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Placement matters in making good decisions sooner: the influence of topology in reaching public utility thresholds

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***Abstract*—Social systems are increasingly being modelled as complex networks, and the interactions and decision making of individuals in such systems can be modelled using game theory. Therefore, networked game theory can be effectively used to model social dynamics. Individuals can use pure or mixed strategies in their decision making, and recent research has shown that there is a connection between the topological placement of an individual within a social network and the best strategy they can choose to maximise their returns. Therefore, if certain individuals have a preference to employ a certain strategy, they can be swapped or moved around within the social network to more desirable topological locations where their chosen strategies will be more effective. To this end, it has been shown that to increase the overall public good, the cooperators should be placed at the hubs, and the defectors should be placed at the peripheral nodes. In this paper, we tackle a related question, which is the time (or number of swaps) it takes for individuals who are randomly placed within the network to move to optimal topological locations which ensure that the public utility satisfies a certain utility threshold. We show that this time depends on the topology of the social network, and we analyse this topological dependence in terms of topological metrics such as scale-free exponent, assortativity, clustering coefficient, and Shannon information content. We show that the higher the scale-free exponent, the quicker the public utility threshold can be reached by swapping individuals from an initial random allocation. On the other hand, we find that assortativity has negative correlation with the time it takes to reach the public utility threshold. We find also that in terms of the correlation between information content and the time it takes to reach a public utility threshold from a random initial assignment, there is a bifurcation: one class of networks show a positive correlation, while another shows a negative correlation. Our results highlight that by designing networks with appropriate topological properties, one can minimise the need for the movement of individuals within a network before a certain public good threshold is achieved. This result has obvious implications for defence strategies in particular.**

***Index Terms*—complex networks, mixing patterns, assortativity**

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I. Introduction

Game theory is the science of strategic decision making among autonomous players[1]. Evolutionary game theory is the adaptation of game theory in populations of players, where game theory is used to explain the evolution of strategies over time[2]. Most real-world populations are not 'well-mixed', but are restricted by spatial limitations. Thus, the players distributed in heterogeneous networks provide an interesting premise to study evolutionary games[3], [4], [5]. Social structures of people have often been modelled as complex networks. While the 'well-mixed' or random models have been used earlier to characterise social interactions, the heterogeneous nature of some interactions, whereby some individuals have more links than others, is nowadays taken into account[6], [3]. It has been found that most social networks are, in fact, the so-called 'scale-free' networks, with power law degree distributions[7], [8]. As such, networked game theory has come into prominence, to analyse the payoff of individuals in such scenarios. Meanwhile, public good games have begun to be studied as a branch of games where the individual pay-offs for agents are less important than the overall payoff (utility) for the community[5], [9]. A good real-world example of this is the welfare systems that are in place in many financial environments to safeguard the financially weaker individuals or organisations.

Normally, evolution within the context of game theory or networked game theory is taken to mean that individual agents adopt or evolve strategies with the view of maximising their individual payoff. This is indeed often the case: each deer in the forest adapts strategies to maximize its lifetime and food intake, and such strategies are passed on to the next generation, either by observation or as some kind of genetic memory. However, environmental pressures may also dictate collective evolution, whereby each individual tries to adapt the best strategy for the collective gain of the society, as opposed to its individual gain. For example, a society of deers may be forced to evolve collective strategies to better survive against a pride of lions. The strategy adapted by each deer, then, is dictated not so much by its individual gain but the collective gain of the society. It is easy to find similar examples in the human society as well[4].

