. Networks generated using three different α values were compared. This makes it possible to observe how the perfectly rational and bounded rational nodes would construct overlay networks under varying cost-benefit ratios of creating links.

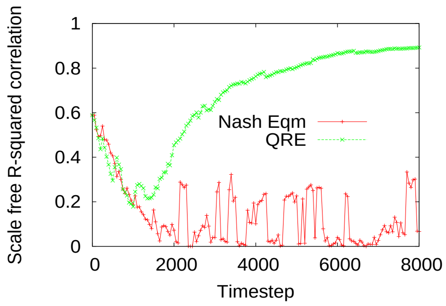
Results and Discussion

Figure A.2 depicts the comparison of networks formed as a result of nodes operating at Nash equilibrium and topologically distributed rationality induced quantal response equilibrium, respectively. The two parameters compared are the scale-free exponent and the R-squared correlation to the respective power-law degree distribution. The scale-free exponent is observed to be between 2 and 3 in most real-world scale-free networks, including peer-to-peer overlay networks [9, 227]. The correlation gives an indication on the proximity of the network to an actual scale-free network with a power-law degree distribution. Three α values, 0.6, 1.5 and 10, are considered in network generations to observe how the topologies vary under varying cost-benefit ratios of link creation.

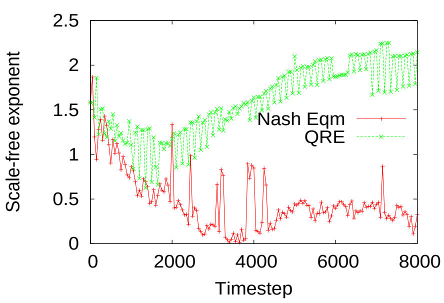
As evident from Fig.A.2, the networks that result from quantal response equilibrium with topological distributed bounded rationality demonstrate scale-free topological features, reminiscent of real-world peer-to-peer overlay networks [227, 9]. The higher R-squared correlation in QRE-based networks indicates that they fit better with a power-law degree distribution. The relatively higher scale-free exponent is also characteristic of real-world



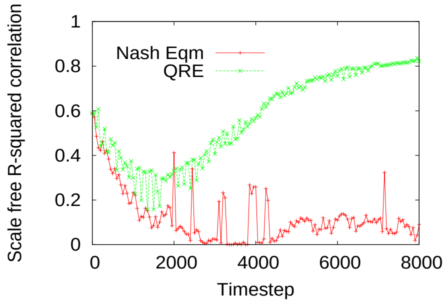
(a) Scale-free exponent, $\alpha = 0.6$



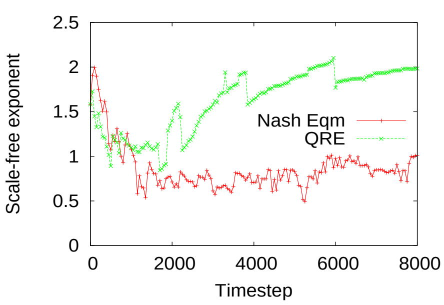
(b) R-squared correlation, $\alpha = 0.6$



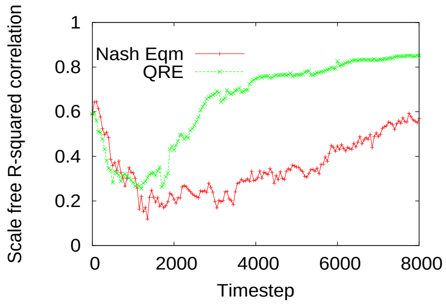
(c) Scale-free exponent, $\alpha = 1.5$



(d) R-squared correlation, $\alpha = 1.5$



(e) Scale-free exponent, $\alpha = 10$



(f) R-squared correlation, $\alpha = 10$

Figure 7.1: The evolution of the scale-free exponent and the R-squared correlation of the Nash equilibrium and the QRE networks with topologically distributed bounded rationality. The α value was set to 0.5, 1.6 and 10, respectively.

peer-to-peer overlay networks. It has been observed that for very large α values, the resulting Nash equilibrium network may show scale-free characteristics [57]. However, in real-world peer-to-peer networks, it is unlikely that the cost of creating a link would be substantially large when compared to the reduction of distance to the other nodes [54]. The results suggest that the QRE-based network may show scale-free characteristics for lower α values as well, suggesting that a topologically interpreted bounded rationality, along with QRE, could be used to model network formation among peers more accurately than Nash equilibrium based-network formation games where players are assumed to have perfect rationality. On the other hand, this application and the results could be used as evidence of the validity of a topological distribution of bounded rationality in a networked populations of strategic players.

7.3.2 Protection against security threats in networks

In the second case study, a scenario is considered where game theory is used to model the level of security of the nodes in a computer network. Setting the security level of a particular network agent is a compromise between the amount of resources that can be allocated for providing security and the level of security desired [132]. Moreover, there could be a compromise between computational and data transfer time and the security level provided [132]. Since these are often conflicting interests, game theory could be effectively used to model to identify the security level set at each node within the network.

Protection against security threats has previously been interpreted in a game-theoretic setting [55]. In this case study, attention is paid to how the nodes of a network would setup their security against potential Distributed Denial of Service (DDoS) attacks. A DDoS attack initiates with an attacker looking for vulnerable machines to get hold of and subsequently use to launch a larger scale attack. The vulnerability of nodes to be susceptible to such invasion would depend on the security level set at each node. Even though the attack is automated, the setting up of security is often a decision that is made with human cognition and experience, which makes it a decision made with bounded rationality.

Consider network of size n, where each user is vulnerable against the initial stage of a DDoS attack. The level of computer security adopted by each user is denoted by s_i. The nodes that would be compromised would be those with the lowest security level $min(s_i) = s_{min}$. An assumption is made such that the cost born by each user i to implement his security policy is a monotonically increasing function of s_i. All compromised users would incur a fixed cost of $P \geq s_i$, irrespective of the minimum security level s_{min}. It is assumed that users probe the security levels of other users and adjust their security levels accordingly.

While this representation of a security model against the initial stage of a DDoS attack is fairly simple, it has been used in modelling similar scenarios [55]. It can also be used to accurately model the DDoS attacks that have been carried out in real networks [55]. The network size does not play a significant role in this model, unlike in real-world attacks.

Thus, this model would be more appropriate for networks of reasonably small size, such as corporate and university networks.

The Nash equilibrium derived for this game occurs when all users within the network choose an identical security level $s_i = P$. The proof of this can be given as follows [55].

Suppose users $1, .., k$ such that $1 \geq k \leq n$ apply a security level of $s_{min} \leq s_i$ for all $i \epsilon i, ..., n$. Therefore, each user $i \epsilon 1, .., k$ is compromised, thus their utility would be $u_i = -P$.

Now suppose user i in 1,....,k increases their security level to $s_i = s_{min} + h$ where $h \geq 0$. Then, user i's utility would be $-s_{min} - h$. Since the original distribution of security levels form a Nash equilibrium, any change in strategy should decrease the utility of player i for any $h \geq 0$, which brings out the inequality:

$$- s_{min} - h \leq -P \tag{7.5}$$

This can be reduced to $s_{min} \geq P$ by continuity. However, it was originally hypothesised that $s_{min} \leq P$. For both these inequalities to hold s_{min} should be equal to P. Since for any i, $s_{min} \leq s_i \leq P$, suggesting that s_i has to be P. Thus, the Nash equilibrium occurs when $s_i = P$ for all i users. Therefore, for this particular network security game, the Nash equilibrium occurs when all users have identicial security level of P and thus the identical utility of $-P$. A more detailed explanation of this proof has been presented by Christin et al. [55].

Although the Nash equilibrium predicts that all users within a network would have an identical security level, real-world network security systems operate otherwise [249, 89]. Security levels tend to follow a heterogeneous distribution within a network of users. Topological bounded rationality may be used as a means to account for this heterogeneity.

In order to incorporate bounded rationality to this game setting, this game is extended so each user decides whether to apply a security level or not. Accordingly, the opponent of each potential player is the network itself. If a user does not have any security applied, $s_i = 0$ and thus the payoff u_i would be 0. The players who engage in the security game probe others and adjust their security levels accordingly. Thus, such players would have the identical utility of $u_i = U$ as other users who participate in the network security game. If $U > 0$, again the Nash equilibrium would occur when all players have identical security level of U. Now, suppose that each user would make this decision based on a rationality level λ_i, which is proportional to the degree d_i of that user. Thus, the probability of a player being part of the security game is:

$$p_i = \frac{e^{\lambda_i . U}}{e^{\lambda_i . U} + e^{\lambda_i . 0}} \tag{7.6}$$

where, $\lambda_i = f(d_i)$, where f is a monotonically increasing function of user i's degree, and $0 \geq \lambda_i \leq \infty$. When the user is not part of the network, they would still have the probability of 0.5 of participating in the network game and earning a utility of $U/2$. However, when the node degree increases, the probability of participation would reach 1, with an expected utility of U. Thus, this model suggests that the users have a utility of $u_i \epsilon [U/2, U)$ and thus the security level s_i distributed in the range $s_i \epsilon [-U/2, -U)$. Therefore, with a topological interpretation of bounded rationality, it could be argued that the heterogeneity of security levels in real-world networks could be accounted for. Moreover, it is common that the servers that are highly connected are more secure than individual workstations that may not have high level of security [146]. Similarly, a topological distribution of bounded rationality suggests that the highly connected users in a network are more likely to have a higher level of security than less connected users.

7.3.3 Routing in a peer-to-peer overlay network

The next application that is considered is routing in a peer-to-peer overlay network. As in the case of peer-to-peer network formation, the peers would have non-perfect and heterogeneous rationality in a peer-to-peer routing scenario. This is evident from the existence of free loading or free-riding in peer-to-peer overlay networks [37]. In a peer-to-peer resource-sharing network, each node relies on other nodes to forward its requests, and in turn it is expected to forward the requests sent by other nodes [37]. However, the self-interested nodes may refuse to forward requests in order to conserve local bandwidth.

It is suggested that incentive mechanisms and reputations systems can be used to facilitate cooperation as a robust and subgame-perfect equilibrium in a network of self-interested players. However, in real-world peer-to-peer networks, collaboration seems to sustain as a strategy even without any particular incentive or reputation mechanisms in place [208].

A variation of the random matching game is adopted as a means of modelling peer-to-peer routing. This game has been previously used to model peer-to-peer routing in the literature [37]. In each round, players are randomly matched, and then each pair plays a single round of the prisoner's dilemma game [216]. Since the prisoner's dilemma game has been widely used to model the behaviour of self-interested agents, this model could be used to model peer-to-peer networks that consists of self-interested players. Based on the same premise, it is possible to define a peer-to-peer routing game. When each request is propagated, the intermediate nodes would play a single instance of the prisoner's dilemma game with the original node that generated the request.

Once a request originates from the node s, the request is forwarded to the immediate next node in the routing path, then a single instance of the prisoner's dilemma game is played between those two nodes. If the second node cooperates, that is if it forwards a request, it gets a pay-off of -2 while it gets a pay-off of 0 if it defects and ignores the request. Node s gets a pay-off of 0 in both instances. If the request is forwarded, then game is repeated with s and the next node in the routing path until the destination node is reached. Thus, if each routing node is at Nash equilibrium, it would always choose to defect as it gives a higher pay-off of 0. This creates the scenario where there is 'tragedy of the commons', where the collaborative environment cannot function at all due to all players behaving in a self-interested manner [189]. However, this is contrary to observed peer-to-peer routing networks where collaboration is sustained even without incentives or a reputation system in place.

Next, this game is further modified by introducing topologically distributed bounded rationality and QRE. The non-zero probability of an intermediate node deciding to forward a request is given by:

$$p_f = \frac{e^{-2.\lambda_n}}{e^{-2.\lambda_n} + e^{0.\lambda_n}} \tag{7.7}$$

where p_f is the probability of forwarding the request and λ_n is the rationality of node n. With this probability distribution, a purely non-rational node with $\lambda_n = 0$ would forward a request with 0.5 probability while a fully rational node, where $\lambda_n -> \infty$, would never cater for another's request. If the rationality is topologically interpreted, this model would suggest that the probability of forwarding a request is distributed in a probability distribution of (0:1/2], where hubs would have a higher tendency of not forwarding incoming requests than leaf nodes.

Methodology

In order to test the implications of a topologically influenced bounded rationality on the peer-to-peer routing game described above, the following experiment was devised. The random matching routing game was run on a scale-free network of nodes by generating broadcast requests from each node. Each node sends requests to all other nodes within the network along the shortest paths. An assumption was made that nodes have the necessary routing information contained in them, in order to forward the requests. The fraction of such requests generated from all the nodes in the network that successfully made it to the destination, was measured. The rationality was calculated based on a convex function of node degree. The network rationality parameter was varied to observe how the routing game responds to the overall rationality level of the network. Then, the results obtained were compared with those of a structured lattice network with comparative link-to-node ratio and a real-world Gnutella peer-to-peer overlay network.

Results and discussion

The Figure A.3 depicts the results obtained by comparing the lattice and scale-free networks on their ability to facilitate peer-to-peer routing under varying network rationality conditions. The network rationality parameter is shown in logarithmic scale. As the figure shows, the scale-free topology facilitates a considerable fraction of requests under a topologically influenced bounded rationality and over a wide range of network rationality. On the other hand, lattice topology allows a significantly smaller fraction of messages to be routed, due to its homogeneous and low average degree and the higher average path length. Most real-world peer-to-peer overlay networks show scale-free topology [227]. Thus, this result may indicate that by being distributed in a scale-free topology, overlay networks obtain the ability to sustain message routing, provided that the nodes have heterogeneous and topologically distributed rationality levels. The real-world Gnutella network, which also shows scale-free characteristics, facilitates message forwarding over a significant range of network rationality. The scale-topology goes through a sudden transition in the fraction of requests allowed when the network rationality parameter increases beyond a particular point. Both structured and non-structured topologies are applied in peer-to-peer overlay networks [148]. These results suggest that if the rationality of agents is topologically distributed, non-structured topologies such as scale-free topologies may be more robust against free-riding behaviour. This may be due to the heterogeneity of rationality that they would facilitate and the lower average path lengths of that they would encompass, limiting the number of iterations in the random matching game.

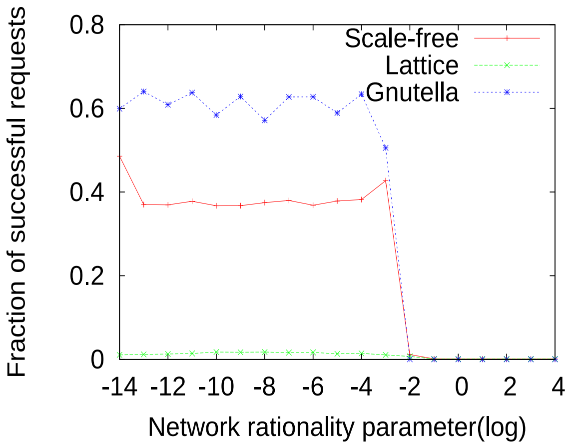


Figure 7.2: The variation of the fraction of overlay requests that are successfully routed against the network rationality parameter. The results of scale-free and lattice topologies are compared with that of a Gnutella network.

7.4 Discussion

Nash equilibrium predicts that there exists a unique equilibrium or multiple equilibria in a strategic decision-making scenario from which no player would benefit deviating. Quantal response equilibrium is a generalisation of Nash equilibrium where the rationality of a player is taken into account to accommodate for the errors made in making decisions. QRE encapsulates a rationality parameter that can be used to manipulate the level of rationality of a given player.

Combining the above two theoretical frameworks, it has been suggested that the rationality parameter could be regarded as being proportional to the amount of social interactions a node may have. Thus, in a weighted network, the rationality of a node would be proportional to the sum of weights of links connected to that node, while in an unweighted network, it could regarded to be proportional to the node degree. In this work, only the unweighted networks are considered. According to this model, the sensitivity of rationality to network topology is denoted by a network rationality parameter r.

In order to validate and demonstrate the applicability of a topologically distributed rationality model, three different real-world case studies were considered. The first case study was peer-to-peer network formation. A game theoretic network formation algorithm that operates with the assumption of Nash equilibrium was adopted. Next, it was extended using QRE and topologically distributed bounded rationality. The resulting network properties demonstrate that the QRE-based network is closer to the scale-free nature of the real-world peer-to-peer network formations. This suggests that the topologically distributed rationality model is useful in modelling peer-to-peer network formation more accurately. This is presented as evidence of the validity and applicability of the topologically distributed bounded rationality model discussed in the previous chapter.

In the second case study, the topologically distributed bounded rationality model was applied to a scenario where game theory is used to model the security levels of nodes in a network. Under a topologically distributed bounded rationality model, it can be shown that the highly connected nodes or hubs have more stringent security policies than the leaf nodes, reminiscent of the real-world networks. In comparison, the Nash equilibrium solution suggests that each node in a network would have identical security level, which is not the case in real-world networks.

In the final case-study, network routing was simulated using the randomly matched prisoner's dilemma game. Accordingly, it was demonstrated that, under topologically distributed bounded rationality, a scale-free network would facilitate the exchange of messages over a broad range of network rationality levels better than a well-mixed population. The results suggest that a topological bounded rationality could be useful in developing game theoretic models that would represent real-world scenarios more accurately than models that assume perfect rationality. Thus, a topologically distributed rationality model may be a useful tool in modelling real-world strategic decision making scenarios among populations of players. More real-world applications and empirical studies are essential to explore the applicability of the topologically distributed bounded rationality model.

In the previous chapter, we proposed a topologically distributed bounded rationality model. In this chapter, we discussed the potential applications of the topologically distributed bounded rationality model. The next chapter proposes an information transfer-based bounded rationality model as a more refined and dynamic bounded rationality model, where the bounded rationality of a node with respect to a given interaction is assumed to be proportional to the incoming information flow.

Chapter 8

Information theoretical interpretation of bounded rationality in network-based games

The previous two chapters elaborated on a topologically distributed bounded rationality model, where the bounded rationality implicitly captures the network topology as a vital aspect in determining the output of strategic interactions. In this chapter, this model is extended to define the bounded rationality of a node with respect to a particular strategic interaction as being proportional to the directed incoming information flow among each pair of players.

8.1 Introduction

The bounded rationality of a player is dependent on the level of information available, the amount of computational time available and the cognitive capacity. Thus, if all players have uniform cognitive capacity and computational time, then it is the amount of information available that determines the bounded rationality of a player. Previous chapters focused on a bounded rationality model that used the spatial arrangement of nodes as a basis for deducing the rationality of nodes. While that approach can be justified by approximating the social interaction of nodes based on their topological characteristics, such as the node degree, it is still a static property of nodes for a particular topological structure. However, what actually matters in determining the rationality of a node with respect to a particular opponent in a strategic interaction is the incoming information flow from that opponent. Thus, it is more accurate to take into account the incoming directed information flow rather than considering the static topology of the network. This would mean that the rationality of nodes would not only depend on the spatial dimension but also the temporal dimension. Thus, an information flow based rationality could be considered a dynamic rationality rather than a static rationality. In this chapter, an attempt is made to model the bounded rationality of players using information theoretic measures. As a measure of bounded rationality, an information theoretic measure called transfer entropy [143] is used. Transfer entropy is used to measure the directed information flow between two processes; thus, it makes an ideal candidate to quantify the bounded rationality arising from the directed information flow from a potential opponent, within a strategic interaction. In this work, random boolean networks [74] are used as the underlying information network to quantify the transfer entropy. They allow the generation of state sequences for each node in the network, which can then be used to measure the directed information flows among interacting nodes. Information flows can be thought of as a more refined form of network topology, where the actual interconnections among nodes are represented by directed information exchanges.