Research in networked game theory has already shown that for the maximization of 'public good', the strategies chosen by individual agents should depend on their topological placement in the network[10], [11], [12], [13]. For example, in a population of agents playing the Prisoners Dilemma game, it has been shown that the hubs should play the cooperation strategy whereas the peripheral nodes should play the Defection strategy to maximise the total network utility[10], [12], [14], [15], [16]. Similarly, recent work[11], [17] has considered several pairs of strategy, and in a scenario where agents playing both strategies are present in the population, identified which agents (or which strategy) should occupy the hubs and which agents (or which strategy) should occupy the peripheral nodes for maximization of network utility. Such a result makes sense in scenarios where individual players or agents have fixed strategies which they cannot change, but can swap positions with neighbours, so that the nodes in the network will appear to be changing their strategies (by swapping players or agents which occupy these nodes) while the overall percentage of players playing a certain strategy in the network will not change. In this work, we are similarly interested in pairs of strategies, but rather than deciphering which strategies should occupy the hubs and which strategies should occupy the peripheral nodes, which has already been established by previous work, we are interested in understanding how long it takes for the agents playing these strategies to move to the optimised locations for maximum public good, from a random initial assignment. In particular, we set a public good threshold, and we swap agents playing fixed strategies to neighbouring nodes, and we measure the number of swaps it takes for the network to reach the desired public good threshold. Then we analyse how this number depends on the topological parameters of the network, such as assortativity, clustering coefficient, scale-free exponent, and information content. That is, we analyse the influence of the topological parameters in determining how quickly an acceptable public good threshold can be reached. A question could be posed as to why this public threshold can not be achieved by individual agents changing their strategies, rather than moving their positions within the network. However, in many scenarios, it is difficult to teach or convince individuals to change their decision making process. For example, in a defence scenario where a group of people defend a structure with an underlying topological pattern, the leader of the defenders often tries to strengthen the defences with their limited resources by placing appropriately skilled people at appropriate locations, rather than training people at each location to help them develop skills needed at that location. At the same time, it is often imperative that no position can be left unmanned for a relatively long time, so that movement of personnel is only possible by moving them to nearby positions. For instance, consider a scenario where a group of soldiers defend a connected system of fortifications, or a group of villagers defend their village against an external foe like bandits or wild animals. Often, the alarm goes off at an unexpected time so that people have to rush to the closest position and man it, so that the initial assignment is random. After this, the leader can only move people by swapping them with people who are immediately next to them in the predefined defensive system, so that no position is left unoccupied for a long time. Nevertheless, the leader may want to move the strongest people to the front, and the smartest people to the rear or the centre of the fortification where they may attend a strategic discussion, by affecting one swap at a time. In this way, it is possible to increase the overall effectiveness (payoff or utility) of the defence, without actually changing the abilities of individuals or the strategies each individual employ or the organisational structure (topology), and achieve a global outcome (better defence) by relying only on local movements (swaps of people). This is the scenario we try to mimic in this work.

This paper is organised as follows. In the next section, we elaborate on the game theoretical background used in this work. Also, we give a brief introduction into topological features of networks. In the following section, we describe our methodology and experimental set-up. In the next section we describe our results and observations. The last section contains the conclusions and indications towards future work.

II. Background

A. Evolutionary Game Theory

Evolutionary game theory is an outcome of the adaptation of game theory into the field of evolutionary biology[2].

1) Iterated Prisoners Dilemma: Prisoner's dilemma is a game that is found in classical game theory[18], which describes the 'dilemma' faced by two prisoners, caught by authorities in the act of committing a lesser crime but suspected to have committed a more serious crime, and being separately offered incentives to confess to the greater crime and thus betray their colleague. Given the payoff matrix in Table. 1, the inequality $T > R > P > S$ should be satisfied in a prisoner's dilemma game. In other words, in the prisoner's dilemma game, the highest combined payoff is obtained by the players when both players cooperate. However, if one player cooperates while the other defects, the defector would obtain a higher payoff individually, while the cooperator gets the least payoff. The dilemma is that the Nash Equilibrium of this game, which occurs when both players defect, does not provide the optimum payoff for either player.

In Iterated Prisoner's Dilemma(IPD), the prisoner's dilemma game is iterated over many timesteps, over a population of players[16]. Each player would play a single iteration of the game with each of their neighbours in each time-step. Iterated prisoner's dilemma game is widely used to model the autonomous decision making behaviour of self-interested players. It has been demonstrated that the topology of the network is significant in the evolution of cooperation of strategies in the IPD game[16], [15], [6]. For example, when the iterated prisoner's dilemma game is played by players using pure cooperation and pure defection strategies, cooperation evolves to be the dominant strategy in a population of players that are distributed in a scale-free topology. In this work, we use Iterated Prisoners Dilemma, with payoffs given by Table I, in our simulation experiments.

2) Memory-one strategies in iterative games: During an iterated game, each player can play each of the 'pure' strategies available to them, or a subset thereof, according to a certain probability distribution. Such a combination of strategies over time according to a probability distribution is called a 'mixed strategy'. In fact, a pure strategy in an iterative game can be regarded as a special case of mixed strategy, where a certain strategy is always chosen (with probabality of 1.0), and the other strategies are chosen with probability zero. The probability of choosing a particular strategy for a particular round could be decided from the 'memory' of the player about the past actions of themselves and their opponents, such as the strategies they

TABLE I: Payoff matrix for the Prisoner's Dilemma, as used for simulation experiments in this paper. Note that ($T > R > P > S$).

	PLAYER 1: Cooperate	PLAYER 1: Defect
PLAYER 2: Cooperate	R (3), R (3)	S (0), T (5)
PLAYER 2: Defect	T (5), S (0)	P (1), P (1)

and their opponents used in previous iterations. Such a mixed strategy is called a finite-memory strategy, where the current mixed strategy would be dependent on n number of historical strategy selections between the two players involved [11]. A memory-one strategy is a finite-memory strategy, where $n = 1$: that is, only the actions of the players in the previous iteration determine the strategy selection of a player.

For example, if two players are playing IPD and employing a memory-one strategy, this strategy could be denoted (p_1, p_2, p_3, p_4) whereby p_1 denotes the probability of cooperation if both players cooperated in the previous round, p_2 denotes the probability of cooperation if the player in question cooperated while the other defected in the previous round, p_3 denotes the probability of cooperation if the player in question defected while the other cooperated in the previous round, and p_4 denotes the probability of cooperation if the player in question defected while the other also defected in the previous round. It is elementary to note that the probabilities of defection do not need to be noted explicitly, and would be given by $(1-p_1, 1-p_2, 1-p_3, 1-p_4)$. Therefore, a strategy (1,1,1,1) by player A would imply that the Player A would cooperate with player B, irrespective of their strategies during the previous encounter between Player A and B. Similarly, (0,0,0,0) would represent that player A would defect regardless of the strategies of players A and B in the last round.

By varying the probabilities of cooperation depending on the actions of the players in the previous encounter, it is possible to define any number of mixed 'memory-one' strategies. Some of the well-known memory-one strategies include the Pavlov strategy (1,0,0,1), Tit-for-Tat strategy (1,0,1,0), and the General Cooperator (0.935, 0.229, 0.266, 0.42) strategy. Pavlov is sometimes also called 'win-stay-lose-shift', because, as can be seen from the cooperation probabilities, the player in question sticks to the action of the previous round in this round only if both players cooperated in the last round or this player defected while the other cooperated in the last round, in both of which scenarios this player would have not lost out in the last round. On the other hand, if this player cooperated while the other defected in the last round, this player would have lost out (that is, gained payoff less than the other player), therefore in this round this player defects: similarly, if they both defected in the last round, it is also considered a 'loss', according to the payoff matrix in Table 1, in the sense that the payoff would have been relatively low, so this time this player tries cooperation. So, essentially, Pavlov represents a strategy where the player sticks to the previous strategy if the payoff in the last round was relatively good (5 or 3 according to Table 1), and shifts strategy if the payoff in the last round was relatively poor (1 or 0 according to Table 1). Tit-for-Tat, is another well knowns strategy where a player would only cooperate if the opponent cooperated in the previous interaction. General Cooperator is the evolutionarily dominating strategy that evolved at low mutation rates from a pure cooperator (1,1,1,1) as demonstrated by Iliopoulos et al[20], [21]. In our simulation experiments in this paper, we make use of Pure Cooperator (1,1,0,0), General Cooperator (0.935, 0.229, 0.266, 0.42), Pure Defector (0,0,0,0), General Defector (0.42, 0.266, 0.229, 0.935), Tit-for-Tat (1,0,1,0) and Pavlov (1,0,0,1) mixed strategies.

3) public good games and public good thresholds: Typically, games are taken to be played for the self-interest of the players. On the other hand, the concept of public-good (sometimes also written 'public-goods') games has recently gained prominence[5], where the emphasis is not on the individual gains of agents but the overall payoff for the society or system. Therefore, in a public-good game, players cooperate, willingly or otherwise, not to maximise their individual gains but to maximise the payoff of the system as a whole. Prisoners Dilemma is by definition a competitive game, but here we treat iterated Prisoners Dilemma as a public good game, focussing on the total payoff for the system (network) on each iteration and how to maximise it, rather than maximising individual payoffs. We define a 'public good threshold' as a value of total network payoff, above which we assume that the expected 'public good' has been achieved. In our experiments below, the public good threshold is set at 1.5 per player per game, unless otherwise stated.