8.2 Background

8.2.1 Information transfer and transfer entropy

The information transfer between a source and a destination is defined as the information provided by the source about the destination's next state that was not contained in the destination's own past. The information transfer is measured by transfer entropy [143] and it is mainly proposed to address the issue of symmetry in the measure of mutual information. Transfer entropy is a directed measure that takes into account the directed information flow from a source to a destination. The transfer entropy from a source Y to destination X is defined as the average mutual information between the previous state of the source y_n and the next state of the destination x_{n+1}, conditioned on the semi-infinite past of the destination x_n.

$$T_{Y \to X} = \lim_{k \to \infty} \left\langle log_2 \frac{p(x_{n+1}|x_n^k, y_n)}{p(x_{n+1}|x_n^k)} \right\rangle \tag{8.1}$$

This formulation is also known as the apparent transfer entropy. Apparent transfer entropy measures the effect of information transfer from a single source only. If the information transfer is affected by the interaction of multiple sources, it does not account for that cumulative effect. Within the scope of this work, the focus is placed on the apparent transfer entropy as it is the information transfer from a single source to a destination is considered in quantifying the rationality level.

8.2.2 Random boolean networks

Random boolean networks (RBNs) were originally developed as genetic regulatory networks [74]. They are also known as Kauffman networks. Random boolean networks are generic since they are not bound by any particular functionality or connectivity of the nodes that are contained in them. They can be used to model the fundamentals of living systems in an integrated and holistic manner. RBNs can be considered as generalisations of boolean cellular automata (CA), where the state of each node is affected by all the other nodes in the network and not necessarily by its immediate neighbours.

The classical RBN (CRBN) model proposed by Kauffman suggests that living organisms could be constructed from interconnected random elements without using specifically programmed elements. An RBN consists of N nodes with either zero or one state and each node would have average K number of connections. The connections are wired randomly. These connections do not change within the phase where the states of the nodes are dynamically changed. The states are updated synchronously, where the states at t+1 depend on the states of the nodes at t.

The random network is initialised with a random state. Then the states are dynamically changed until a stable state is reached. The state space of finite value 2^N, making the system repeat a state eventually. At that state, the RBN is said to have reached an attractor. If the attractor is of a single state it is called a point attractor and if the attractor it consists of two or more states, it is called an attractor cycle.

For a given RBN, each node would have 2^{2^k} possible boolean functions. Also, each node would have possible $\left(\frac{N!}{(N-K)!}\right)^N$ combinations for K different links. Therefore, all possible networks for given N nodes with average degree K would be [86]:

$$\left(\frac{2^{2^k} N!}{(N-K)!}\right)^N \tag{8.2}$$

At the inception of the RBN, the nodes are initialised with a probability r of having the state 1. This probability is called the bias of the RBN. If r is closer to 1 or 0, the network has low activity and if r is closer to 0.5, the network has high activity.

8.2.3 Phase transitions in random boolean networks

RBNs have been observed to have three clear types of dynamics: ordered, chaotic and critical. At relatively low connectivity and low activity, the network is at the ordered phase, while at relatively high connectivity and high activity, the network is at the chaotic phase. At the ordered phase, the network demonstrates high stability towards perturbations and strongly converges towards similar macro states in state space. At the chaotic phase, the network demonstrates low stability of states to perturbations and displays divergence of similar macro states. At the critical phase [142], the network displays uncertainty in the convergence or divergence of similar macro states and there is a percolation of nodes remaining static and updating their states.

In order to quantify the phase transitions in the RBNs, a normalised Hamming distance can be used. Suppose a random initial state A of the network is considered, and then the value of a single node is inverted to produce B. Afterwards, if both A and B are run for many time steps, the normalised Hamming distance of the two networks at would be:

$$D(A,B) = \frac{1}{N}\sum_{i=1}^{N}|a_i - b_i| \tag{8.3}$$

The normalised Hamming distance between their initial and final states could be used to obtain the convergence/divergence parameter δ:

$$\delta = D(A,B)_{t\to\infty} - D(A,B)_{t=0} \tag{8.4}$$

Here, $\delta < 0$ suggests the convergence of similar initial states, and $\delta > 0$ implies the divergence of similar initial states. For infinitely large networks, when r is fixed the critical average degree K_c is given by:

$$K_c = \frac{1}{2r(1-r)} \tag{8.5}$$

Accordingly, when r is set to 0.5, the phase transition would occur at the average degree $K = 2.0$. Thus, the phase transition in state dynamics can be observed when the topology is held constant by fixing K (for $K > 2$) and by altering the activity level. Instead, the average degree can be held constant and the activity level can be altered to obtain a phase transition. In the experiments carried out in this work, both the average degree $\overline{K}$ and the bias parameter r were varied to obtain RBNs of varying topological characteristics and activity levels.

8.3 Quantifying bounded rationality as an information theoretic measure

The bounded rationality of players occurs due to the limitation of information that they have on the actions of other players. Provided that the computational time available and the cognitive capacity of players are homogeneous, it is the access to information about the environment that determines the rationality of players.

On the other hand, the information transfer and storage measures used in information theory try to quantify the information exchange and information contained by a particular player. In this work, an attempt is made to utilise this quantification of information as a measure of rationality in a strategic decision-making environment. As the source for quantifying information, random functions are used in the random boolean networks. Since these networks contain varying topological structures, this approach makes it possible to observe the effect of topology on the network games via the information transfer that happens on top of the network structure.

Network topology has previously been used to model the bounded rationality of nodes [125]. Accordingly, if a particular node has a higher degree or connectivity, it is assumed to have a higher bounded rationality and vice versa. Justification for this interpretation can be found in theories such as the social cognitive theory [21]. However, even though a particular node may have a higher degree, what actually matters is the amount of information that flows along those interconnections. Using a social network example, the frequency of interaction of two individuals may vary and the amount of information exchanged would vary accordingly. Thus, using the quantified information flow as a measure of rationality is more viable than using a topological measure. The network topology is the physical network while the actual information flow is the logical network on which the flow of information may occur. By considering the actual information flow on top of the physical network, it is possible to derive a more accurate measure of the bounded rationality of the players in the network.

Another analogy that could be used to explain the difference between the network topology and the information flow are the road and traffic networks. The road network is analogous to the physical network topology that connects the nodes, while the traffic network is the traffic flow that actually operates on top of that physical network. The amount of traffic information contained at a particular junction or a node can only be accurately measured by quantifying the traffic flow, not just be measuring the number of roads that are connected to that particular node. Certain roads may have a lot of traffic while others contain a relatively low amount of traffic. Thus, to accurately measure the amount of traffic-related information and to make traffic-routing decisions based on that traffic information, each node in the road network would actually have to base its rationality on the traffic flow. Following from this analogy, it is possible to further justify quantifying the bounded rationality of a node in a network game on the information transfer and the information content contained in a node, rather than based on the number of physical interconnections that a node may have.

8.3.1 Quantifying bounded rationality based on transfer entropy as a link parameter

In this work, transfer entropy is utilised in two different approaches to quantify rationality. The first approach is to consider the information transfer from specific neighbours. In that approach, the rationality of each neighbour is quantified as a link-related parameter. With this approach, it is possible to evaluate whether it would be effective for a player to consider the rationality of opponents at an individual level and not apply a common rationality level for all the opponents. This could be used to observe whether it is the rationality towards individuals opponents or the common rationality towards all opponents that drives the interactions towards a Nash equilibrium. This approach of quantifying bounded rationality could be considered as a specific rationality approach, where the bounded rationality is specific for each an every opponent of a particular node.

8.3.2 Quantifying bounded rationality based on transfer entropy as a node parameter

The other approach of quantifying rationality that is considered here is averaging the incoming information flows towards a node. In a social network perspective, this would be analogous to a player considering all of the information that they gather from their neighbours and then summarising it to decide on an overall rationality that may be used in interactions with all of the players. Thus, the bounded rationality obtained from this approach is a general bounded rationality that is common to all neighbours of a particular player. By comparing these two approaches of quantifying bounded rationality on information transfers, it is possible to deduce whether it is more beneficial for a player to have specific bounded rationality towards all opponents or whether it is more beneficial to have a general rationality by considering all incoming information from all opponents.

8.3.3 Transfer entropy from direct neighbours

In the experiments conducted in this work, the information transfer obtained only from the direct neighbours was considered. However, in a social network, information flow can occur from any of the nodes within the network. Nodes that are multiple hops away may contribute to the decision-making of a node. Even though the impact of the direct neighbours may be more significant, the contribution of the indirect neighbours may also still be significant.

8.3.4 Transfer entropy from indirect neighbours

One of the advantages of having an information theoretic approach to quantifying bounded rationality is that information transfers can be measured between any two nodes in a network that are not necessarily physically connected. Thus, it is possible to easily extend the neighbour-based bounded rationality calculation model to encompass the multilevel neighbours until the very edge of the network is reached. This approach could be used to predict the strategic interactions of nodes that are not currently linked with each other.

8.3.5 Quantifying bounded rationality based on transfer entropy from multi-hop neighbours

Theoretically, it is possible to consider the information flow from all neighbours within the network to a particular node in concern in order to determine its rationality from information flow. If the rationality of a particular node is R_n, and if the information transfer from a particular node i to node n is depicted by $T(i \rightarrow n)$ and the topological distance from node i to node n is $d_{i,n}$:

$$R_n = \sum_{i}^{N} f(T(i \rightarrow n), d_{i,n}) \tag{8.6}$$

Thus, the accumulation of the rationality contribution from each neighbour within the network provides the cumulative rationality derived from the information flow from each node within the network. The scenario that is considered in this work can be regarded as a special instance of this model where $d_{i,n} = 1$ and the function $f = 1/deg(n).T(i \rightarrow n)$, where the information flows of each immediate neighbour are averaged over the number of neighbours.

8.4 Using information theoretic measures to predict the equilibria of potential interactions

One of the key advantages of using the information transfer as a rationality metric is that it can be used to infer the rationality of an interaction that has not already happened. If the network is not static and is growing, there may be new interactions that are taking place among nodes that are not already connected with each other. Suppose the information transfer measured using transfer entropy from a node i to node n is is denoted by $T_{i \rightarrow n}$, the rationality of the potential interaction could be quantified as $R_{i \rightarrow n} = f(T_{i \rightarrow n})$, even though these two nodes are not physically connected. Thus, for an external observer, it is possible to predict the interaction that could occur between node n and node i before the actual link is established using the quantal response equilibrium model. Thus, this is another potential advantage of deriving the bounded rationality of a node with respect to a potential opponent, based on the information flow from the opponent to the node in concern.

In the following section, the methodology that was used to test the effect of network topology on the local information dynamics and thereby the equilibria of the interactions is described.

8.5 Methodology

In this work, random boolean networks are used as the basis for generating information flows and measuring information flows. For the experiments conducted in this research, networks with two hundred and fifty nodes were used. Two hundred and fifty networks were generated to normalise the effect of randomisation. Since the objective is to observe how the topological changes affect the information flows and therein the bounded rationality of the nodes, the average degree of the networks was varied from the ordered phase to the chaotic phase. The directed information flow derived based on the transfer entropy normalised over the average transfer entropy of the network was used as the rationality parameter.

Next, the equilibrium of the prisoner's dilemma game along each link was measured using the quantal response equilibrium model. The divergence of each interaction from Nash equilibrium was measured using the Jensen-Shannon divergence measure. By measuring the average divergence, it is possible to compare networks on their deviation from the optimal behaviour shown in the Nash equilibrium. Thus, this approach helps to compare and contrast the divergence from the Nash equilibrium at each phase; namely, the ordered, critical and chaotic phases of random boolean networks. Since random boolean networks are used to model living organisms, it is possible to infer the optimality of the strategic interactions that may occur at different phases of complex systems.

Then, the divergence values obtained from the link based rationality and node based rationality was compared at each phase in the random boolean networks. This helps to determine whether it is the information transfer along the links or averaged over nodes that would be more effective in optimising the outcome of a game.

The payoff values of the prisoner's dilemma game were set to T = 5, R = 3, P = 1 and S = 0, respectively. When the rationality parameter is gradually increased, the probability of cooperation decreases from 0.5 to 0, reaching Nash equilibrium. Figure 8.1 depicts the variation of the probability of cooperation when the rationality parameter is increased. As shown in the figure, the rationality value range of [0:5] adequately captures this variation of rationality.

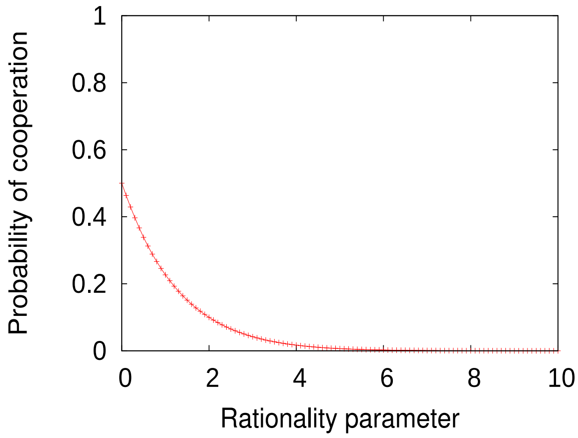
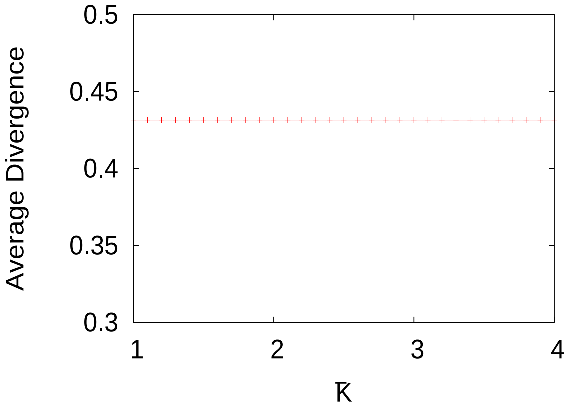


Figure 8.1: Variation of divergence based on the average degree and the bias parameter.

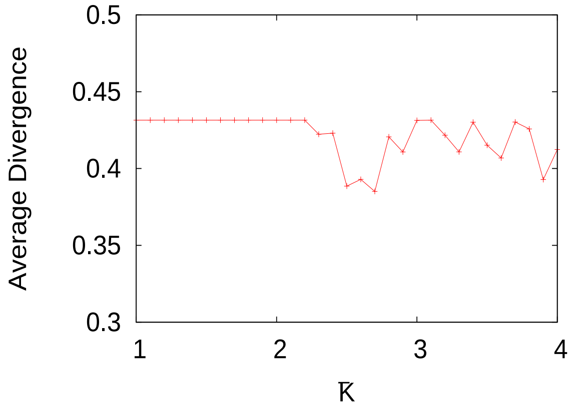
Thus, the transfer entropy values are rescaled to the range of 0 to 5 to account for the variation of players from complete random behaviour to completely rational behaviour.

8.6 Results

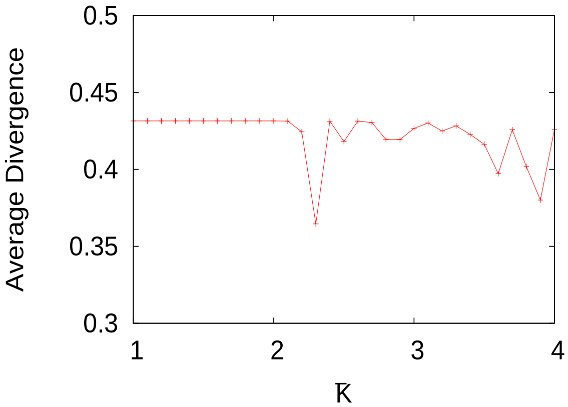
Fig. 8.2 depicts the variation of the average divergence from the Nash equilibrium in each interaction, when the average degree $\overline{K}$ and the bias parameter of the random boolean networks are changed.



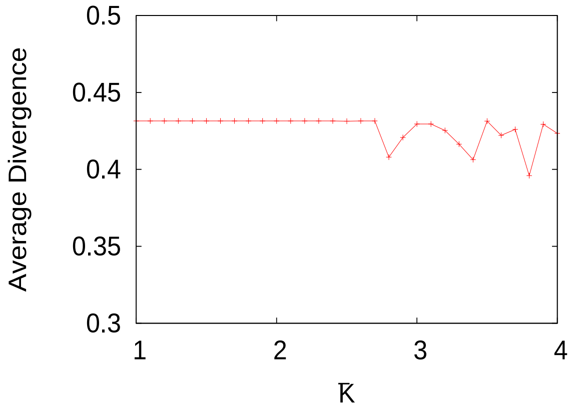
(a) r = 0.1



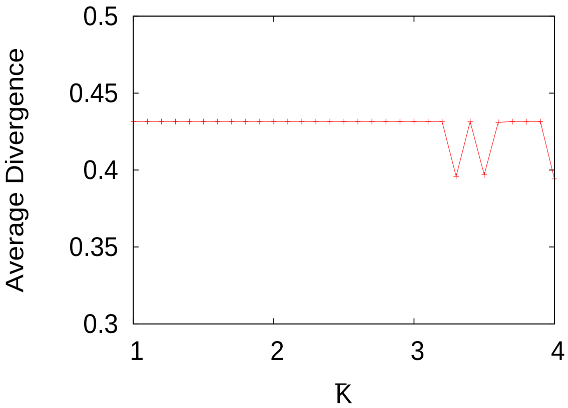
(b) r = 0.3



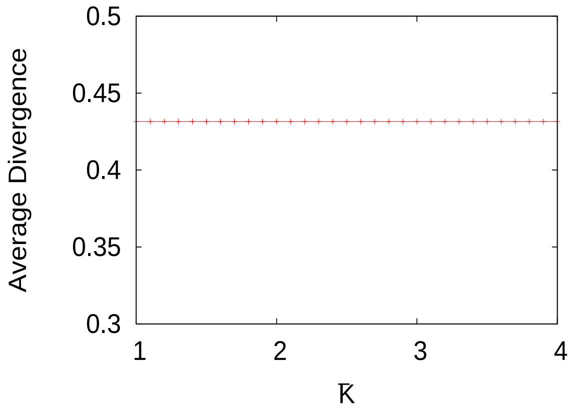
(c) r = 0.4



(d) r = 0.6



(e) r = 0.7



(f) r = 0.9

Figure 8.2: Variation of divergence based on the average degree $\overline{K}$ and the bias parameter r.

8.7 Discussion

Based on the results depicted, it is evident that the average divergence of a node is the least when the random boolean network is at the chaotic phase. It is at the chaotic phase that the complexity of a system emerges. In RBNs, the chaotic phase emerges when the average degree K is greater than 2 and the network has high activity, with the bias parameter is around 0.5. Thus, it can be argued that it is a tendency to optimise the strategic interactions that makes the complexity emerge in real-world systems. The two parameters that are varied, the bias parameter and the network topology, both affect the divergence from Nash equilibrium of each interaction. The results find that when the topology and the activity of the network reach the chaotic phase, the strategic interactions tend to be optimal, rather than when the network operates at an ordered phase. Thus, the results indicate that the complexity that emerges in the real-world systems may be a result of their tendency to optimise the strategic interactions. As future work, the robustness of this result could be studied, particularly in relation to the applied boolean networks such as gene regulatory networks.