B. Games on networks

Games played on the so-called 'well-mixed' populations can be contrasted to games played on networks, where each player can only play games with their immediate neighbours and thus the opportunity to play games is restricted by the topology[6], [3]. Networked game theory has progressed significantly since the introduction of the so-called small-world and scale-free topologies into the more general field of network theory more than a decade ago[8], [7]. Thus, several studies have looked at how network topology influences the stability of strategies, for Prisoners Dilemma and other games[17], [10], [12], [15]. In Prisoners dilemma, even though defectors always win against cooperators and the Nash equilibrium is when both players defect, in networked games it was shown that cooperation was a stable strategy, and a portion of cooperating agents were able to persist indefinitely in the system[6].

In the present work, we also attempt to understand the role of topology in game dynamics, except that we focus on the time (or number of strategy swaps) taken to achieve a certain public good threshold and how this time is influenced by topology, rather than focussing on evolutionary stability of strategies or the topological placement of strategies necessary for public good, as some previous studies have done [6], [5], [11]. Therefore, it is necessary for us to briefly define the topological concepts used in this study. In the following subsections, we provide some definitions for key network science concepts that we will use.

1) Scale-free networks: Scale-free networks are ubiquitous in real world, and for this reason often used as model networks in networked game theory[6]. In a scale-free network, the degree distribution follows a power law, and the probability of a node to have a degree of k is given by[8], [22] $p_k = Ak^{-\gamma}$ where

A is a constant and γ is the power law exponent (also referred as scale-free exponent). A higher value of γ results in a degree distribution with a steeper slope, while a lower value of γ results in a flatter degree distribution.

2) Clustering Coefficient: The clustering coefficient of a node represents the ratio between the number of links between the neighbours of that node, and the number of all possible links between those neighbours. It is defined as[23]:

$$c_i = \frac{2y_i}{k_i(k_i - 1)} \tag{1}$$

where k_i is the degree of node i, and y_i is the number of edges between the neighbours of node i. The network clustering coefficient $\bar{C}$ is defined as the average of the clustering coefficients of all nodes in that network.

3) Assortativity: Assortativity measures the tendency for nodes to connect to similar nodes, and degree assortativity quantifies this tendency when similarity is defined in terms of node degree. As a Pearson correlation it ranges from 1.0 (for perfect assortativity) to -1.0 (for perfect disassortativity), and is defined (for undirected networks) as:

$$r = \frac{M^{-1}\sum_i j_i k_i - [M^{-1}\sum_i \frac{1}{2}(j_i + k_i)]^2}{M^{-1}\sum_i \frac{1}{2}(j_i^2 + k_i^2) - [M^{-1}\sum_i \frac{1}{2}(j_i + k_i)]^2} \tag{2}$$

where M is the number of edges, and j_i, k_i are the degrees of the vertices at either end of the i^{th} edge, with $i = 1...M$.

4) Information Content: In information theory, mutual information measures the amount of information that can be obtained about one random variable by observing another random variable[24].

$$I(X;Y) = \sum_{x \in X}\sum_{y \in Y} P(x,y) \log \frac{P(x,y)}{P(x)P(y)} \tag{3}$$

where $P(x,y)$ is the joint probability distribution of X and Y. $P(x)$ and $P(y)$ are the marginal probability distribution functions of X and Y. In the context of a network, the Shannon information $I(q)$ of the network topology, also called the mutual information of a network, can be expressed as[7]:

$$I(q) = \sum_{j=0}^{k_{max}-1}\sum_{k=0}^{k_{max}-1} e_{j,k} \log \frac{e_{j,k}}{q_j q_k} \tag{4}$$

where the $e_{j,k}$ and q_k are as defined above in section II-B3. The Shannon information is a relative quantity that expresses the amount of information that is hidden within the network topology. Therefore, a relatively homogenous graph has a lower information content, while a more heterogeneous graph, such as a scale-free network, has a higher information content.

In this paper, we use the scale-free exponent γ, network clustering coefficient $\bar{C}$, degree assortativity r as well as information content $I(q)$ of networks, as defined above, as the topological metrics to quantify the topology of a network, and study the relationship between these metrics and the time it takes for a network where IPD is played to reach payoff thresholds.

III. Methodology

Here we describe how the simulation experiments were set-up and run.

A. Generating the scale-free networks

We generated scale-free networks with size $N = 1500$ nodes and $M = 2250$ links, thus having an average degree of three. To do so, we first generated the desired degree distribution, which is given by $p_k = Ak^{-\gamma}$, where A and γ are parameters. Since the degree distribution is a probability distribution which should sum up to one, A can be computed for a given γ and average degree (which, in this case, is three). Once the degree distribution is generated, the 1500 nodes were generated and assigned degrees according to the degree distribution, and then randomly wired according to their degrees. Since the average degree of 3.0 was used in the calculation of A, this process always results in $M = 2250$ links, as desired (subject to finite-state effects which may result in a few more or few less edges).

We first generated a range of scale-free networks with a range of γ values from 2.0 to 3.0, since this is the range of scale-free exponents on most naturally occurring scale-free networks[7]. We also generated scale-free networks with $\gamma = 1.0$, since this special case can sometimes reveal interesting observations, as we will see below. Then, we generated networks with a range of clustering coefficients, by fixing the γ and for each γ generating several scale-free networks so that the randomness in the link generation process results in a range of network clustering coefficients. In order to generate graph with varying levels of assortativity, we first generated a connected scale-free graph for a particular γ then applied degree-preserving rewiring (DPR) which changes assortativity[25] (also known as the Xulvi-Brunet–Sokolov algorithm) until we reach the desired value.

The Xulvi-Brunet–Sokolov algorithm as implemented by us, which can be used to increase the assortativity of a network while preserving the degree distribution (and thus, the scale-free exponent, γ) can be described briefly as follows. It is an iterative algorithm, and on each iteration, two pairs of linked nodes (i.e, nodes at the ends of two non-adjacent links) are randomly selected. Then these four nodes are sorted by degree. If the two nodes with higher degrees are already connected (which also means the two nodes with lower degrees are already connected), the iteration is completed. If not, then the two links between the two selected pairs are deleted, and instead, two new links are made, one between the two nodes with the highest degrees, and the other between the two nodes with the lowest degrees. Thus, the degree of each node is preserved, and the iteration is completed. This process is repeated until the desired level of assortativity is reached. The process is illustrated in Fig. 1.

The process of decreasing the assortativity is similar. The only difference, at each iteration, is to delete the links between the two pairs of nodes selected, and then pair up the highest degree node to the lowest degree node and the other two nodes, forming two new links. Thus, assortativity can be reduced while the degree distribution and scale-free exponent are preserved.

We vary the information content, or mutual information, of networks in the following manner. Consider equation 4 again, from which we can see that the information content depends on the two distributions, $e_{j,k}$ and $q_j q_k$. As equation 4 shows, the maximal information content could be obtained when the product $q_j * q_k$ diverges the most from $e_{j,k}$. In contrast, minimal information $I(q)$ is attained when $q_j * q_k$ and $e_{j,k}$ diverge the least. Therefore, we varied the information content by varying

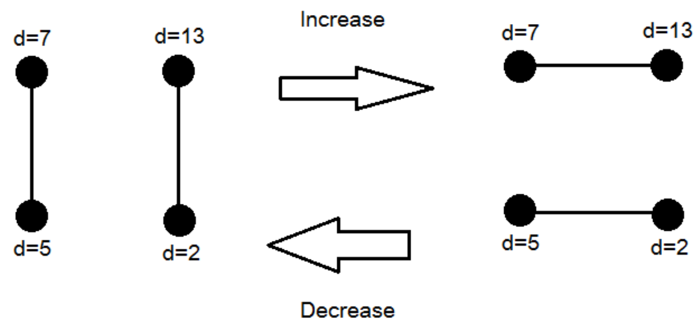

Fig. 1: The Xulvi-Brunet–Sokolov algorithm. To increase assortativity, at each iteration two pairs of nodes (that is, two links) are randomly selected, and after these two links are deleted, two new links are created, one between the nodes with the highest degrees ($d = 13$ and $d = 7$ in the example), another between the nodes which have the lowest degrees ($d = 5$ and $d = 2$ in the example). This iterative process increases the assortativity of a scale-free network while preserving the degree distribution, and thus, preserving the scale-free exponent. The reverse process results in the reduction of assortativity.