Apart from these findings that were based on random boolean networks, this work suggests the possibility of using the directed information transfer metric of transfer entropy as a basis for deducing the rationality of a player with respect to a particular opponent. The bounded rationality derived based on transfer entropy is a dynamic rationality instead of the static rationality distribution suggested in the topologically distributed bounded rationality model. Since information flows depict a more refined form of interconnections among nodes, the transfer entropy-based bounded rationality has the potential to be a more accurate bounded rationality measure. While topological bounded rationality presents a rationality distribution that is distributed over space, the transfer entropy based rationality model results in a bounded rationality measure that is distributed over both space and time. Since transfer entropy has been effectively used to measure the directed information flow in myriad real-world social networks [69, 188], this would open up a novel approach in modelling rationality in strategic interactions such as online auctions, trading in financial markets and political campaigns.

This chapter concludes the second segment of thesis where the bounded rationality is conceptualised as a topological and information transfer-based measure. The modelling of bounded rationality provides an implicit mechanism to capture the influence of topology and information diffusion on networked game dynamics. The following chapter concluded the thesis.

Chapter 9

Conclusions

Socio-economic systems and evolving populations are often modelled as networked games. Such networks consist of non-trivial topological features and demonstrate emergent properties such as scale-free and small-world behavior.

In this work, the effects of the complex network topology and information diffusion on the strategic interactions of autonomous agents in socio-economic systems are analysed. Further, the reciprocal effect of such strategic interactions on the topology of the complex networks is also investigated.

This work is based on three fundamental scientific fields, namely; Information Theory, Game Theoretic Analysis and Social Network Analysis. Based on these three pillars, the question of how the network topology and information diffusion affects the outcomes of strategic games is addressed. This fundamental research question is further broken down into a number of sub questions and is respectively addressed in each of the subsequent chapters. Following is a brief summary on the existing gaps and challenges in these fields, that are addressed in this work.

1. Topological effect on the evolution of cooperation Even though self-interested players should never cooperate according to the Darwinian worldview, the real-world populations do demonstrate cooperation. One of the key issues addressed in this work is how the network topology and the information diffusion among players affect the evolution of cooperation in a population of players.

2. Topological effect on the evolutionary stability of strategies The evolutionary stability of strategies defines a particular strategy's ability to eradicate a competing strategy. In this work, we study how the network topology affects the evolutionary stability of strategies.

3. Optimising the topological distribution of strategies to maximise the global payoff In a population of self-interested players, the collective payoff may depend on the distribution of strategies among the network topology. In this work, we study the optimum placement of competing strategies to maximise the collective payoff of a population.

4. Topological distribution of bounded rationality While Nash equilibrium assumes that the players in a network are perfectly rational, in the real world players demonstrate non-optimal or bounded rationality. In this thesis, we study the implications of the hypothesis that the heterogeneous rationalities of players in a network can be inferred by the network topology.

5. Applications of Topologically distributed bounded rationality While most of the real-world applications of behavioural game theory are modelled with players with perfect rationality, in the actual scenarios, the players demonstrate heterogeneous rationality levels. Thus, in this work we attempt to answer the question whether incorporating bounded rationality helps in improving the existing game theoretic models.

6. Information Theoretical interpretation of bounded rationalityDirected information transfer can be measured using transfer entropy in information theory. On the other hand, behavioural game theory suggests that the rationality of a player is proportional to the information availability. Combining these areas of science, we try to bridge the gap between game theory and information theory by suggesting that the rationality of a player is proportional to the cumulative incoming transfer entropy.

These existing gaps and challenges are addressed with the following set of contributions.

1. Network topology under information diffusion constraints plays a key role in the evolution of coordination.

2. The evolutionary stability of strategies is dependent on (i) The network topology (ii) The evolutionary method used and (iii) The initial configuration of strategies.

3. The global collective payoff of a network of players is optimal when the evolutionarily stable strategies occupy the hubs and the evolutionarily weak strategies occupy the leaf nodes.

4. When the bounded or non optimal rationality of players are defined as a function of network topology, the networks of players evolve into scale-free and small-world topologies, in their attempt to optimise the strategic interactions.

5. The prevalence of multiple equilibria in strategic interactions is affected by the network topology.

6. Topologically distributed bounded rationality may be used to improve the accuracy of the real-world applications of behavioural game theory.

7. Information theoretic measures, such as transfer entropy can be used to quantify the bounded rationality of strategic players in a network. This would give rise to an interpretation of rationality, which varies both spatially and temporally.

A more detailed explanation on the contributions of this thesis is given as follows.

9.1 Summary of contributions

9.1.1 The influence of network topology on the evolution of coordination in networked games

As the first component of the study on how the network topology affects the dynamics of networked games, the evolution of coordination is studied in complex networks under varying topologies. Specifically, the evolution of coordination is studied by varying the information flow and the topology of the network.

In this regard, the evolution of coordination in social systems was analysed by simulating the coordination game on an ensemble of complex network topologies. A comparative study was conducted, for network topologies that are commonly found in social systems, using four different classes of well-known network models. Namely, these included; scale-free, small-world, hierarchical-modular and Erdős-Rènyi random networks. As a reference, an approximation of well-mixed population/lattice was considered.

In all classes of networks considered, it was observed that if nodes are unaware of the payoffs of their neighbours and cannot adapt, the relative payoff for coordination has to be relatively high, for the average payoff of coordinators to be higher than the average payoff of non-coordinators. However, when the nodes are aware of the payoffs of their neighbours and can evolutionarily adapt, coordination emerges as the winning strategy, even for relatively lower levels of coordination payoffs. Therefore, this work suggests that not only the network topology, but also the information diffusion among nodes is critical in the prevalence of the coordination in self-organising networks. The effect of network topology and information flow on coordination may provide a possible explanation to the dilemma on the existence of coordination in real-world networks where self-interested and competitive agents interact with each other.

This study produced a number of key observations and findings. For instance, it was observed that when there is no evolution, the relative coordinator payoff, β, has to be above 2 for coordinating nodes to have higher average payoff than non-coordinators. However, after the payoff information-based evolution and adaptation, there emerges a range of β less than 2 for which coordinators still remain a majority. Another interesting observation was that in most topologies, after sufficient evolution and adaptation, the proportion of coordinators goes through a phase transition when the relative coordinator payoff, β, is increased. Further, it was noted that it was the peripheral hubs that first completely adopt coordination and drive the evolution of coordination. Another key observation was that noise and time lags in payoff information was found to adversely affect the evolution of coordination, although the level of this effect depends on topology.

In addition to the aforementioned general findings, there were some topology-specific findings that were uncovered within the scope of the evolution of coordination. Most importantly, the evolution of coordination is most pronounced and the transition in terms of relative coordinator pay-off β is sharpest in small-world networks. This may explain why most collaboration networks depict small-world nature. On the other hand, the emergence of coordination after evolution is least rapid in scale-free networks. Scale-free networks are the most sensitive to noise in pay-off information and the evolution of coordinators is most affected by noise, while small-world networks are not as sensitive. Similarly, scale-free networks are the most sensitive to time lags in information regarding the pay-off. After the evolution of coordination, the average pay-off of coordinators is higher than the initial stage in scale-free and hierarchical networks, while it is lower than the pay-off at initial stage for small-world and ER random networks. It is important to note that the proportion of coordinators at the end was found to be higher than during the initial stages in all classes. The degree of hierarchy in hierarchical-modular networks and the degree of small-worldness in small-world networks both seem to aid the emergence of coordination.

Comparing these topology-specific findings, we could argue that scale-free networks and small-world networks display contrasting characteristics in terms of the evolution of coordination. The hierarchical-modular class tends to display features similar to scale-free networks, while the ER random networks display features similar to small-world networks. Nevertheless, it can be concluded that topological features, qualified here by the four classes of networks, influence the evolution of coordination in social systems in non-trivial ways.

There are several key implications from the results obtained in the attempt to study the evolution of coordination. It is widely known that both small-world and scale-free features are observed in real-world social systems, although in various degrees. Based on the simulation results obtained in this work, it is shown that while the emergence of coordination can be aided equally readily by both features, scale-freeness increases the sensitivity of the system to noise and time lags in information diffusion, while networks which are exclusively small-world are relatively unaffected. This would imply that systems, that are small-world but not scale-free are likely to evolve into being dominant in coordination and sustain it under difficult information-diffusion conditions. This argument is further supported by the observation that the small-worldness itself, measured by the clustering coefficient and the diameter of the network, seems to aid the transition in terms of relative coordinator pay-off. This observation has been further corroborated by other studies in different game contexts. Experiments were conducted with several network densities in different classes of networks, and it was shown that the sparser the network, the easier the emergence of coordination, other parameters being unchanged. Therefore, the smaller the number of games played within a network, the easier it seems for coordination to evolve as the winning strategy. Thus, these results are useful in understanding the behaviour of spatially connected social systems.

In conclusion, based on the results obtained in the first component of the research conducted, it is possible to conclude that the network topology under information diffusion constraints plays a key role in the evolution of coordination in networked populations of players. Further, the results observed strongly suggest one arc of the cyclic interdependence between form and function, which indicates that the function of the network, in terms of coordination, follows the form or the topology of the network.

While coordination is a significant strategy in evolving games, the effect of network topology on generic strategies is relevant in all strategic interactions. Thus, as the next logical step of this study, the influence of network topology on the evolutionary stability of strategies was examined.

9.1.2 The influence of network topology on the evolutionary stability of strategies

This subcomponent of the research focuses on how the network topology of a population of players affects the evolutionary stability of a strategy. In order to simulate this, a particular subclass of strategies known as zero-determinant strategies, which has been demonstrated to be evolutionarily unstable against the Pavlov strategy, were played against each other. From the perspective of the relationship between the form and function, we attempt to test whether the function in the form of evolutionary stability is affected by the form or the topology of the social structure.

Based on the results gathered from the simulations conducted, it is evident that network topology has an effect on the evolutionary stability of a particular strategy. Another important finding of this research is that the topologically influenced evolutionary stability is a weak evolutionary stability, not a strong evolutionary stability. In other words, the stable strategy would not be able to completely eradicate the competing strategy and the competing strategy would still be able to survive within the confines of the network.

Further, the results obtained in this study suggest that the topological effect of evolutionary stability is determined by the evolutionary process used. When using the death-birth Moran process to evolve the population, topology does not seem to have a significant effect on the evolutionary stability of strategies. However, when the strategy adoption process suggested by Santos et al. [223] is applied, topology does seem to have a significant effect on the evolutionary stability of a strategy within a population. The strategy adoption process takes into account the cumulative payoff of each node in determining whether a strategy should be replaced or not. As such, this result suggests that an evolutionarily unstable strategy could survive when it occupies the hubs surrounded by leaf nodes assigned with an evolutionarily stable strategy. In a heterogeneous network of players, hubs tend to have more strategic interactions with their opponents in comparison to leaf nodes. Thus, a hub with an evolutionarily unstable strategy would continue to be irreplaceable by the neighbouring nodes' strategies, as it would continue to have a higher pay-off than its immediate neighbours.

The evolutionary process adopted may have significant implications in the real-world networks of strategic players. The death-birth Moran process is more suitable in a biological context where the lifetime of a player is significantly less than the evolutionary time span. It could be effectively used to model the evolution of species where the strategies are hard-wired to the players and the evolution occurs through the replacement of players with replicas of better performing players. However, in a social context, the evolution of strategies may be driven by the adoption of strategies by the players based on the performance of their neighbouring players. In other words, a stochastic strategy adoption process could be used to model the evolution of strategies when the lifetime of a player maybe considerably larger than the time-span of evolution. Examples of such situations include the interactions that occur in corporate sectors and financial markets. In these instances, it is often observable that, in their struggle to survive, the players continually adopt the strategies of other players. Thus, the strategy adoption evolutionary update process may be more relevant when the evolution of strategies is applied in a social context. Accordingly, the topological effect on the evolutionary stability of strategies may be more prevalent in a social context, than a biological context.

Further, it is important to note that not only the topology but also the initial distribution of the strategies within the network plays a significant role in shaping the evolution of the strategies. For instance, when an evolutionarily unstable strategy occupies hubs as opposed to the leaf nodes at the initiation of the evolution, it manages to become an even more prominent strategy within the network over time, resembling a weak evolutionarily stable strategy. Again, this particular observation indicates that the function, which is the evolutionarily stability of strategies, is partially determined by the form or the topology of the network.

Even though the main focus of this work was on the Zero Determinant and Pavlov strategies, it was possible to replicate similar observations with other well-known strategies such as the general cooperator and cooperator strategies, competing against the Pavlov strategy. This could mean that the variation of evolutionary stability due to topological stability of strategies is a more general phenomenon which may be applicable to most strategies that are competing with each other.

This study suggests that it is possible to identify three basic factors that determine the topological stability of strategies in a non-homogeneous network. These factors are the network topology, the evolutionary process and the initial distribution of the strategies. By varying these three factors, an evolutionarily unstable strategy may be able to survive and may even operate as a weak evolutionarily stable strategy in a population of players connected in a non-homogeneous topology. Based on the observations in this work, the topological stability of strategies may be more prevalent in a social context of the evolution of strategies than in a biological context. Further, all three of the factors indicate that the function of a socio-economic system, in terms of the evolutionary stability, is determined by the network structure, which in an abstract form is the network topology of the system. These findings indicate that it is paramount to account for the network structure in determining the evolutionary stability of a strategy among a population of players.

Though the evolution of strategies is usually studied from an individualistic point of view, the optimisation of common or public utility of a population is also a significant problem to address. Accordingly, the next research subquestion attempts to optimise the strategy placement with the objective of maximising the public utility in a network.

9.1.3 The influence of network topology on the optimisation of public good in complex networks

Under this research subquestion, focus was given to the influence of network topology in maximising the common or public good in a population of interacting agents. In order to simulate such a scenario, the iterative prisoner's dilemma game was adopted as a public goods game in which agents play prisoner's dilemma repeatedly and adapt their strategies with the goal of increasing the total network utility. Evolution was simulated by implementing a version of the genetic algorithm optimisation, where each member of the population is a network with a particular distribution of strategies. Thus, the evolution of networks were considered as social structures, rather than the evolution of individuals.

It was found that networks, which prefer a certain type of strategy to be at their hub over another type, evolve for high network utility. As such, the evolved networks preferred cooperation over defection, general cooperation over zero-determinant, Pavlov over general cooperation, and Pavlov over zero-determinant at their hubs. This indicates that when societies compete, those that can efficiently order individuals according to their strategies have better chances of gaining higher overall payoffs. This is a significant result in understanding cooperation for public good.

As a future step in this direction, similar experiments can be conducted with other optimisation techniques, such as simulated annealing and the ant-colony optimisation. A broader range of memory-one and other strategies can be considered (tit-for-tat, for example). Furthermore, experiments could be performed on particular application domains, such as defence and project management, to better demonstrate the utility of these results. Overall, the findings presented in this chapter demonstrate that the network topology or the structure is key in determining the optimum global wealth of a population based on the placement of strategies.

The initial segment of the thesis attempted to study the effect of network topology on different aspects of networked games such as the evolution of coordination and the evolutionary stability of strategies. The next segment mainly focuses on defining the non-optimality or bounded rationality of strategic players based on network topology and information transfer. Bounded rationality implicitly captures the effect of network topology and information flow in strategic interactions.

9.1.4 The influence of network topology on the bounded rationality of networked players

Under the purview of the topological effects on networked games, this work evaluates how the network topology affects the rationality of strategic players. In real-world socio-economic systems, interactions occur among players whose rationality is bounded. There are several theories and hypotheses that suggest that the rationality of a player may be correlated to the amount of social interactions they undertake. These theories include social cognitive theory, social brain hypothesis and the cognitive hierarchical model. Accordingly, it was assumed that there exists a correlation between the rationality of a player and the amount of social interactions they undertake. Based on this fundamental and reasonable assumption, a topological model of bounded rationality is proposed. Further, this model is used to understand the relationship between the topology of socio-economic systems and their dynamics. Particularly, focus is given to the scale-free characteristic and its influence on social network dynamics. Given a particular heterogeneous distribution of rationality among players, how the topological characteristics encourage system rationality is considered. The Nash equilibrium predicts the strategies that players with perfect rationality would choose. Accordingly, the Jensen-Shannnon divergence of the Nash and quantal response equilibrium states are computed for each pair of players in a given social system. Subsequently, the average of these divergences was used as a measure to quantify the 'system rationality'. In this regard, a number of well-known games were considered, including the prisoner's dilemma, the stag-hunt, the meeting game and the matching pennies game to simulate scenarios where cognitive decisions must be made.

The topological analysis of bounded rationality points to some interesting implications. In the first step of this analysis, a number of network classes were compared, including scale-free, Erdős-Rènyi random and lattice networks (representing well-mixed populations). In comparison, it was shown that among these classes, it is the scale-free networks which facilitate the best convergence towards Nash equilibrium (highest system rationality) on average. Based on this observation, it may be argued that this might be one reason why many real-world social systems are scale-free. This result suggests that the function of socio-economic systems in terms of bounded rationality is affected by the form or topology of socio-economic systems.

Further exploring the possible relationship between the scale-free networks and bounded rationality, the variation of the average Jensen-Shannon divergence was measured while a network was grown according to the Barabàsi-Albert model. The resulting observations suggest that network growth has a negative correlation with the Jensen-Shannon divergence, suggesting that networks converge towards Nash equilibrium as they grow. This observation suggests that there could be a game theoretical explanation on why there is preferential attachment in networks. Exploring the same argument further, a network growth model that interprets preference in preferential attachment as the tendency to optimise a strategic interaction by converging towards Nash equilibrium is proposed. The resulting network of this growth model displays both scale-free and assortative tendencies. This is a key observation in the underlying argument that the structure of a socio-economic system could be influenced by the strategic interactions, which is implicitly defined by the bounded rationality of players.

Building further evidence for this conjecture, the topological evolution of social systems was simulated using the simulated annealing technique, beginning from a random network topology. It became apparent that when evolutionary pressure is applied on social systems to converge towards Nash equilibria, scale-free and small-world features emerge. This finding could be significant in its implications, since it provides an alternative explanation for the prevalence of scale-free networks in many real-world systems and societies. Thus, this result strongly suggests that the function of socio-economic structures influences their form.

Subsequently, the topological analysis of bounded rationality was extended to games with multiple equilibria. Again, it was demonstrated that when evolutionary pressure is applied on systems to converge towards Nash equilibria (regardless of which equilibrium state a particular pair of players converge towards), scale-free and small world features emerge. Further, the likelihood of the existence of multiple equilibria among the players of a system with a bounded heterogeneous rationality distribution was considered. It was observed that a delicate balance exists: when the average rationality (as distinguished from what system rationality, which is computed from the Jensen-Shannon divergence between quantal response equilibrium and Nash equilibria) is low, the scale-free nature of the system encourages the emergence of multiple equilibria, while when the average rationality is high, the scale-free property in fact hinders the existence of multiple equilibria. Therefore, the number of rational choices available to players from which they cannot deviate without loss depends on the social network topology as well as the level of rationality prevalent in the system. Thus, it is evident that the form or the topology of a socio-economic structure affects the function of the network in terms of its capacity to facilitate multiple equilibria.