the $e_{j,k}$ for a particular pre-defined degree distribution of a scale-free network (which means the expected degree distribution q_k, and scale-free exponent γ, are also fixed), while ensuring that $\sum_{jk} e_{j,k} = 1$. Once a particular $e_{j,k}$ distribution is obtained, we used it to 'wire' the given number of nodes in the scale-free network, and then we repeated the same process for the same q_k distribution but for a different $e_{j,k}$ distribution. This process resulted in a range of scale-free networks which all had the same degree distribution and expected degree distribution (and thus, the same scale-free exponent $gamma$), while differing in the mutual information (information content) of the topology.

This process is illustrated in Fig. 3, for the sample network illustrated in Fig. 2. The divergence between $q_j * q_k$ and $e_{j,k}$ is represented by the vertical distance between the two surfaces shown in figure 3. While the blue surface remains unmoving, the red surface can be moved (subject to the constraint that the distribution should sum up to unity), resulting in the divergence between $q_j * q_k$ and $e_{j,k}$ varying. Thus, by 'moving' the red surface we can vary the information content of a network. Note however that the sample network in Fig. 2 is much smaller than the networks used in our experiments, which all had $N = 1500$ nodes and $M = 2250$ links.

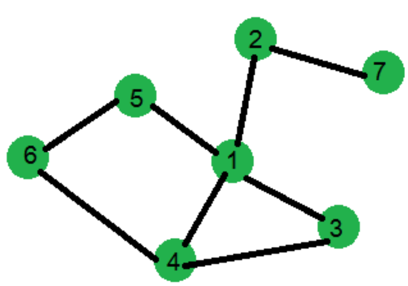

Fig. 2: Example network to illustrate the concept of divergence between distributions $q_j * q_k$ and $e_{j,k}$, as shown in Fig. 3.

IV. Results

Here we present the results of our simulation experiments. We analysed how the time to reach a certain public good utility threshold varies against various topological attributes. This time was measured by counting the number of 'strategy swaps' needed between neighbouring nodes to reach the given public good threshold (in brief, swaps to reach threshold, or hereafter STRT). Each node was allowed to swap its strategy only with one of its immediate neighbours in the network topology. Each swap was performed by randomly selecting a link, and swapping the strategies of the nodes on either end of the link, in such a way that the strategy which needs to be at the hubs for maximum public good (as identified in [11], [17]) would be placed at the node with the higher degree: thus, the swap was not performed if this rule is already satisfied for the given pair of nodes. For example, if a particular link connected a node with degree 3 playing Cooperator and another node with degree 7 playing Defector, these strategies were swapped, so that the Cooperator strategy would end up with the higher-degreed node. If the strategy assignment was the other way round to begin with, then the strategies were not swapped. Thus, the STRT represented the level of effort needed before the public utility threshold was reached. Four topological attributes were considered, namely i) scale-free exponent γ ii) network clustering coefficient iii) network assortativity iv) network mutual information. In the figures presented below, each datapoint in each figure represents an average of 20 simulation runs on the same network, and the public good threshold is set at 1.5 per player per game with respect to Table 1, unless otherwise stated.

A. STRT against Scale-free exponent

The results for STRT against scale-free exponent are shown in figures 4, 5, 6 and 7, below. In general, STRT displays a linear negative correlation with scale-free exponent. That is, as the scale-free exponent increases, the number of strategy swaps needed before the network reaches a certain public good threshold decreases; that is, the threshold can be reached quicker. This result is true across all pairs of strategies that we considered, and regardless of the actual value of the public good threshold, as long as the threshold is sufficiently high. For example, Fig. 4 shows the case where cooperator and defector strategies are swapped in a prisoners dilemma game, and the public utility threshold was set at 1.5 per player per game on average. Fig. 5 shows the 'general cooperator' strategy, as described before, being swapped with 'general defector' strategy, while Fig. 6 shows 'Pavlov' strategy being swapped with 'Tit-for-tat' strategy, and Fig. 7 shows 'Pavlov' strategy being swapped with 'General defector' strategy all for varying public utility thresholds. The results, it can be seen, are very similar: in all cases, there is a very linear and negative correlation between the STRT and the scale-free exponent γ.