Within the context of this work, rational players are regarded as those who try to maximise their average individual pay-offs. If players attempted this within a heterogeneous system, they may make choices that are contrary to the Nash equilibrium. Therefore, a system which converges towards Nash equilibrium will not necessarily have increasing average pay-offs. Indeed, in the case of prisoner's dilemma game, the convergence towards Nash equilibrium results in decreasing average pay-offs. Thus, it could be argued that such a system is, on average, not becoming more rational. However, in an environment where there is a lot of mistrust and/or competition, the priority of the players will be to ensure that their average pay-offs are better than other players with whom they compete; that is, they would want to ensure that they are not cheated by others. The self-interest, and the relative wellbeing in the system, therefore, gains prominence over the absolute wellbeing, represented by the cumulative pay-off. In such systems, the convergence towards Nash equilibria, on average, means the players are getting better at preserving their relative self-interest, and thus becoming more rational in a selfish sense. The findings that are presented here are related to this sense of rationality, and not the common good of the system [122]. However, in games other than prisoner's dilemma (for example, in the stag-hunt game), it was found that the average pay-off could indeed increase as the system converges towards (multiple) Nash equilibria, depending on the actual values of pay-offs for each scenario. Thus, the public good of the system corresponds to the selfish rationality of players. Therefore, it is important to realise that the results that were obtained are applicable in terms of average selfish rationality of players, which may or may not correspond to the common good of the system. In any case, it is quite conceivable that players would put their relative wellbeing over their absolute wellbeing, since human beings perceive their level of wellbeing primarily by comparing themselves with their local neighbours. In summary, it remains a vital research question of great scientific and practical significance to understand how the cognitive decision-making of players and the resultant dynamics in socio-ecological systems are shaped by both the topology and the bounded rationality of actors in such systems.

It is widely known that network topology does affect the strategic decision making scenarios of self-interested players. On the other hand, game theoretic models have been proposed to model network growth, suggesting that network formation can be regarded as a strategic decision that a node makes. This topological rationality model suggests that there could be a simultaneous and cyclic interdependency between network topology and strategic decision-making of self-interested agents based on heterogeneous bounded rationality. Thus, strategic games maybe more sensitive to the local context when they operate over a spatially distributed network of players than is generally assumed. This is highlighted by identifying such games as network-based games, in comparison to network game models where a two-player game is simply iterated over a network. Thus, understanding how node rationality is affected by network topology could be significant in contextualising an abstract gaming model. There may also be other centrality measures and topological characteristics that may serve as better indications of the rationality of a node in its social context. Further, quantifying the information flow along a link with an opponent may provide a better indication of a node's rationality of that particular opponent, instead of merely depending on the physical topology. More empirical studies are required to confirm the possible correlation between the bounded rationality of players and their network topological placement and properties.

Topologically distributed bounded rationality poses an interesting avenue to study the cyclic relationship between the network topology and strategic interactions. The fact that the bounded rationality of players promotes the scale-free and small-world nature of socio-economic structures suggest that the topology of the network is shaped by strategic interactions. On the other hand, the variation of the existence of multiple equilibria under different topologies suggests that the strategic interactions are affected by the network topology. Thus, the topological distribution of bounded rationality model indicates that the topology and the strategic interactions of socio-economic structures are cyclically interdependent.

9.1.5 Applications of topologically distributed bounded rationality

Following the proposition of the topologically distributed bounded rationality model, three different real-world case studies were considered in order to validate and demonstrate the applicability of this concept. The first case study is peer-to-peer network formation. A game theoretic network formation algorithm that operates with the assumption of Nash equilibrium is adopted. Next, it is extended using quantal response equilibrium and topologically distributed bounded rationality. The resulting network properties demonstrate that the quantal response equilibrium-based network is closer to the scale-free nature of the real-world peer-to-peer network formations. This may suggest that the topologically distributed rationality model is useful in modeling peer-to-peer network formation more accurately. This also suggests that the function of a socio-economic structure is vital in determining the form of that particular socio-economic structure.

Next, the topologically distributed bounded rationality model is applied to a scenario where game theory is used to model the security levels of nodes in a network. Under a topologically distributed bounded rationality model, it can be shown that the highly connected nodes or hubs would have more stringent security policies than the leaf nodes, reminiscent of the real-world networks. In comparison, the Nash equilibrium solution suggests that each node in a network would have identical security levels, which is not the case in real-world networks.

As the final case study, network routing is simulated using the randomly matched prisoner's dilemma game. Accordingly, it is shown that a scale-free network would facilitate the exchange of messages over a broad range of network rationality levels when compared to a well-mixed population under topologically distributed bounded rationality. The results suggest that a topological bounded rationality could be useful in developing game theoretic models that would represent real-world scenarios more accurately, in comparison to the models that assume perfect rationality. Thus, topologically distributed rationality model can be a useful tool in modelling real-world strategic decision-making scenarios among populations of players. More real-world applications and empirical studies are essential to explore the applicability of the topologically distributed bounded rationality model.

Both the network security and network routing applications demonstrate how the topology of a network influence the strategic interactions of its nodes. Thus, the applications of topologically distributed bounded rationality shows how the structure and strategic interactions of socio-economic structures are interdependent with each other.

9.1.6 Information theoretical analysis of bounded rationality in network-based games

This work extends the topological model of bounded rationality to an information flow-based interpretation, where the directed information flow is used to quantify bounded rationality of a particular interaction. Random boolean networks were used to generate the information flows based on which the bounded rational agents interact. The optimal behaviour of the strategic interactions were compared under varying topologies and activity levels of random boolean networks. Based on the observations, it is evident that the average divergence of a node is the least when the random boolean network is at the chaotic phase. It is at the chaotic phase that the complexity of a system emerges. In random boolean networks, the chaotic phase emerges when the average degree is greater than 2 and the network has high activity, with the bias parameter around 0.5. Thus, it can be argued that it is the tendency to optimise the strategic interactions that makes the complexity emerge in real-world systems. The two parameters that are varied, the bias parameter and the network topology, both affect the divergence from Nash equilibrium of each interaction. The results depict that when the topology and the activity of the network reaches the chaotic phase, the strategic interactions tend to be optimal, rather than when the network operates at an ordered phase. Thus, the results indicate that the complexity that emerges in the real-world systems may be a result of their tendency to optimise the strategic interactions.

Apart from these findings, that were based on random boolean networks, this work mainly suggests the possibility of using the directed information transfer metric of transfer entropy as a basis for deducing the rationality of a player, with respect to a particular opponent. The bounded rationality derived based on transfer entropy is a dynamic rationality instead of the static rationality distribution suggested in the topologically distributed bounded rationality model. Since information flows depict a more refined form of interconnections among nodes, the transfer entropy based bounded rationality has the potential to be a more accurate bounded rationality measure. While topological bounded rationality presents a rationality distribution that is distributed over space, the transfer entropy based rationality model results in a bounded rationality measure that is distributed over both space and time. Since transfer entropy has been effectively used to measure the directed information flow in myriad real-world social networks [69, 188], this would open up a novel approach in modeling rationality in strategic interactions such as online auctions, trading in financial markets and political campaigns.

9.1.7 Epilogue

The network topology and information flow of socio-economic systems play a critical role in determining the dynamics and function of socio-economical systems modelled as networked games. Whether it is the evolution of coordination, evolutionary stability of strategies of the distribution of bounded rationality, the network topology and the information flow of socio-economic structures are critical in determining the outcome of the strategic interactions.

This work tries to analyse the different aspects of how network topology and information flow affect the strategic interactions of populations of players that are topologically distributed. Firstly, the effect of topology on the evolution of coordination is studied. Even though players in a population are thought to be self-interested, coordination is abundant in populations of strategic players. It is shown the network topology, particularly scale-free topology, plays a critical role in the emergence of coordination in populations of players. Next, the effect of network topology on the evolutionary stability of strategies was observed. Evolutionary stability is the tendency of a strategy to dominate a population against any competing strategy. The results suggest that the network topology, initial distribution of strategies and the evolutionary update process is critical in determining whether a strategy is strongly or weakly evolutionarily stable. Next, it was shown that network topology and the placement of strategies is critical in optimising the common public good in a population of players.

The rest of the thesis focuses on modelling the bounded rationality of agents in a network. Two different models of bounded rationality are proposed that are based on network topology and directed information flow, respectively. The topological model of bounded rationality proposes a spatial distribution of bounded rationality while the information flow-based bounded rationality model proposes a rationality distribution that is dispersed both over space and time. A result of particular significance that the topological bounded rationality models suggests is that the random populations evolve to be scale-free and small-world in nature under strategic interactions with bounded rationality. The information theoretic interpretation of bounded rationality proposes a more refined version of bounded rationality that is dynamic in nature and is applicable on real-world socio-economic systems such as financial markets and political campaigns. Though the topological and information theoretic bounded rationality models introduced here are applied only in computer science related applications in this work, they could be applicable in other domains and sectors such as healthcare, finance and education where social interactions and strategic decision making is interwoven.

In conclusion, each of the subquestions addressed in this work confirms that the topology and information diffusion in socio-economic systems modelled as networked games are critical in determining the dynamics and evolution of such systems. Further, this study suggests that there exists a cyclic interdependency between the topology, information diffusion and the strategic interactions in socio-economic systems.

Appendix A

Modelling of social influence using bounded rationality

A.1 Introduction

Influence modelling in social networks is a key research problem with many applications over different domains. As a motivating example, consider the scenario where the present discussion on global-warming is operating in online social media and in social networks in general. With the issue of global warming, the actions of individuals, organisations and governments are deeply influenced by several key individuals who may be scientists, political figures and social figures. Thus, modelling the influence of such key players over the rest of the network would be an important research problem as it affects the spread of information over the network. This information spread may be key in determining the subsequent actions that would affect the resolution or the aggravation of the issue at hand.

Numerous attempts have been made to model the influence in a social context. Two classical models are linear threshold model and the independent cascade model [133]. Both these models take into account the neighbourhood effect of adopting a particular state by a node in the social network. Social influence modelling tries to address the optimisation problem of finding the optimum configuration of seeds to maximise the social influence. Under both these models, the optimisation problem of selecting the most influential nodes has been shown to be an NP-hard problem[133, 134]. Therefore, greedy algorithm is often used to come up with an approximated solution [134]. Another approach to model social influence has been to use the Page-rank algorithm based models, especially with respect to measuring the influence of micro blogs[111]. Game theoretic influential models too have been suggested to model social influence, where social influence is modelled as a strategic game[58, 251]. However, these models assume the prevalence of perfect rationality in players making their decision to adopt a particular state, even though in real-world players are bounded rational [88]. In this work, we present a social influence model that is based on the bounded rationality of players in a social network. In the proposed model, the rationality of following an influencing node or adopting a strategy would be negatively proportional on the distance from the seed.

Bounded rationality in social network suggests that players make non optimum decisions due to the limitations of access to information. Based on the premise that adopting a state or an idea can be regarded as being 'rational', this chapter proposes an influence model is proposed based on the heterogeneous bounded rationality of players in a social network[121]. The quantal response equilibrium model can be employed to incorporate the bounded rationality in the context of social influence. The bounded rationality of following a seed or adopting the strategy of a seed would be negatively proportional to the distance from that node. This model may be used in scenarios where there are multiple types of influencers and varying payoffs of adopting a state. Different seed placement mechanisms compared and contrasted to identify the optimum method to minimise the existing social influence in a network when there are multiple and conflicting seeds. This is ascertained by placing opposing seeds according to a measure derived from a combination of the betweenness centrality values from the seeds and the closeness centrality of the network would provide the maximum negative influence. Further, this model is further extended to a strategic decision making scenario where each seed would be operating a strategy in a strategic game.

The rest of this chapter is organised as follows. In the next section, the relevant background knowledge is discussed. In particular, the focus is given on social networks, existing social network influence models and game theory. Then, an influence model is proposed based on bounded rationality and quantal response equilibrium model. In the subsequent section, this model is extended into a strategic decision making scenario. Next, the propagation of social influence is simulated when seeds are placed at different configurations. Then an efficient is proposed mechanism to find the optimum placement of seeds to counter the influence of existing seeds, when there are multiple types of contending seeds. Finally, the results are presented with the derived conclusions.

A.2 Background

A.2.1 Influence modelling in social networks

Modelling of influence in social networks have gained much interest in the recent past. This is partly due to the potential that the emergence of online social networks present, in myriad of fields from online marketing of products to political campaigns[51, 71]. Especially due to advent of 'viral marketing' where word of mouth is used as a form of advertising through social media, the importance of social influence modelling has become even more prevalent[42, 71]. One key advantage in online social networks is it is possible to harness the meta information about the social network such as the underlying topology and the weights of the links, based on the data that is captured from the social interactions[71]. The key challenge in social influence modelling would be to identify the placement of 'seeds' or the influencing agents that would be able to create a cascading effect in the network, where the maximum possible number of nodes in the network are affected. This problem becomes even more complex when there are multiple types of contending seeds are in operation[50, 58]. Two main classes of influence or diffusion models are found in the literature, namely the linear threshold model and independent cascade model. Apart from that, recent interest has emerged on network topological influence models based on the Page-rank algorithm, and even based on game-theoretic models[111, 58]. Following is a brief introduction to some of the common social influence models found in the literature.

A.2.2 Linear threshold model

One of the most common models used to model social influence is the linear threshold model[133]. The assumption that is made in this model is that a node has a binary state of being active or inactive, with respect to a particular state that it is under influenced. Each node would have a random variable that dictates the fraction of nodes based on whose state which, it will switch or keep its current state. Formally put, each node v

would have a threshold $\theta_v \epsilon [0, 1]$ that is randomly selected, which denotes the fraction of neighbours of node v that has to be active in order for node v to be active and vice-versa. Each node is affected by each neighbour w according to a weight $b_{v,w}$ such that, $\sum_{w\, neighbour\, of\, v} b_{v,w} \leq 1$. A node is activated when the total weight of its active neighbours is at least θ_v:

$$\sum_{w\, neighbour\, of\, v} b_{v,w} \geq \theta_v$$

The random assignments of threshold θ_v account for the lack of knowledge of intrinsic latent tendencies of nodes to adopt neighbour strategies.

The classical linear threshold model is designed to show a single binary positive state. However, in a social network, there could be opposing or conflicting influences in place. To account for this possible negative influence, as extension to the linear threshold model has been suggested called the Competitive linear threshold model (CLT) [105] that account for the possible negative influences that maybe present in a network. Thus, instead of the two states inactive and +active, there are three possible states in a CLT model, namely inactive, +active and -active.

Notice that both the LT and CLT model assume that nodes switch in binary states and not in a probabilities manner. The stochasitc nature is captured in the randomness of the threshold. Also, even under the CLT model, it is not possible to model influencing scenarios where more than one positive or negative influence is present. In other words, there may be scenarios where multiple options are available for an individual in a social network (such as an election), where the CLT model could not be applicable.

A.2.3 Independent Cascade model

In the independent cascade model [133], when a node v becomes active, it has a single chance of activating each currently inactive neighbour w. Each activation attempt would succeed with probability p_{vw}. Here too, the agent states are defined as binary states while multiple influence types are not considered. An extension for the independent cascade model has been proposed which allow the inclusion of negative opinions[50].

A.2.4 Page-Rank based influence models

One key feature of both the linear threshold model and independent cascade model and their variants is that they consider only the local topology of a particular node and do not directly consider how the global topology would affect the influence. However, another interesting influence model that has come about recently is the Page-rank based influence model, which considers the non-local topology in modelling influence. Page rank algorithm was initially used by Google to rank the web pages based on rankings of their neighbourhood[192]. It can effectively be used to measure social influence, particularly in online social networks such as the blogsphere. Page-rank with prior has been suggested as one such possible influence model[266], which has been used to measure social influence in collaboration networks. It has also been used to evaluate microblog users' influence[111]. However, Page-rank is generally sought after to quantify the influence or the rank of each node rather than to identify to find the optimum seed arrangement to maximise or minimise social influence.

A.2.5 Game theoretic influence models

There have been some recent influence models developed based on game theoretic principles. These models assume the adoption of a state as a strategic decision in a social network. One such model is a model Dynamic Influence in competitive environments (DICE) [58]. It has been used to model scenarios where competing ideas operate simultaneously as well as sequentially. Further, the DICE model has been demonstrated to be a generalisation of the classic linear threshold model and the independent cascade model. Competitive diffusion process [251], which is another game theoretical social influence model, models the diffusion of information under competing seeds by extending the linear threshold model on 2-player games. Both these models assume perfect rationality of players, which may not be the case with real-world players. Thus, in the proposed model, an attempt is made to extend the existing game theoretical models to incorporate bounded rationality in following the seeds.

A.3 Modelling social influence using bounded rationality

Based on the background theoretical knowledge, a social network influence model is proposed based on game theory and bounded rationality of nodes. In order to do that, first the social influence of nodes is modelled as an influence game, where there would be influencing nodes or 'seeds' and followers operating in a network of players. The seeds would continue to operate with a permanent binding to a particular state. This inclination may be due to some external knowledge or an incentive the seed may have from the external environment. In the context of a influencer-follower scenario, the bounded rationality of a follower would be a 'rationality of following'. Higher the rationality of a follower with respect to a seed, higher the probability of it following the state of the seed. In this social influence model, the fundamental assumption is that the rationality parameter of a particular follower is negatively proportional to its distance from the seed. This assumption would account for the random noise that would be accumulated as the followers move further from a seed. Based on the rationality of following, it is possible to measure the probability of a follower being at the state of the influencing node or the seed node. Thus, our model does not produce a binary outcome where the followers would be active or inactive in binary states, rather the result would be a probability on which a follower would adopt the probability of the influencer. In a game theoretic terminology, the follower's probability distribution would be a mixed strategy equilibrium, where the two strategies would be whether to adopt the strategy of the seed or not. Formally put, the follower probability $p_{n,s}$ of adopting the active state s of the seed would be,

$$P_{n,s} = \frac{e^{\beta_{n,i}.U_s.P_{i,s}}}{e^{\beta_{n,i}.U_s.P_{i,s}} + e^{\beta_{n,i}.U_{-s}.P_{i,-s}}} \tag{A.1}$$

where,

$P_{n,s}$ - Probability of the follower node n being at state s (active state)
$\beta_{n,i}$ - Following rationality of node n with respect to node i
U_s - Utility of adopting the state s
$P_{i,s}$ - Probability of the influencer i being in state s (this is always 1)
$P_{i,-s}$ - Probability of the influencer i being not in state s (this is always 0)
U_{-s} - Utility of not being in state s (inactive state)

This model captures the random noise of the followers with the assumption that the rationality of a node of following the influencer is negatively proportional to the distance from the seed or the influencer. Thus, $\beta_{n,i} \alpha \frac{1}{d_{n,i}}$ where $d_{n,i}$ is the distance along the shortest path from the influencer i to node n along the shortest path. As the follower moves further from the seed, the rationality parameter reaches 0, making them behave randomly. If the followers are placed closer to the seed, then there would be higher rationality and thus a higher probability of following the state of the seed. Another important factor to note is the not only the distance from the seed, but also the reward or utility of adopting the state too play a significant role in determining whether a follower would adopt the state of the seed. Thus, the above model may be extended to accommodate for a scenario where there are multiple seeds or influencers instead of a single influencing node.

$$P_{n,s} = \frac{\sum_{i=1}^{N} e^{\beta_{n,i}.U_s.P_{i,s}}}{\sum_{i=1}^{N} e^{\beta_{n,i}.U_s.P_{i,s}} + \sum_{i=1}^{N} e^{\beta_{n,i}.U_{-s}.P_{i,-s}}} \tag{A.2}$$

where N is the total number of influencers in the network. In the above model, each node would have a separate rationality parameter for each influencer, based on the distance to them. Thus, it would capture the varying network distances from each influencer to more accurately predict the status of the follower.

This would imply that in a population that is closely knitted would have a higher tendency of following a seed compared to a population that is sparsely connected. Further, small-world networks[9] would tend to leverage social influence as they have relatively low average path lengths[9].

A.3.1 Modelling social influence under opposing influencers using bounded rationality

This particular bounded rationality based social influence model could also be used to model the influence of a single type of influencers. Yet, in most real-world social influence scenarios, there would be conflicting interests at play. There would be influencers with a negative influence as well as positive influence on the same state. In addition to that, there could be instances where there are multiple influencers that are mutually exclusive from each other. A good example of this are political campaigns where there would be more than two candidates running. Thus, the above model can be easily extended to account for two opposing types of influencers of states S_1 and S_2 as follows.

$$P_{n,S_1} = \frac{\sum_{i=1}^{N} e^{\beta_{n,i}.U_{S_1}.P_{i,S_1}} + \sum_{j=1}^{M} e^{\beta_{n,j}.U_{-S_2}.P_{j,-S_2}}}{\begin{array}{c}\displaystyle\sum_{i=1}^{N} e^{\beta_{n,i}.U_{S_1}.P_{i,S_1}} + \sum_{i=1}^{N} e^{\beta_{n,i}.U_{-S_1}.P_{i,-S_1}} + \\ \displaystyle\sum_{j=1}^{M} e^{\beta_{n,j}.U_{S_2}.P_{i,S_2}} + \sum_{j=1}^{M} e^{\beta_{n,j}.U_{-S_2}.P_{i,-S_2}}\end{array}} \tag{A.3}$$

Here,

P_{n,S_1} - Probability of the follower node n being at state S_1

$\beta_{n,i}$ - Following rationality of node n with respect to the influencer i

$\beta_{n,j}$ - Following rationality of node n with respect to the influencer j

U_{S_1} - Utility of adopting the state S_1

P_{i,S_1} - Probability of the influencer i being in state S_1 (this is always 1)

$P_{i,-S_1}$ - Probability of the influencer i not being in state S_1 (this is always 0)

P_{i,S_2} - Probability of the influencer j being in state S_2 (this is always 1)

$P_{i,-S_2}$ - Probability of the influencer j not being in state S_2 (this is always 0)

U_{-S_1} - Utility of not being in state S_1

U_{S_2} - Utility of adopting the state S_2

U_{-S_2} - Utility of not being in state S_2

The rationality parameters with respect to each influencer would again be dependent on the distance of the node in concern from each of the influencers. Note that the followers can take either of the two states S_1 or S_2 under the influence of the two types of influencers. However, it does not account for a neutral state where the followers may not follow either of the two types of influencers. If a neutral state is considered, then the numerator should only contain the exponent of S_1, as in that case a node being influenced to be in state $-S_2$ does not mean it would automatically adopt S_1. Further, it is possible to extend the same model to take into account multiple types of influencers and not just two opposing types, since every influencer state can be regarded as a possible strategy a follower could adopt with heterogeneous rationality levels.

A.3.2 Modelling strategic influence using bounded rationality

In addition to social influence, the same approach could be used to measure the influence of strategic decision making scenarios. Game theoretic models are often used in observing the evolution of populations of players[83, 223]. The evolutionary dynamics of the populations help to understand how each strategy would perform under different network topologies[223]. If an assumption is made that certain nodes continue to stick to a particular strategy irrespective of its environment, due to some external knowledge or influence external to the network, then they can be modelled as influencers while the rest of the population can be regarded as followers that get affected by those influencing nodes. For instance, when the prisoner's dilemma game is player over a network, depending on the topological position and arrangement of each node, different nodes would adopt cooperation or defection[223]. However, if it is assumed that the cooperation or defection tendency of each follower is affected by the influence by the seeds that stick to a particular strategy, then the social influence of strategic games can be modelled using heterogeneous bounded rationality and quantum response equilibria.

The Eq.A.4 and Eq. A.5 depict the two logit functions based on QRE with a strategically influenced rationality parameter to derive the probability of cooperation in a networked PD game.

$$p_{1,c} = \frac{e^{\beta_{1,c}(p_{2,c}(u_{111}+(1-p_{2,c})(u_{121})))}}{e^{\beta_{1,c}(p_{2,c}(u_{111}+(1-p_{2,c})(u_{121})))} + e^{\beta_{1,d}(p_{2,c}(u_{211}+(1-p_{2,c})(u_{221})))}} \tag{A.4}$$

$$p_{2,c} = \frac{e^{\beta_{2,c}(p_{1,c}(u_{112}+(1-p_{1,c})(u_{212})))}}{e^{\beta_{2,d}(p_{1,c}(u_{112}+(1-p_{1,c})(u_{212})))} + e^{\beta_{2,d}(p_{1,c}(u_{122}+(1-p_{1,c})(u_{222})))}} \tag{A.5}$$

Here, $p_{1,c}$ and $p_{2,c}$ are the probability of cooperating for player 1 and 2, respectively. The rationalities $\beta_{1,c}$ and $\beta_{1,d}$ can be used to quantify the influence on player 1 on cooperating and defecting respectively. These influences would be determined by the distances from the influencers or seeds adopting each strategy. Similarly, $\beta_{1,d}$ and $\beta_{2,d}$ signify the influence based rationalities of defection for player 1 and 2 respectively. Formally put,

$$\beta_{n,c}\, \alpha \sum_{i=1}^{N} 1/d_{n,i_c} \tag{A.6}$$

where $\beta_{n,c}$ is the influence based rationality of cooperating in node n, d_{n,i_c} is the distance of node n from the influencer i (which is a pure cooperator) and N would be the total number of cooperator influencers within the network. The influence based rationality of defecting can also be calculated in a similar fashion. Thus, each follower would capture the influence of the cooperator and defector influence nodes within the network, through the distance based bounded rationality for each strategy. Similar to the influence game, this model can be extended to incorporate multiple types of influencers with multiple strategies.

A.4 Optimizing social influence using bounded rationality models

In this sub-section, a method to place the influencers in order to maximise their influence on the population, based on the bounded rationality based influence models that were suggested in the previous sub-section. For this purpose, two scenarios are considered. One is where the network has only a single type of influencers and the requirement is to select the placement of influencers to maximise their influence. This is termed as the influence maximisation problem in the literature[133]. The other is the scenario where the network two kinds of opposing influencers. Supposing the network is already occupied with one type of influencers, and it is necessary to identify the optimum way of placing the rivalling set of influencers, so that the influence of the originally placed influencers is minimised. These optimisations would be applicable to the influencer-follower game that was discussed earlier and also more general strategic decision making situations.

Firstly, let us consider a scenario where there is only one type of seeds or influencers in a network in a influencing game. As discussed previously, the fundamental assumption that is made here is that the bounded rationality of following is inversely proportional to the distance of the followers from the influencing node. Thus, the influencers are best placed in a network where the distance to the followers is minimum. The natural candidate to locate that placement would be the closeness centrality of the network, since closeness centrality[9] is used to identify the nodes that have average minimum shortest path distance to the other nodes within the network. The Eq.A.7 depicts the equation for calculating the closeness centrality of a node. When the influencers are placed according to the closeness centrality of network, the influence on the rest of the network would be highest, when there is only one type of positive influence at play.

$$C_H(x) = \sum_{y \neq x} \frac{1}{d(y,x)} \tag{A.7}$$

where $d(y, x)$ would be the distance along the shortest path from node x to node y.

Based on this premise, it is possible to formulate the most optimum method to place the rivalling or conflicting influencers when there are multiple opposing influencers in operation. Assuming that the original influencer or influencers are already placed in a network, there would be two factors that affect the effectiveness of the rivalling influencers. Those two factors are,

Active factor The distance from the rivalling influencers to the follower nodes in the network

Passive factor The ability of the rivalling influencers to 'block' the influence of the original influencers

The first factor could be measured using the closeness centrality of the network. This would be called the 'active factor' within the context of this chapter, since the opposing nodes would directly influence the other nodes to follow them. The second factor becomes relevant since the rationality is spread through the network topology. If a node is occupied by an opposing influencer, then it no longer would take part in spreading the influence of the original influencers. From the perspective of the original set of influencers, the opposing influencers would cease to exist in the network, in their quest to spread their influence. Thus, simply by being placed in positions where the distance from the original influencers to the followers could be increased, the rivalling influencers can minimise the influence of the original set of influencers. In other words, the opponent influencers are best placed in 'between' the original influencers and the followers of the network. Thus, another well-known centrality measure, betweenness centrality[9] can be employed to identify the topological positions where the influence of the original influencers would be minimised. However, it is not necessary to measure the betweenness centrality values of all nodes in the network to identify these positions. Only the betweenness centrality values for the original influencers would be sufficient to identify the topological positions where there influence can most effectively be interfered. This would be called the 'passive factor' of negative influence since the opponent influencers reduce the influence of the original influencers simply by passively occupying the high betweenness centrality nodes.

By combining these two active and passive negative factors, it is possible to derive a method that could be used to identify the positions where it would be most effective to place the negative influence seeds. The algorithm 11 depicts this method that is used to optimally place the negative influencers in a network where there are already existing positive influencers.

Algorithm 11: Optimum influence arrangement of negative influencing seeds, when a network is already occupied with positive influencing seeds.

Data: Network topology, Seeds
Result: Optimum arrangement of opposing seeds
Measure the betweenness centrality values of the nodes from the original set seeds;
Measure the closeness centrality of the entire network;
Calculate the average of these two measures;
Order the nodes in descending order based on this averaged measure;
Place the opposing seeds in the order of the combined measure;

A.4.1 Complexity analysis

Influence maximisation is an optimisation problem that is tries to optimally place the seeds in order to the influence spread. Under the independent cascade model and linear threshold model, the influence maximisation becomes an NP-hard problem[133]. Therefore, in order to solve them, it is necessary to employ a greedy algorithm to find the optimum seed arrangement. The greedy algorithm of optimising influence spread outperforms the degree and centrality based heuristics[133, 134]. Still, it requires an approximation using the Monte-Carlo simulations of the influence cascade model over a number of interactions to obtain an accurate estimation of the influence spread. Thus, it may be inefficient in a sufficiently large network with multiple seeds involved.

Using the proposed bounded rationality based influence model, it is possible to devise a deterministic algorithm that can be used to identify the optimum set of influencers without depending on heuristics. The time complexity of finding the optimum placement of a single type of influencers would be $O(V, E)$ where V would be the number of vertices and E would be the number of edges in the network, which is the time complexity of calculating the closeness centrality. The time complexity of optimising the negative influence when there are opposing influencers would be $O(V, E) + O(V_S, E)$, where V_S would be the initial set of seed nodes. Based on the proposed influence model, it is possible to measure the actual optimisation problem in polynomial time. Also, instead of an approximation, it is possible to derive an exact solution using the bounded rationality based influence model.

A.5 Methodology

In this sub-section, the methodology that is used to perform simulations to demonstrate the applicability of the bounded rationality based influence model is discussed. First, three different network topologies are compared, scale-free, ER random and well-mixed, in how they would facilitate the influence spread under a bounded rationality influence model. In order to do that, a seed was placed at the hub of each network and measure the average probability of following the seed, over the network. The network with the lowest diameter would be the one that facilitates most influence. The two states that are considered are either active or inactive states where active refers to adopting the state of the influencer. However, since the QRE model gives a probability distribution of whether a node would follow the influencer or not, the outcome would not be a binary state distribution, rather a probability distribution of being in the active state. The payoff of adopting the state of the seed is set to be 1 and not adopting the state is set to 0.

$$p_{n_1} = \frac{e^{(\beta_{n_i} \cdot p_i^1 .1)}}{e^{(\beta_{n_i} \cdot p_i^1 .1)} + e^{(\beta_{n_i} \cdot p_i^0 .0)}} \tag{A.8}$$

Here p_{n_1} would be the probability of node n being in state 1, where state 1 denotes being active and 0 denotes being inactive. β_{n_i} would be node n's 'rationality of following' with respect to the influencer i, which would be dependent on node n's distance along the shortest path from the influencer i. Therefore, $\beta_{n_i} = c/d_{n_i}$ where c is a constant and d_{n_i} is the distance of node n from node i along the shortest path. 1 and 0 are the payoffs of state 1 and 0 respectively.

Next, in order to test how the payoff of following affects the probability distribution of following the seed, the same experiment was performed while varying the payoff of the active state. A scale-free topology was used for this simulation and the variation of the average probability of following over the payoff of the active state was observed.

Then, the effectiveness of the algorithm to reduce the influence of the hub by placing opposing influencers in the network was evaluated. To test this, two different scenarios were considered. In the first one, the hub is placed with the original influencing node and 4 opposing influencers are distributed according to different configurations. In the first configuration, they are placed in the remaining hubs in the order of the degree. In the next two configurations, the opponents are placed in the order of betweenness from the hub and the closeness centrality of the network, respectively. In the fourth configuration, the opponents are placed in the order of the combined measure that we presented in algorithm 11. The variation of negative influence was compared and contrasted by observing the adoption probability under these four configurations. The same experiment was then repeated for multiple conflicting influencers on both sides. The original set of influencers were randomly distributed and four opposing influencers are placed according to the four configurations mentioned above. This experiment was repeated over twenty iterations to account for the effect of randomness in the initial configuration of influencers. It is assumed that the followers would adopt either of the active or the negatively active states, thus they wouldn't be in an inactive state. It should be noted that an inactive state could also be incorporated by extending the QRE model that is used. The payoffs of following each type of influencers were set to 2 and 1 respectively. In each type of influencer, the payoff of not following the influencer would be a negative payoff of -2 and -1 respectively. Eq. A.9 shows how the probability of a node being in state s, which is the state of the original set of influencers, being calculated using QRE and distance induced bounded rationality.

$$p_{n_s} = \frac{\sum_{i=1}^{N} e^{(\beta_{n_i} \cdot p_i^s . 2)} + \sum_{j=1}^{M} e^{(\beta_{n_j} \cdot p_j^s . -1)}}{\sum_{i=1}^{N} e^{(\beta_{n_i} \cdot p_i^s . 2)} + e^{(\beta_{n_i} \cdot p_i^{-s} . -2)} + \sum_{j=1}^{M} e^{(\beta_{n_j} \cdot p_j^s . -1)} + e^{(\beta_{n_j} \cdot p_j^{-s} . 1)}} \tag{A.9}$$

Here, p_{n_1} would be the probability of node n adopting state s. β_{n_i} would be the rationality of following the influencers of state s where N is the number of such influencers. Also, β_{n_j} would be the rationality of following the influencer of state $-s$ where M would be the number of such influencers. The values +2 and -2 are the payoffs of either following or not following the influencers of state 1. Similarly, +1 and -1 are the payoffs of either following or not following the influencers of state $-s$. The rationalities of following would be inversely proportional to the distance from each influencer. For instance, the rationality β_{n_j} of following an influencer of type j is set as c/d_{n_j} where c is a constant and d_{n_j} is the distance from node n to influencer j.

Then it was observed how the strategic game based influence would operate in a bounded rationality based influence model. In particular, the optimum method to reduce the influence of already existing influencers was evaluated. The Prisoner's dilemma game was used with four cooperators distributed randomly in the network while four defectors try to negatively influence to minimise cooperation in the network. The rationality of cooperation and defection would be negatively proportional to the distance to each influencer. As with the previous experiment, different opponent placement strategies were compared with the optimum placement strategy that was discussed in 11. The payoffs of the PD game were set such that $u_{111}, u_{122} = 4, u_{121}, u_{212} = 0, u_{122}, u_{211} = 5$ and $u_{221}, u_{222} = 1$. The equations Eq. A.4 and Eq. A.5 are used to calculate the probability of cooperation in each iteration. For each node, the rationality of cooperation and defection are calculated by taking into the cumulative effect of influencers of each type. For example, for node n the rationality of cooperation would be $\sum_{i=1}^{N} fraccd_{n_i}$, where c is a constant and N would be the number of cooperators in the network while d_{n_i} would be the distance to each cooperator from the node.

These experiments could be used evaluate how the bounded rationality based influence modelling can be applied in a influencer-follower scenario or a strategic game scenario, where there are a dedicated set of influencers or seeds and the rest of the population is following them. It should be noted that although we consider only two types of rivalling influencers or strategies, the bounded rationality based influence model could be expanded for multiple types of influencers and strategies as well.

A.6 Results

The Table A.1 shows the comparison of the average following probability under the three network topologies considered. As the table shows, the scale-free topology facilitates the highest average following probability compared to the ER random and lattice topologies. This is due to the fact that a scale-free network of a comparative size and average degree may have a lower diameter compared to a ER-random or a lattice network.

Table A.1: The average probability of following the seed in different topologies

Network topology	Average probability of following
Scale-free	0.95
ER Random	0.89
Lattice	0.51

The Fig. A.1 depicts the variation of the following probabilities in a scale-free network, when the seed is placed at the hub and when the payoff for adopting the state of the seed is increased. As shown in the figure, there is a clear positive correlation between the payoff of the active state and the influence spread. Thus, this shows that not only the topology of the network and the positioning of the seed(s) but also the payoff of the active state is critical in determining the spread of influence.

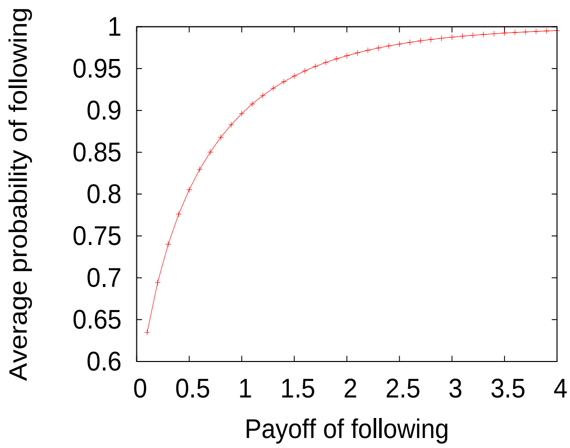


Figure A.1: The variation of the probability of following against the payoff of the active state.

Next, the results of the experiment when two conflicting types of influencers are placed in a scale-free network, are discussed. The original influencer is placed in a hub and four opposing influencers are place according to four different configurations. Namely, they are placed according to the degree centrality of the network, betweenness centrality from the hub, closeness centrality of the network and a combination of the second and third measures (as discussed in algorithm11). Fig. A.2 shows the comparison when the influencing seed is placed the hub in opposition to four counter influencers in the same four configurations. As the figure shows, the combined measure method proposed in algorithm 11 gives the best results in terms of maximising the negative influencer, thereby reducing the average probability of following. The table A.2 shows the average probability of following when the opposing seeds are placed in the four different configurations. The results reiterate that it is the negative seed placement method discussed in the algorithm 11 that provides the optimum reduction of the influence from the original seed.

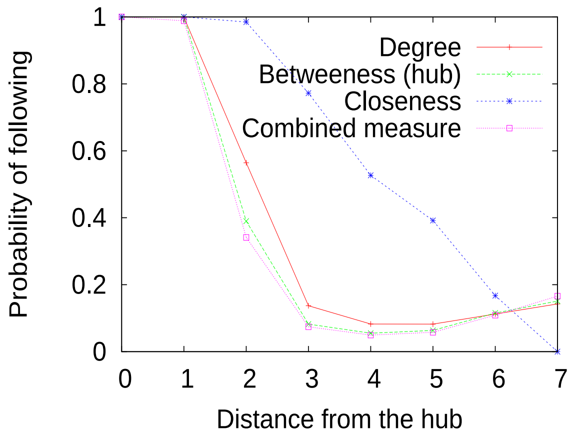


Figure A.2: The variation of the probability of following against the distance from seed placed at hub in a scale-free network. Four opposing seeds are placed in the order of (i) degree, (ii) betweenness from the hub, (iii) closeness centrality, (iv) combination of both of (ii) & (iii).

Table A.2: The average probability of following the seed in different topologies. The opposing seeds are placed in four different configurations.

Opposing influencer configuration	Average following probability
Degree	0.29
Betweenness (hub)	0.21
Closeness	0.79
Combined measure (2 & 3)	0.19

Fig. A.3 depicts the variation of following probability against the average distance from the influencing seeds when multiple original influencers are placed randomly. The results are averaged over twenty independent runs. As the figure depicts, it is the combined measure used to determine the placement of the opposing seeds that mitigate the influence of the original set of influencers, most effectively. This is further confirmed by the comparison of average probabilities of following, given in table A.3.

Next, the results for the same set of experiments repeated for a strategic decision making scenario are presented, where the PD game is played over a scale-free network. Fig. A.4 shows the results for the scenario when the coordinator seed is placed at the hub and the defecting seeds are placed according to the four different configurations discussed

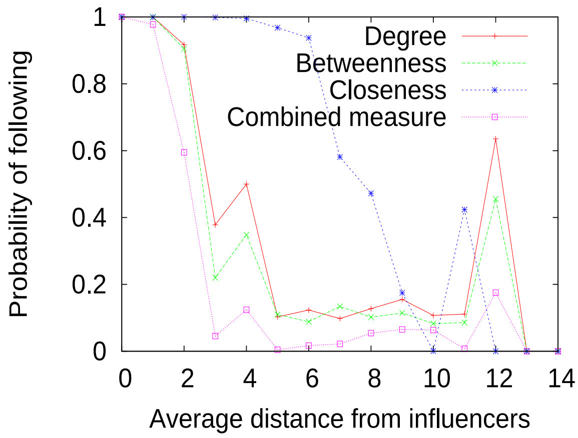


Figure A.3: The variation of the probability of following against the average distance from seeds placed at random positions in a scale-free network. The opposing seeds are placed in the order of (i) degree, (ii) betweenness from the hub, (iii) closeness centrality, (iv) combination of both (ii) & (iii). The results are averaged over twenty independent runs.

Table A.3: The average probability of following the randomly placed seeds in different topologies. The opposing seeds are placed according to four different configurations.

Opposing influencer configuration	Average following probability
Degree	0.30
Betweenness (seeds)	0.19
Closeness	0.95
Combined measure (2 & 3)	0.04

above. Similar to the influencer-follower game, the strategic decision making scenario too is most affected when the opposing strategies are placed according to the combined measure. Table A.4 shows the average probability of cooperation in those four types of configurations of placing the defectors. The placement of opponent strategies purely based on the betweenness centrality from the hub adopting the pure coordinator strategy too facilitates an effective reduction of cooperation in the overall network.

The Fig. A.5 depicts the variation of cooperation against the average distance from multiple cooperator seeds that are placed randomly. The defector seeds are placed according to the four configurations discussed previously. Here too, the combined method of placing the opposing strategies make the highest reduction in the cooperator strategy in the network, further emphasised by the average cooperator probabilities shown in Table A.5.

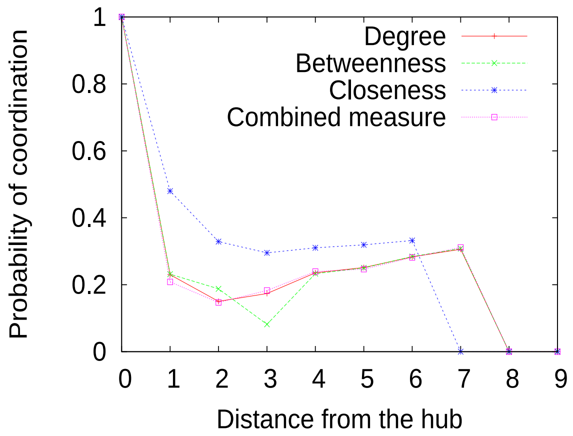


Figure A.4: The probability of cooperation against the distance from the cooperator seed placed at hub in a scale-free network. Four defector seeds are placed in the order of (i) degree, (ii) betweenness from the hub, (iii) closeness, (iv) combination of both of (ii) & (iii).

Table A.4: The average probability of cooperation in different topologies when the cooperator seed is placed at the hub. The defector seeds are placed in four different topologies.

Opposing strategy configuration	Average probability of cooperation
Degree	0.226
Betweenness (hub)	0.209
Closeness	0.335
Combined measure (2 & 3)	0.206

A.7 Discussion

In this work, a novel social influence model is proposed based on the bounded rationality of agents in a social network. First, the social influence is modelled as an influence game where there are two types of players, influencers and followers. The followers are assumed to be following the influencers based on a bounded rationality, which is inversely proportional to their distance from the influencers. Based on this model, it is shown that scale-free networks facilitate social influence compared to ER random and lattice networks.

Then this model is extended to scenarios where there are multiple and opposing seeds. A method was proposed to optimally place the negative seeds to minimise the influence of the original set of positive influencers. In this method, the opponents are placed according to the order of a combined centrality measure, which is the average of the betweenness centrality from the original seeds and the closeness centrality of the nodes within the network. It is found that in general, placing the opponents in the order of betweenness

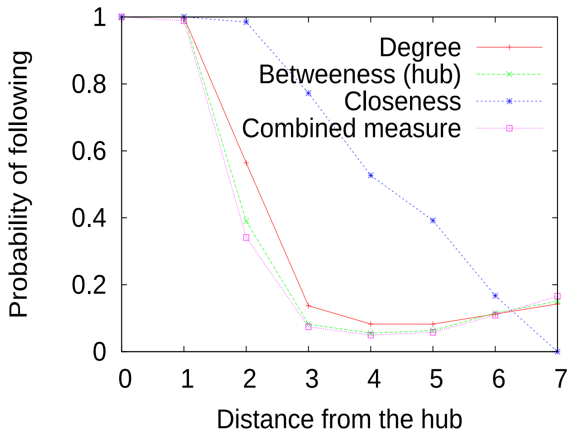


Figure A.5: The variation of coordination against the average distance from cooperators that are randomly placed in a scale-free network. Four opposing seeds are placed in the order of (i) degree, (ii) betweenness from the hub, (iii) closeness, (iv) combination of both of (ii) & (iii).

Table A.5: The average probability of cooperation when the cooperator seeds are randomly placed in different topologies. The opposing seeds are placed according to four different configurations.

Opposing strategy configuration	Average probability of cooperation
Degree	0.36
Betweenness (seeds)	0.35
Closeness	0.44
Combined measure (2 & 3)	0.25

centrality from the original seeds, thereby 'interfering' their influence to the rest of the network, is more effective than choosing the nodes that have higher closeness centrality within the entire network to place the rivalling influencers. Moreover, the proposed combined centrality measure performs best in maximising the negative influence, when the original seed is placed at the hub, or when multiple original seeds randomly distributed.

Further, this influence model is extended to strategic decision making scenarios in a social network. Here, each node is assumed to have a rationality of following each strategy and that rationality is negatively proportional to the distance from each seed with that strategy within the network. This approach makes it possible to model the network with heterogeneous rationalities in nodes that are dependent on the number of seeds and the distances from them. Using this model it is possible demonstrate that as with the influence game, strategic games like the prisoner's dilemma game could be simulated in a social network with bounded rationalities that are influenced by the seeds that have permanently adopted a particular strategy. The followers would adopt their respective strategies based on the bounded rationality of following each strategy and their respective payoffs. Based on this model, it could be shown that the most efficient way to counter an existing strategy is to place the opponent strategy seeds in the order of the combined centrality measure of betweenness from the original strategy seeds and the closeness centrality of the network.

To our understanding, this is the first attempt to model the social influence using bounded rationality and the QRE model. The applications of such a model could be myriad, especially it allows the computation of social influence in a computationally efficient manner. Moreover, as it is based on game theoretic principles, it allows the payoff of following an influencer or adopting a strategy to be a key variable in the modelling, which is not present in the standard social influence models. Countering the influence of an existing network is a critical problem that may have many applications in scientific, social and political networks. Thus, the combined centrality measure of placing opponent nodes and strategies may be quite useful in negatively affecting existing social influence. This model can be even applied to model social influence in scenarios where there are multiple types of influencers and strategies, such as in a political campaign. Further research is needed to explore the applicability of this model in real-world social networks.

Bibliography

[1] "Konisberg bridges problem," http://physics.weber.edu/carroll/honors/konigsberg.htm, accessed: 2015-02-03.

[2] "Seven bridges of knigsberg," http://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg, accessed: 2015-02-03.

[3] E. Aarts and J. Korst, "Simulated annealing and boltzmann machines: a stochastic approach to combinatorial optimization and neural computing," 1988.

[4] C. Adami and A. Hintze, "Evolutionary instability of zero-determinant strategies demonstrates that winning is not everything," *Nature communications*, vol. 4, no. 2193, 2013.

[5] E. Ahmed and A. Elgazzar, "On coordination and continuous hawk-dove games on small-world networks," *The European Physical Journal B-Condensed Matter and Complex Systems*, vol. 18, no. 1, pp. 159–162, 2000.

[6] Y.-Y. Ahn, J. P. Bagrow, and S. Lehmann, "Communities and Hierarchical Organization of Links in Complex Networks," no. arXiv:0903.3178, Mar 2009. [Online]. Available: http://cds.cern.ch/record/1167633

[7] R. Albert, I. Albert, and G. L. Nakarado, "Structural vulnerability of the north american power grid," *Physical review E*, vol. 69, no. 2, p. 025103, 2004.

[8] R. Albert and A.-L. Barabási, "Emergence of scaling in random networks," *Science*, vol. 286, pp. 509–512, 1999.

[9] R. Albert and A.-L. Barabási, "Statistical mechanics of complex networks," *Reviews of Modern Physics*, vol. 74, pp. 47–97, 2002.

[10] R. Albert, H. Jeong, and A.-L. Barabási, "Internet: Diameter of the world-wide web," *Nature*, vol. 401, no. 6749, pp. 130–131, 1999.

[11] M. Aldana, E. Balleza, S. Kauffman, and O. Resendiz, "Robustness and evolvability in genetic regulatory networks," *Journal of theoretical biology*, vol. 245, no. 3, pp. 433–448, 2007.

[12] U. Alon, *Introduction to Systems Biology: Design Principles of Biological Circuits*. London: Chapman and Hall, 2007.

[13] E. Altman, T. Boulogne, R. El-Azouzi, T. Jiménez, and L. Wynter, "A survey on networking games in telecommunications," *Computers & Operations Research*, vol. 33, no. 2, pp. 286–311, 2006.

[14] L. A. N. Amaral, A. Scala, M. Barthelemy, and H. E. Stanley, "Classes of small-world networks," *Proceedings of the national academy of sciences*, vol. 97, no. 21, pp. 11 149–11 152, 2000.

[15] H. Amini, R. Cont, and A. Minca, "Resilience to contagion in financial networks," *Mathematical Finance*, 2013.

[16] R. J. Aumann, "Backward induction and common knowledge of rationality," *Games and Economic Behavior*, vol. 8, no. 1, pp. 6–19, 1995.

[17] R. Axelrod, "The evolution of strategies in the iterated prisoner's dilemma," *The dynamics of norms*, pp. 1–16, 1987.

[18] R. Axelrod, D. E. Axelrod, and K. J. Pienta, "Evolution of cooperation among tumor cells," *Proceedings of the National Academy of Sciences*, vol. 103, no. 36, pp. 13 474–13 479, 2006.

[19] P. Bajari and A. Hortacsu, "Economic insights from internet auctions: A survey," National Bureau of Economic Research, Working Paper 10076, November 2003.

[20] W. E. Baker, "The social structure of a national securities market," *American journal of sociology*, pp. 775–811, 1984.

[21] A. Bandura, "Social cognitive theory: An agentic perspective," *Annual review of psychology*, vol. 52, no. 1, pp. 1–26, 2001.

[22] A. Bandura and D. C. McClelland, "Social learning theory," 1977.

[23] Y. Bar-Yam, "General features of complex systems," *Encyclopedia of Life Support Systems (EOLSS), UNESCO, EOLSS Publishers, Oxford, UK*, 2002.

[24] A.-L. Barabási, "Scale-free networks: A decade and beyond," *Science*, vol. 325, no. 5939, pp. 412–413, 2009.

[25] A.-L. Barabási and R. Albert, "Emergence of scaling in random networks," *science*, vol. 286, no. 5439, pp. 509–512, 1999.

[26] A.-L. Barabási, R. Albert, and H. Jeong, "Scale-free characteristics of random networks: The topology of the world-wide web," *Physica A*, vol. 281, pp. 69–77, 2000.

[27] A.-L. Barabási and E. Bonabeau, "Scale-free networks," *Scientific American*, vol. 288, pp. 50–59, 2003.

[28] E. N. Barron, *Game theory: an introduction.* John Wiley & Sons, 2013, vol. 2.

[29] D. S. Bassett and E. Bullmore, "Small-world brain networks," *The neuroscientist*, vol. 12, no. 6, pp. 512–523, 2006.

[30] M. B. Baxter, *A Darwinian worldview: Sociobiology, environmental ethics and the work of Edward O. Wilson.* Ashgate Publishing, Ltd., 2012.

[31] G. S. Becker, "A theory of competition among pressure groups for political influence," *The Quarterly Journal of Economics*, pp. 371–400, 1983.

[32] N. Bellomo, *Modeling complex living systems: a kinetic theory and stochastic game approach.* Springer, 2008.

[33] J. Bendor and P. Swistak, "Types of evolutionary stability and the problem of cooperation." *Proceedings of the National Academy of Sciences*, vol. 92, no. 8, pp. 3596–3600, 1995.

[34] W. Bennett, "Communicating global activism," *Information, Communication & Society*, vol. 6, no. 2, pp. 143–168, 2003.

[35] N. Biggs, E. K. Lloyd, and R. J. Wilson, *Graph Theory, 1736-1936.* Clarendon Press, 1986.

[36] B. Bimber, “The internet and political transformation: Populism, community, and accelerated pluralism,” *Polity*, pp. 133–160, 1998.

[37] A. Blanc, Y.-K. Liu, and A. Vahdat, “Designing incentives for peer-to-peer routing,” in *INFOCOM 2005. 24th Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings IEEE*, vol. 1, pp. 374–385. IEEE, 2005.

[38] P. Bonacich, “Eigenvector-like measures of centrality for asymmetric relations,” *Social Networks*, vol. 23, no. 3, pp. 191–201, July 2001.

[39] R. Boyd and P. J. Richerson, “Culture and the evolution of the human social instincts,” *Roots of human sociality*, pp. 453–477, 2006.

[40] Y. Bramoullé and R. Kranton, “Public goods in networks,” *Journal of Economic Theory*, vol. 135, no. 1, pp. 478–494, 2007.

[41] A. Brandenburger, “Cooperative game theory,” *Teaching Materials at New York University*, 2007.

[42] J. J. Brown and P. H. Reingen, “Social ties and word-of-mouth referral behavior,” *Journal of Consumer research*, pp. 350–362, 1987.

[43] D. M. Cable and S. Shane, “A prisoner’s dilemma approach to entrepreneur-venture capitalist relationships,” *Academy of Management Review*, vol. 22, no. 1, pp. 142–176, 1997.

[44] G. Caldarelli, “Scale-free networks: complex webs in nature and technology,” *OUP Catalogue*, 2007.

[45] C. Camerer, *Behavioral game theory: Experiments in strategic interaction.* Princeton University Press, 2003.

[46] C. F. Camerer, T.-H. Ho, and J.-K. Chong, “A cognitive hierarchy model of games,” *The Quarterly Journal of Economics*, pp. 861–898, 2004.

[47] A. Cavagna, A. Cimarelli, I. Giardina, G. Parisi, R. Santagati, F. Stefanini, and M. Viale, “Scale-free correlations in starling flocks,” *Proceedings of the National Academy of Sciences*, vol. 107, no. 26, pp. 11 865–11 870, 2010.

[48] H. Chen, P. De, Y. J. Hu, and B.-H. Hwang, "customers as advisors: the role of social media in financial markets," in *3rd Annual Behavioural Finance Conference, Queen's University, Kingston, Canada.¡ http://www. bhwang. com/customers. pdf*, 2013.

[49] P. Chen, H. Xie, S. Maslov, and S. Redner, "Finding scientific gems with google's pagerank algorithm," *Journal of Informetrics*, vol. 1, no. 1, pp. 8–15, 2007.

[50] W. Chen, A. Collins, R. Cummings, T. Ke, Z. Liu, D. Rincon, X. Sun, Y. Wang, W. Wei, and Y. Yuan, "Influence maximization in social networks when negative opinions may emerge and propagate." in *SDM*, pp. 379–390. SIAM, 2011.

[51] W. Chen, Y. Wang, and S. Yang, "Efficient influence maximization in social networks," in *Proceedings of the 15th ACM SIGKDD international conference on Knowledge discovery and data mining*, pp. 199–208. ACM, 2009.

[52] X. Chen and L. Wang, "Promotion of cooperation induced by appropriate payoff aspirations in a small-world networked game," *Physical Review E*, vol. 77, no. 1, p. 017103, 2008.

[53] S. Choi, D. Gale, and S. Kariv, "Social learning in networks: a quantal response equilibrium analysis of experimental data," *Review of Economic Design*, vol. 16, no. 2-3, pp. 135–157, 2012.

[54] N. Christin and J. Chuang, "On the cost of participating in a peer-to-peer network," in *Peer-to-Peer Systems III.* Springer, 2005, pp. 22–32.

[55] N. Christin, J. Grossklags, and J. Chuang, "Near rationality and competitive equilibria in networked systems," in *Proceedings of the ACM SIGCOMM workshop on Practice and theory of incentives in networked systems*, pp. 213–219. ACM, 2004.

[56] R. M. Christley, G. Pinchbeck, R. Bowers, D. Clancy, N. French, R. Bennett, and J. Turner, "Infection in social networks: using network analysis to identify high-risk individuals," *American journal of epidemiology*, vol. 162, no. 10, pp. 1024–1031, 2005.

[57] B.-G. Chun, R. Fonseca, I. Stoica, and J. Kubiatowicz, "Characterizing selfishly constructed overlay routing networks," in *INFOCOM 2004. Twenty-third AnnualJoint*

Conference of the IEEE Computer and Communications Societies, vol. 2, pp. 1329–1339. IEEE, 2004.

[58] A. Clark and R. Poovendran, "Maximizing influence in competitive environments: a game-theoretic approach," in *Decision and Game Theory for Security.* Springer, 2011, pp. 151–162.

[59] A. Clauset, C. R. Shalizi, and M. E. Newman, "Power-law distributions in empirical data," *SIAM review*, vol. 51, no. 4, pp. 661–703, 2009.

[60] D. L. Cogburn and F. K. Espinoza-Vasquez, "From networked nominee to networked nation: Examining the impact of web 2.0 and social media on political participation and civic engagement in the 2008 obama campaign," *Journal of Political Marketing*, vol. 10, no. 1-2, pp. 189–213, 2011.

[61] A. M. Colman, *Game theory and its applications: In the social and biological sciences.* Psychology Press, 2013.

[62] R. Costanza, L. Wainger, C. Folke, and K.-G. Mäler, "Modeling complex ecological economic systems," *BioScience*, pp. 545–555, 1993.

[63] T. M. Cover and J. A. Thomas, "Entropy, relative entropy and mutual information," *Elements of Information Theory*, pp. 12–49, 1991.

[64] T. M. Cover and J. A. Thomas, *Elements of information theory.* John Wiley & Sons, 2012.

[65] N. De Silva, "Constructive relativism," in *10th International Conference on Sri Lanka Studies, University of Kelaniya*, p. 109, 2005.

[66] I. de Sola Pool and M. Kochen, "Contacts and influence," *Social networks*, vol. 1, no. 1, pp. 5–51, 1979.

[67] E. Dekel, D. Fudenberg, and D. K. Levine, "Learning to play bayesian games," *Games and Economic Behavior*, vol. 46, no. 2, pp. 282–303, 2004.

[68] F. R. Denton and P. J. Heald, "Random walks, non-cooperative games, and the complex mathematics of patent pricing," *Rutgers L. Rev.*, vol. 55, p. 1175, 2002.

[69] T. Dimpfl and F. J. Peter, "Using transfer entropy to measure information flows between financial markets," *Studies in Nonlinear Dynamics and Econometrics*, vol. 17, no. 1, pp. 85–102, 2013.

[70] Y. Ding, "Scientific collaboration and endorsement: Network analysis of coauthorship and citation networks," *Journal of informetrics*, vol. 5, no. 1, pp. 187–203, 2011.

[71] P. Domingos and M. Richardson, "Mining the network value of customers," in *Proceedings of the seventh ACM SIGKDD international conference on Knowledge discovery and data mining*, pp. 57–66. ACM, 2001.

[72] S. N. Dorogovtsev and J. F. Mendes, *Evolution of networks: From biological nets to the Internet and WWW.* Oxford University Press, 2013.

[73] S. N. Dorogovtsev and J. F. F. Mendes, *Evolution of Networks: From Biological Nets to the Internet and WWW.* Oxford: Oxford University Press, January 2003.

[74] B. Drossel, "Random boolean networks," *Reviews of nonlinear dynamics and complexity*, vol. 1, pp. 69–110, 2008.

[75] R. I. Dunbar, "The social brain hypothesis," *brain*, vol. 9, no. 10, pp. 178–190, 1998.

[76] R. I. Dunbar, "The social brain hypothesis and its implications for social evolution," *Annals of human biology*, vol. 36, no. 5, pp. 562–572, 2009.

[77] J. A. Dunne, R. J. Williams, and N. D. Martinez, "Network structure and biodiversity loss in food webs: robustness increases with connectance," *Ecology letters*, vol. 5, no. 4, pp. 558–567, 2002.

[78] D. Easley and J. Kleinberg, "Networks, crowds, and markets," 2012.

[79] V. M. Eguiluz, D. R. Chialvo, G. A. Cecchi, M. Baliki, and A. V. Apkarian, "Scale-free brain functional networks," *Physical review letters*, vol. 94, no. 1, p. 018102, 2005.

[80] M. Elliott and B. Golub, "A network approach to public goods," in *Proceedings of the fourteenth ACM conference on Electronic commerce*, pp. 377–378. ACM, 2013.

[81] P. Erdős and A. Rényi, "On random graphs," *Publicationes Mathematicae Debrecen*, vol. 6, pp. 290–297, 1959.

[82] R. Ferrero, S. Shahidehpour, and V. Ramesh, "Transaction analysis in deregulated power systems using game theory," *Power Systems, IEEE Transactions on*, vol. 12, no. 3, pp. 1340–1347, 1997.

[83] D. B. Fogel, "Evolving behaviors in the iterated prisoner's dilemma," *Evolutionary Computation*, vol. 1, no. 1, pp. 77–97, 1993.

[84] B. Fuglede and F. Topsoe, "Jensen-shannon divergence and hilbert space embedding," in *IEEE International Symposium on Information Theory*, pp. 31–31, 2004.

[85] A. Galeotti, S. Goyal, M. O. Jackson, F. Vega-Redondo, and L. Yariv, "Network games," *The review of economic studies*, vol. 77, no. 1, pp. 218–244, 2010.

[86] C. Gershenson, "Introduction to random boolean networks," *arXiv preprint nlin/0408006*, 2004.

[87] G. Ghoshal and A.-L. Barabási, "Ranking stability and super-stable nodes in complex networks," *Nature communications*, vol. 2, p. 394, 2011.

[88] G. Gigerenzer and R. Selten, *Bounded rationality: The adaptive toolbox.* Mit Press, 2002.

[89] R. E. Gleichauf, W. A. Randall, D. M. Teal, S. V. Waddell, and K. J. Ziese, "Method and system for adaptive network security using network vulnerability assessment," Oct. 9 2001, uS Patent 6,301,668.

[90] J. K. Goeree, C. A. Holt, and T. R. Palfrey, "Quantal response equilibrium and overbidding in private-value auctions," *Journal of Economic Theory*, vol. 104, no. 1, pp. 247–272, 2002.

[91] J. K. Goeree, C. A. Holt, and T. R. Palfrey, "Quantal response equilibrium," *The New Palgrave Dictionary of Economics. Palgrave Macmillan, Basingstoke*, 2008.

[92] D. E. Goldberg and J. H. Holland, "Genetic algorithms and machine learning," *Machine learning*, vol. 3, no. 2, pp. 95–99, 1988.

[93] R. Golman, "Homogeneity bias in models of discrete choice with bounded rationality," *Journal of Economic Behavior & Organization*, vol. 82, no. 1, pp. 1–11, 2012.

[94] M. Granovetter, "Network sampling: Some first steps," *American Journal of Sociology*, pp. 1287–1303, 1976.

[95] D. T. Green and J. M. Pearson, "Social software and cyber networks: Ties that bind or weak associations within the political organization?" in *System Sciences, 2005. HICSS'05. Proceedings of the 38th Annual Hawaii International Conference on*, pp. 117b–117b. IEEE, 2005.

[96] M. Greiff, "Learning with whom to interact: A public good game on a dynamic network," *Etica E Politica*, vol. 15, no. 2, pp. 58–81, 2013.

[97] P. A. Haile, A. Hortaçsu, and G. Kosenok, "On the empirical content of quantal response equilibrium," *The American Economic Review*, vol. 98, no. 1, pp. 180–200, 2008.

[98] M. Harre and A. Snyder, "Transitions in learning about your opponent," *IEEE Conference on Intelligent Computing and Intelligent Systems*, 2011.

[99] M. S. Harré, "Navigating catastrophes: Local but not global optimisation allows for macro-economic navigation of crises," in *Journal of Physics: Conference Series*, vol. 410, no. 1, p. 012045. IOP Publishing, 2013.

[100] M. S. Harré, S. R. Atkinson, and L. Hossain, "Simple nonlinear systems and navigating catastrophes," *The European Physical Journal B*, vol. 86, no. 6, pp. 1–8, 2013.

[101] J. C. Harsanyi, "Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points," *International Journal of Game Theory*, vol. 2, no. 1, pp. 1–23, 1973.

[102] J. C. Harsanyi, "Games with incomplete information played by bayesian players, i–iii: Part i. the basic model&," *Management science*, vol. 50, no. 12_supplement, pp. 1804–1817, 2004.

[103] C. Hauert, A. Holmes, and M. Doebeli, "Public goods games," *University of Vienna*, 2005.

[104] C. Haythornthwaite, "Social network analysis: An approach and technique for the study of information exchange," *Library & information science research*, vol. 18, no. 4, pp. 323–342, 1996.

[105] X. He, G. Song, W. Chen, and Q. Jiang, "Influence blocking maximization in social networks under the competitive linear threshold model." in *SDM*, pp. 463–474. SIAM, 2012.

[106] S. Hick and J. G. McNutt, *Advocacy, activism, and the Internet: Community organization and social policy.* Lyceum Books, Incorporated, 2002.

[107] W. Hines, "Evolutionary stable strategies: a review of basic theory," *Theoretical Population Biology*, vol. 31, no. 2, pp. 195–272, 1987.

[108] K. Hölttä, E. S. Suh, and O. de Weck, "Tradeoff between modularity and performance for engineered systems and products," in *International Conference on Engineering Design (ICED05)*, A. Samuel and W. Lewis, Eds., p. DS35_60.1. Melbourne: The Design Society, 2005.

[109] H. Hotz, "A short introduction to game theory," 2006.

[110] P. N. Howard, A. Duffy, D. Freelon, M. Hussain, W. Mari, and M. Mazaid, "Opening closed regimes: What was the role of social media during the arab spring?" 2011. [Online]. Available: http://pitpi.org/index.php/2011/09/11/opening-closed-regimes-what-was-the-role-of-social-media-during-the-arab-spring/

[111] L. Huang and Y. Xiong, "Evaluation of microblog users influence based on pagerank and users behavior analysis," 2013.

[112] D. Iliopoulos, A. Hintze, and C. Adami, "Critical dynamics in the evolution of stochastic strategies for the iterated prisoner's dilemma," *PLoS computational biology*, vol. 6, no. 10, p. e1000948, 2010.

[113] S. Izmalkov, "Bayesian-nash games," *Online, MIT Open Courseware. Available: ocw.mit.edu*, 2005.

[114] A. B. Jaffe and M. Trajtenberg, *Patents, citations, and innovations: A window on the knowledge economy.* MIT press, 2002.

[115] R. Jain, "Designing a strategic bipartite matching market," in *Decision and Control, 2007 46th IEEE Conference on*, pp. 139–144. IEEE, 2007.

[116] H. Jeong, B. Tombor, R. Albert, Z. N. Oltvai, and A.-L. Barabási, "The large-scale organization of metabolic networks," *Nature*, vol. 407, no. 6804, pp. 651–654, 2000.

[117] L.-L. Jiang and M. Perc, "Spreading of cooperative behaviour across interdependent groups," *Scientific reports*, vol. 3, no. 2483, 2013.

[118] M. Jovanović, F. Annexstein, and K. Berman, "Modeling peer-to-peer network topologies through small-world models and power laws," in *IX Telecommunications Forum, TELFOR*, pp. 1–4, 2001.

[119] F. Karinthy, *Chains. Everything is different.* Atheneum Press Budapest, 1929.

[120] D. Kasthurirathna, A. Dong, M. Piraveenan, and I. Y. Tumer, "The failure tolerance of mechatronic software systems to random and targeted attacks," in *ASME 2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, pp. V005T06A036–V005T06A036. American Society of Mechanical Engineers, 2013.

[121] D. Kasthurirathna, M. Harre, and M. Piraveenan, "Influence modelling using bounded rationality in social networks," in *Proceedings of the 2015 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining 2015*, pp. 33–40. ACM, 2015.

[122] D. Kasthurirathna, H. Nguyen, M. Piraveenan, S. Uddin, and U. Senanayake, "Optimisation of strategy placements for public good in complex networks," in *Proceedings of the 2014 International Conference on Social Computing*, p. 1. ACM, 2014.

[123] D. Kasthurirathna and M. Piraveenan, "Cyclic preferential attachment in complex networks," *Procedia Computer Science*, vol. 18, pp. 2086–2094, 2013.

[124] D. Kasthurirathna and M. Piraveenan, "Topological stability of evolutionarily unstable strategies," in *Evolving and Autonomous Learning Systems (EALS), 2014 IEEE Symposium on*, pp. 35–42. IEEE, 2014.

[125] D. Kasthurirathna and M. Piraveenan, "Emergence of scale-free characteristics in socio-ecological systems with bounded rationality," *Scientific Reports*, vol. 5, no. 10448, 2015.

[126] D. Kasthurirathna, M. Piraveenan, and M. Harre, "Evolution of coordination in scale-free and small world networks under information diffusion constraints," in *Advances in Social Networks Analysis and Mining (ASONAM), 2013 IEEE/ACM International Conference on*, pp. 183–189. IEEE, 2013.

[127] D. Kasthurirathna, M. Piraveenan, and M. Harre, "Influence of topology in the evolution of coordination in complex networks under information diffusion constraints," *The European Physical Journal B*, vol. 87, no. 1, pp. 1–15, 2014.

[128] D. Kasthurirathna, M. Piraveenan, and G. Thedchanamoorthy, "Network robustness and topological characteristics in scale-free networks," in *Evolving and Adaptive Intelligent Systems (EAIS), 2013 IEEE Conference on*, pp. 122–129. IEEE, 2013.

[129] D. Kasthurirathna, M. Piraveenan, and G. Thedchanamoorthy, "On the influence of topological characteristics on robustness of complex networks," *Journal of Artificial Intelligence and Soft Computing Research*, vol. 3, no. 2, pp. 89–100, 2013.

[130] D. Kasthurirathna, M. Piraveenan, and S. Uddin, "Evolutionary stable strategies in networked games: the influence of topology," *Journal of Artificial Intelligence and Soft Computing Research*, vol. 5, no. 2, pp. 83–95, 2015.

[131] D. Kasthurirathna, M. Piraveenan, and S. Uddin, "Modelling networked systems using the topologically distributed bounded rationality framework," 2016.

[132] C. Kaufman, R. Perlman, and M. Speciner, *Network security: private communication in a public world*. Prentice Hall Press, 2002.

[133] D. Kempe, J. Kleinberg, and É. Tardos, "Maximizing the spread of influence through a social network," in *Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining*, pp. 137–146. ACM, 2003.

[134] D. Kempe, J. Kleinberg, and É. Tardos, "Influential nodes in a diffusion model for social networks," in *Automata, languages and programming*. Springer, 2005, pp. 1127–1138.

[135] F. Kepes (Ed), *Biological Networks.* Singapore: World Scientific, 2007.

[136] P. Kohli, Y. Bachrach, T. Graepel, G. Smyth, M. Armstrong, D. Stillwell, R. Herbrich, and M. Kearns, "Behavioral game theory on online social networks: Colonel blotto is on facebook," mimeo, Tech. Rep., 2011.

[137] D. P. Kraines and V. Y. Kraines, "Natural selection of memory-one strategies for the iterated prisoner's dilemma," *Journal of Theoretical Biology*, vol. 203, no. 4, pp. 335–355, 2000.

[138] R. Kumar, J. Novak, and A. Tomkins, "Structure and evolution of online social networks," in *Link mining: models, algorithms, and applications.* Springer, 2010, pp. 337–357.

[139] V. Latora and M. Marchiori, "Efficient behavior of small-world networks," *Physical Review Letters*, vol. 87, no. 19, p. 198701, 2001.

[140] S. Le and R. Boyd, "Evolutionary dynamics of the continuous iterated prisoner's dilemma," *Journal of theoretical biology*, vol. 245, no. 2, pp. 258–267, 2007.

[141] R. J. Leonard, "From parlor games to social science: von neumann, morgenstern, and the creation of game theory 1928-1944," *Journal of Economic Literature*, pp. 730–761, 1995.

[142] J. T. Lizier, M. Prokopenko, and A. Y. Zomaya, "The information dynamics of phase transitions in random boolean networks," in *Artificial Life XI - Proceedings of the Eleventh International Conference on the Simulation and Synthesis of Living Systems*, S. Bullock, J. Noble, R. Watson, and M. A. Bedau, Eds. MIT Press, 2008, pp. 374–381.

[143] J. T. Lizier, M. Prokopenko, and A. Y. Zomaya, "Local information transfer as a spatiotemporal filter for complex systems," *Physical Review E*, vol. 77, no. 2, p. 026110, 2008.

[144] J. T. Lizier, M. Prokopenko, and A. Y. Zomaya, "Local information transfer as a spatiotemporal filter for complex systems," *Physical Review E*, vol. 77, no. 2, p. 026110, 2008.

[145] J. T. Lizier, M. Prokopenko, and A. Y. Zomaya, "Information modification and particle collisions in distributed computation," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 20, no. 3, p. 037109, 2010.

[146] P. A. Loscocco, S. D. Smalley, P. A. Muckelbauer, R. C. Taylor, S. J. Turner, and J. F. Farrell, "The inevitability of failure: The flawed assumption of security in modern computing environments," in *Proceedings of the 21st National Information Systems Security Conference*, vol. 10, pp. 303–314, 1998.

[147] E. K. Lua, J. Crowcroft, M. Pias, R. Sharma, and S. Lim, "A survey and comparison of peer-to-peer overlay network schemes," *Communications Surveys & Tutorials, IEEE*, vol. 7, no. 2, pp. 72–93, 2005.

[148] Q. Lv, P. Cao, E. Cohen, K. Li, and S. Shenker, "Search and replication in unstructured peer-to-peer networks," in *Proceedings of the 16th international conference on Supercomputing*, pp. 84–95. ACM, 2002.

[149] S. J. Majeski, "Arms races as iterated prisoner's dilemma games," *Mathematical Social Sciences*, vol. 7, no. 3, pp. 253–266, 1984.

[150] N. Masuda and K. Aihara, "Spatial prisoner's dilemma optimally played in small-world networks," *Physics Letters A*, vol. 313, no. 1, pp. 55–61, 2003.

[151] J. Maynard Smith, "The theory of games and the evolution of animal conflicts," *Journal of theoretical biology*, vol. 47, no. 1, pp. 209–221, 1974.

[152] R. A. McCain, *Game theory: A nontechnical introduction to the analysis of strategy.* World Scientific, 2010.

[153] R. D. McKelvey and T. R. Palfrey, "Quantal response equilibria for normal form games," *Games and economic behavior*, vol. 10, no. 1, pp. 6–38, 1995.

[154] R. D. McKelvey and T. R. Palfrey, "Quantal response equilibria for extensive form games," *Experimental economics*, vol. 1, no. 1, pp. 9–41, 1998.

[155] J. C. C. McKinsey, *Introduction to the Theory of Games.* Courier Corporation, 2012.

[156] M. McPherson, L. Smith-Lovin, and J. M. Cook, "Birds of a feather: Homophily in social networks," *Annual review of sociology*, pp. 415–444, 2001.

[157] J. Mehta, C. Starmer, and R. Sugden, "The nature of salience: An experimental investigation of pure coordination games," *The American Economic Review*, pp. 658–673, 1994.

[158] S. Milgram, "The small world problem," *Psychology today*, vol. 2, no. 1, pp. 60–67, 1967.

[159] S. Milgram, "The small world problem," *Psychology Today*, vol. 1, p. 61, 1967.

[160] A. A. Mina, D. Braha, and Y. Bar-Yam, "Complex engineered systems: A new paradigm," in *Complex engineered systems.* Springer, 2006, pp. 1–21.

[161] M. Mirchev, I. Mishkovski, and L. Kocarev, "Selective attack in virus propagation processes," in *ICT Innovations 2010.* Springer, 2011, pp. 286–295.

[162] A. Mirshahvalad, J. Lindholm, M. Derlen, and M. Rosvall, "Significant communities in large sparse networks," *PloS one*, vol. 7, no. 3, p. e33721, 2012.

[163] N. Mishra, R. Schreiber, I. Stanton, and R. E. Tarjan, "Clustering social networks," in *Algorithms and Models for the Web-Graph.* Springer, 2007, pp. 56–67.

[164] A. Mislove, M. Marcon, K. P. Gummadi, P. Druschel, and B. Bhattacharjee, "Measurement and analysis of online social networks," in *Proceedings of the 7th ACM SIGCOMM conference on Internet measurement*, pp. 29–42. ACM, 2007.

[165] M. Mitchell, "Complex systems: Network thinking," *Artificial Intelligence*, vol. 170, no. 18, pp. 1194–1212, 2006.

[166] M. Mitchell, "Complex systems: Network thinking," *Artificial Intelligence*, vol. 170, no. 18, pp. 1194–1212, 2006.

[167] J. M. Montoya and R. V. Solé, "Small world patterns in food webs," *Journal of theoretical biology*, vol. 214, no. 3, pp. 405–412, 2002.

[168] D. Mookherjee and B. Sopher, "Learning behavior in an experimental matching pennies game," *Games and Economic Behavior*, vol. 7, no. 1, pp. 62–91, 1994.

[169] P. A. P. Moran *et al.*, "The statistical processes of evolutionary theory." *The statistical processes of evolutionary theory.*, 1962.

[170] Y. Moreno, R. Pastor-Satorras, and A. Vespignani, "Epidemic outbreaks in complex heterogeneous networks," *The European Physical Journal B-Condensed Matter and Complex Systems*, vol. 26, no. 4, pp. 521–529, 2002.

[171] P. Morris, *Introduction to game theory.* Springer Science & Business Media, 2012.

[172] J. D. Morrow, *Game theory for political scientists.* Princeton University Press Princeton, NJ, 1994.

[173] J. Nash, "Non-cooperative games," *Annals of mathematics*, pp. 286–295, 1951.

[174] J. F. Nash *et al.*, "Equilibrium points in n-person games," *Proceedings of the national academy of sciences*, vol. 36, no. 1, pp. 48–49, 1950.

[175] M. E. Newman, "Assortative mixing in networks," *Physical review letters*, vol. 89, no. 20, p. 208701, 2002.

[176] M. E. Newman, "Mixing patterns in networks," *Physical Review E*, vol. 67, no. 2, p. 026126, 2003.

[177] M. E. Newman, "The structure and function of complex networks," *SIAM review*, vol. 45, no. 2, pp. 167–256, 2003.

[178] M. E. Newman and D. J. Watts, "Renormalization group analysis of the small-world network model," *Physics Letters A*, vol. 263, no. 4, pp. 341–346, 1999.

[179] M. E. J. Newman, "Models of the small world," *Journal of Statistical Physics*, vol. 101, no. 3, pp. 819–841, November 2000. [Online]. Available: http://dx.doi.org/10.1023/A:1026485807148

[180] M. E. J. Newman, "Assortative mixing in networks," *Physical Review Letters*, vol. 89, no. 20, p. 208701, 2002.

[181] M. E. J. Newman, "Mixing patterns in networks," *Physical Review E*, vol. 67, no. 2, p. 026126, 2003.

[182] M. E. J. Newman, "The mathematics of networks," in *The New Palgrave Dictionary of Economics*, S. N. Durlauf and L. E. Blume, Eds. Basingstoke: Palgrave Macmillan, 2008.

[183] M. E. J. Newman, A. L. Barabási, and D. J. Watts, Eds., *The Structure and Dynamics of Networks*. Princeton University Press, 2006.

[184] M. Nowak, K. Sigmund, *et al.*, "A strategy of win-stay, lose-shift that outperforms tit-for-tat in the prisoner's dilemma game," *Nature*, vol. 364, no. 6432, pp. 56–58, 1993.

[185] M. A. Nowak, S. Bonhoeffer, and R. M. May, "Spatial games and the maintenance of cooperation," *Proceedings of the National Academy of Sciences*, vol. 91, no. 11, pp. 4877–4881, 1994.

[186] M. A. Nowak and R. M. May, "Evolutionary games and spatial chaos," *Nature*, vol. 359, no. 6398, pp. 826–829, 1992.

[187] H. Ohtsuki, C. Hauert, E. Lieberman, and M. A. Nowak, "A simple rule for the evolution of cooperation on graphs and social networks," *Nature*, vol. 441, no. 7092, pp. 502–505, 2006.

[188] M. Oka and T. Ikegami, "Characterizing autonomy in the web via transfer entropy network," *Artif. Life*, vol. 13, pp. 234–242, 2012.

[189] E. Ostrom, "Tragedy of the commons," *The New Palgrave Dictionary of Economics*, pp. 3573–3576, 2008.

[190] J. M. Pacheco, F. C. Santos, M. O. Souza, and B. Skyrms, "Evolutionary dynamics of collective action in n-person stag hunt dilemmas," *Proceedings of the Royal Society B: Biological Sciences*, vol. 276, no. 1655, pp. 315–321, 2009.

[191] L. Page, S. Brin, R. Motwani, and T. Winograd, "Pagerank: Bringing order to the web," Stanford Digital Libraries Working Paper, Tech. Rep., 1997.

[192] L. Page, S. Brin, R. Motwani, and T. Winograd, "The pagerank citation ranking: Bringing order to the web." 1999.

[193] R. T. Paine, "Food web complexity and species diversity," *American Naturalist*, pp. 65–75, 1966.

[194] J. Park and M. E. J. Newman, "Statistical mechanics of networks," *Physical Review E*, vol. 70, no. 6, pp. 066 117+, Dec 2004.

[195] L. Pavel, "Basics of game theory," in *Game Theory for Control of Optical Networks*. Springer, 2012, pp. 11–26.

[196] M. Perc, "Evolution of cooperation on scale-free networks subject to error and attack," *New Journal of Physics*, vol. 11, no. 3, p. 033027, 2009.

[197] H. Peters, "Extensive form games," *Game Theory: A Multi-Leveled Approach*, pp. 197–212, 2008.

[198] M. Piraveenan, M. Prokopenko, and L. Hossain, "Percolation centrality: Quantifying graph-theoretic impact of nodes during percolation in networks," *PloS one*, vol. 8, no. 1, p. e53095, 2013.

[199] M. Piraveenan, M. Prokopenko, and A. Zomaya, "Local assortativeness in scale-free networks," *EPL (Europhysics Letters)*, vol. 84, no. 2, p. 28002, 2008.

[200] M. Piraveenan, M. Prokopenko, and A. Zomaya, "On congruity of nodes and assortative information content in complex networks," *Networks and Heterogeneous Media (NHM)*, vol. 3, no. 10.3934/nhm.2012.7.441, pp. 441–461, 2012.

[201] M. Piraveenan, M. Prokopenko, and A. Y. Zomaya, "Local assortativeness in scale-free networks," *Europhysics Letters*, vol. 84, no. 2, p. 28002, 2008.

[202] M. Piraveenan, M. Prokopenko, and A. Y. Zomaya, "Assortativeness and information in scale-free networks," *European Physical Journal B*, vol. 67, pp. 291–300, 2009.

[203] M. Piraveenan, M. Prokopenko, and A. Y. Zomaya, "Assortativity and growth of Internet," *European Physical Journal B*, vol. 70, pp. 275–285, 2009.

[204] M. Piraveenan, M. Prokopenko, and A. Y. Zomaya, "Assortative mixing in directed biological networks," *IEEE/ACM Transactions on computational biology and bioinformatics*, vol. 9(1), pp. 66–78, 2012.

[205] M. Piraveenan, K. Wimalawarne, and D. Kasthurirathn, "Centrality and composition of four-node motifs in metabolic networks," *Procedia Computer Science*, vol. 18, pp. 409–418, 2013.

[206] R. Poli, "An analysis of publications on particle swarm optimization applications," *Essex, UK: Department of Computer Science, University of Essex*, 2007.

[207] J. Poncela, J. Gómez-Gardeñes, A. Traulsen, and Y. Moreno, "Evolutionary game dynamics in a growing structured population," *New Journal of Physics*, vol. 11, no. 8, p. 083031, 2009.

[208] J. Pouwelse, P. Garbacki, D. Epema, and H. Sips, "The bittorrent p2p file-sharing system: Measurements and analysis," in *Peer-to-Peer Systems IV*. Springer, 2005, pp. 205–216.

[209] J. L. Powell, P. A. Lewis, R. I. Dunbar, M. García-Fiñana, and N. Roberts, "Orbital prefrontal cortex volume correlates with social cognitive competence," *Neuropsychologia*, vol. 48, no. 12, pp. 3554–3562, 2010.

[210] W. H. Press and F. J. Dyson, "Iterated prisoners dilemma contains strategies that dominate any evolutionary opponent," *Proceedings of the National Academy of Sciences*, vol. 109, no. 26, pp. 10 409–10 413, 2012.

[211] D. d. S. Price, "A general theory of bibliometric and other cumulative advantage processes," *Journal of the American society for Information science*, vol. 27, no. 5, pp. 292–306, 1976.

[212] M. Prokopenko, F. Boschetti, and A. J. Ryan, "An information-theoretic primer on complexity, self-organization, and emergence," *Complexity*, vol. 15, no. 1, pp. 11–28, 2009.

[213] D. Qiu and R. Srikant, "Modeling and performance analysis of bittorrent-like peer-to-peer networks," in *ACM SIGCOMM Computer Communication Review*, vol. 34, no. 4, pp. 367–378. ACM, 2004.

[214] R. Radner, "Collusive behavior in noncooperative epsilon-equilibria of oligopolies with long but finite lives," *Journal of economic theory*, vol. 22, no. 2, pp. 136–154, 1980.

[215] A. Rapoport, "Contribution to the theory of random and biased nets," *The bulletin of mathematical biophysics*, vol. 19, no. 4, pp. 257–277, 1957.

[216] A. Rapoport, *Prisoner's dilemma: A study in conflict and cooperation.* University of Michigan Press, 1965, vol. 165.

[217] W. H. Riker, "The entry of game theory into political science," *Toward a history of game theory*, vol. 24, p. 207, 1992.

[218] F. J. Roethlisberger and W. J. Dickson, *Management and the Worker.* Psychology Press, 2003, vol. 5.

[219] B. W. Rogers, T. R. Palfrey, and C. F. Camerer, "Heterogeneous quantal response equilibrium and cognitive hierarchies," *Journal of Economic Theory*, vol. 144, no. 4, pp. 1440–1467, 2009.

[220] Z. Rong, X. Li, and X. Wang, "Roles of mixing patterns in cooperation on a scale-free networked game," *Physical Review E*, vol. 76, no. 2, p. 027101, 2007.

[221] M. Rubinov, S. A. Knock, C. J. Stam, S. Micheloyannis, A. W. F. Harris, L. M. Williams, and M. Breakspear, "Small-world properties of nonlinear brain activity in schizophrenia," *Human Brain Mapping*, vol. 30, pp. 403–416, 2009.

[222] G. Sabidussi, "The centrality index of a graph," *Psychometrika*, vol. 31, no. 4, pp. 581–603, 1966.

[223] F. Santos, J. Rodrigues, and J. Pacheco, "Graph topology plays a determinant role in the evolution of cooperation," *Proceedings of the Royal Society B: Biological Sciences*, vol. 273, no. 1582, pp. 51–55, 2006.

[224] F. C. Santos and J. M. Pacheco, "Scale-free networks provide a unifying framework for the emergence of cooperation," *Physical Review Letters*, vol. 95, no. 9, p. 098104, 2005.

[225] F. C. Santos, J. F. Rodrigues, and J. M. Pacheco, "Graph topology plays a determinant role in the evolution of cooperation," *PROCEEDINGS OF THE ROYAL SOCIETY B-BIOLOGICAL SCIENCES*, vol. 273, no. 1582, pp. 51–55, Jan. 2006.

[226] S. Sarkar and A. Dong, "Characterizing modularity, hierarchy and module interfacing in complex design systems," in *ASME 2011 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (IDETC/CIE2011)*, vol. 9: 23rd International Conference on Design Theory and Methodology; 16th Design for Manufacturing and the Life Cycle Conference, pp. 375–384. New York: ASME, 2011.

[227] N. Sarshar and V. Roychowdhury, "Scale-free and stable structures in complex ad hoc networks," *Physical Review E*, vol. 69, no. 2, p. 026101, 2004.

[228] C. E. Shannon, "A mathematical theory of communication," *The Bell System Technical Journal*, vol. 27, pp. 379–423, 623–656, July, October, 1948.

[229] C. Shirky, "The political power of social media: Technology, the public sphere, and political change," *Foreign affairs*, pp. 28–41, 2011.

[230] Y. Shoham, "Computer science and game theory," *Communications of the ACM*, vol. 51, no. 8, pp. 74–79, 2008.

[231] H. A. Simon, "A behavioral model of rational choice," *The quarterly journal of economics*, pp. 99–118, 1955.

[232] H. A. Simon, "On a class of skew distribution functions," *Biometrika*, pp. 425–440, 1955.

[233] B. Skyrms, "The stag hunt," in *Proceedings and Addresses of the American Philosophical Association*, vol. 75, no. 2, pp. 31–41. JSTOR, 2001.

[234] B. Skyrms and R. Pemantle, "A dynamic model of social network formation," *Proceedings of the National Academy of Sciences*, vol. 97, no. 16, pp. 9340–9346, 2000.

[235] J. M. Smith, "Evolution and the theory of games: In situations characterized by conflict of interest, the best strategy to adopt depends on what others are doing," *American Scientist*, pp. 41–45, 1976.

[236] J. M. Smith, *Evolution and the Theory of Games.* Cambridge university press, 1982.

[237] J. M. Smith, *Evolution and the Theory of Games.* Springer, 1993.

[238] J. M. Smith and G. Price, "lhe logic of animal conflict," *Nature*, vol. 246, p. 15, 1973.

[239] R. V. Solé and S. Valverde, "Information theory of complex networks: On evolution and architectural constraints," in *Complex networks.* Springer, 2004, pp. 189–207.

[240] R. V. Solé and S. Valverde, "Information theory of complex networks: on evolution and architectural constraints," in *Complex Networks*, ser. Lecture Notes in Physics, E. Ben-Naim, H. Frauenfelder, and Z. Toroczkai, Eds. Springer, 2004, vol. 650.

[241] V. Srivastava, J. O. Neel, A. B. MacKenzie, R. Menon, L. A. DaSilva, J. E. Hicks, J. H. Reed, and R. P. Gilles, "Using game theory to analyze wireless ad hoc networks." *IEEE Communications Surveys and Tutorials*, vol. 7, no. 1-4, pp. 46–56, 2005.

[242] A. J. Stewart and J. B. Plotkin, "Extortion and cooperation in the prisoners dilemma," *Proceedings of the National Academy of Sciences*, vol. 109, no. 26, pp. 10 134–10 135, 2012.

[243] K. Strom, "A behavioral theory of competitive political parties," *American journal of political science*, pp. 565–598, 1990.

[244] M. R. Subramani and B. Rajagopalan, "Knowledge-sharing and influence in online social networks via viral marketing," *Communications of the ACM*, vol. 46, no. 12, pp. 300–307, 2003.

[245] P. D. Taylor and L. B. Jonker, "Evolutionary stable strategies and game dynamics," *Mathematical biosciences*, vol. 40, no. 1, pp. 145–156, 1978.

[246] G. Thedchanamoorthy, "Alternative approaches and their applications, measuring mixing patterns in complex networks," Master's thesis, School of Project Management, Centre for Complex Systems Research Faculty of Engineering and Information Technologies The University of Sydney, 2015.

[247] G. Thedchanamoorthy, M. Piraveenan, D. Kasthuriratna, and U. Senanayake, "Node assortativity in complex networks: An alternative approach," *Procedia Computer Science*, vol. 29, pp. 2449 – 2461, 2014, 2014 International Conference on Computational Science.

[248] G. Thedchanamoorthy, M. Piraveenan, D. Kasthuriratna, and U. Senanayake, "Node assortativity in complex networks: An alternative approach," *Procedia Computer Science*, vol. 29, pp. 2449–2461, 2014.

[249] D. J. Thomsen, R. O'brien, J. Bogle, and C. Payne, "Locally adaptable central security management in a heterogeneous network environment," Dec. 11 2007, uS Patent 7,308,702.

[250] A. Traulsen, J. C. Claussen, and C. Hauert, "Coevolutionary dynamics in large, but finite populations," *Physical Review E*, vol. 74, no. 1, p. 011901, 2006.

[251] V. Tzoumas, C. Amanatidis, and E. Markakis, "A game-theoretic analysis of a competitive diffusion process over social networks," in *Internet and Network Economics.* Springer, 2012, pp. 1–14.

[252] S. Valverde, R. F. Cancho, and R. V. Sole, "Scale-free networks from optimal design," *EPL (Europhysics Letters)*, vol. 60, no. 4, p. 512, 2002.

[253] J. Von Neumann and O. Morgenstern, "Game theory and economic behavior," 1944.

[254] D. Walker, S. Reay Atkinson, and L. Hossain, "Counterinsurgency through civil infrastructure networks," in *SOTICS 2012, The Second International Conference on Social Eco-Informatics*, pp. 7–12, 2012.

[255] M. Walker and J. Wooders, "Mixed strategy equilibrium," *Retrieved December*, vol. 19, p. 2006, 2006.

[256] S.-J. Wang and C. Zhou, "Hierarchical modular structure enhances the robustness of self-organized criticality in neural networks," *New Journal of Physics*, vol. 14, no. 2, p. 023005, 2012.

[257] X. R. Wang, J. M. Miller, J. T. Lizier, M. Prokopenko, and L. F. Rossi, "Quantifying and tracing information cascades in swarms," *PloS one*, vol. 7, no. 7, p. e40084, 2012.

[258] S. Wasserman, *Social network analysis: Methods and applications.* Cambridge university press, 1994, vol. 8.

[259] D. J. Watts, *Six Degrees: The Science of a Connected Age.* New York: Norton, 2003.

[260] D. J. Watts and S. H. Strogatz, “Collective dynamics of small-world networks,” *Nature*, vol. 393, no. 6684, pp. 440–442, June 1998.

[261] J. W. Weibull, *Evolutionary game theory.* MIT press, 1997.

[262] O. E. Williamson, “The economics of organization: The transaction cost approach,” *American journal of sociology*, pp. 548–577, 1981.

[263] D. Wolpert, J. Jamison, D. Newth, and M. Harre, “Strategic choice of preferences: The persona model,” *The BE Journal of Theoretical Economics*, vol. 11, no. 1, 2011.

[264] D. H. Wolpert, “Information theory-the bridge connecting bounded rational game theory and statistical physics,” in *Complex Engineered Systems.* Springer, 2006, pp. 262–290.

[265] D. H. Wolpert, M. Harré, E. Olbrich, N. Bertschinger, and J. Jost, “Hysteresis effects of changing the parameters of noncooperative games,” *Physical Review E*, vol. 85, no. 3, p. 036102, 2012.

[266] B. Xiang, Q. Liu, E. Chen, H. Xiong, Y. Zheng, and Y. Yang, “Pagerank with priors: an influence propagation perspective,” in *Proceedings of the Twenty-Third international joint conference on Artificial Intelligence*, pp. 2740–2746. AAAI Press, 2013.

[267] W. Yoshida, R. J. Dolan, and K. J. Friston, “Game theory of mind,” *PLoS computational biology*, vol. 4, no. 12, p. e1000254, 2008.

[268] B. Zhang, “Quantal response methods for equilibrium selection in normal form games,” *Available at SSRN 2375553*, 2013.

[269] H. Zhang, W. B. Croft, B. Levine, and V. Lesser, “A multi-agent approach for peer-to-peer based information retrieval system,” in *Proceedings of the Third International Joint Conference on Autonomous Agents and Multiagent Systems-Volume 1*, pp. 456–463. IEEE Computer Society, 2004.

[270] Y. Zhang, M. Aziz-Alaoui, C. Bertelle, and J. Guan, “Local nash equilibrium in social networks,” *Scientific reports*, vol. 4, 2014.

[271] Y.-C. Zhang, "Modeling market mechanism with evolutionary games," *arXiv preprint cond-mat/9803308*, 1998.