B. STRT against Clustering Coefficient

We compared STRT against clustering coefficient of the scale-free networks that we studied, and did not find any particular correlation. This is shown in figures 8, 9, and 10 respectively. As before, each figure corresponds to a particular pair of strategies: Coordinator and Defector, General Coordinator and General Defector, Pavlov and Tit-for-Tat. It is clear that regardless of

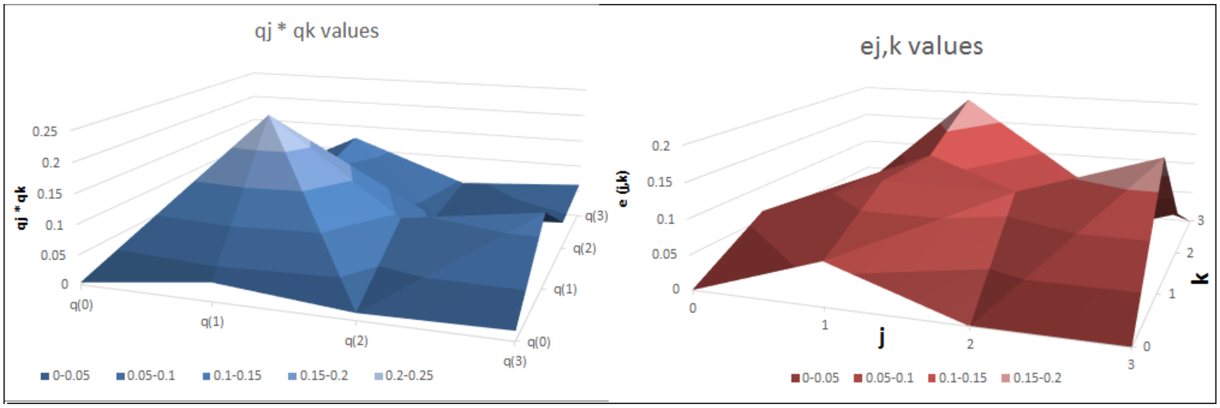

Fig. 3: The (a) $q_j * q_k$ values and the (b) $e_{j,k}$ for the network shown in Fig. 2. Note that if the distribution represented by the red surface is varied while the blue surface is not varied, the network in Fig. 2 will be rewired, and the information content of the resulting network will change, however the degree distribution will remain unchanged.

the pair of strategies which are being swapped by the nodes, increasing or decreasing the clustering coefficient of the network topology does not have a marked impact in how quickly the desired public good threshold can be reached.

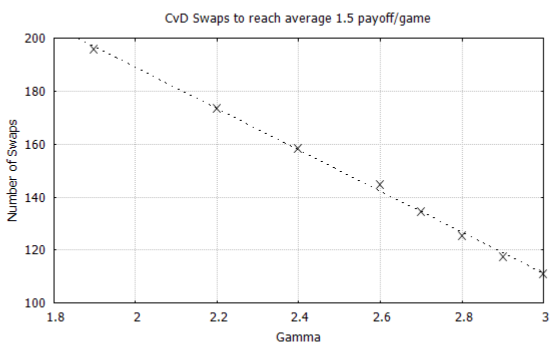

Fig. 4: STRT against scale-free exponent γ for nodes swapping Cooperator and Defector strategies.

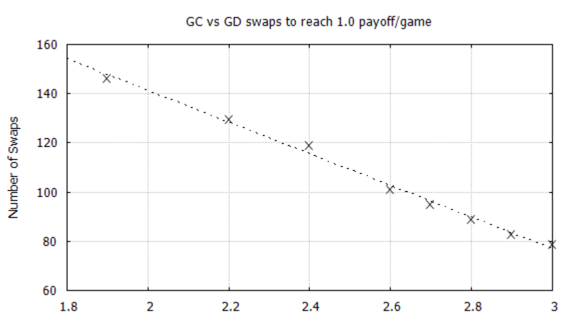

Fig. 5: STRT against scale-free exponent γ for nodes swapping General Cooperator and General Defector strategies.

C. STRT against Assortativity

We then compared STRT against the assortativity of the scale-free networks that we studied. Here we found that there is a positive correlation, but the correlation is exponential rather than linear, as figures 11, 12, and 14 show. The correlation is not much affected by the scale-free exponent of the scale-free networks, as

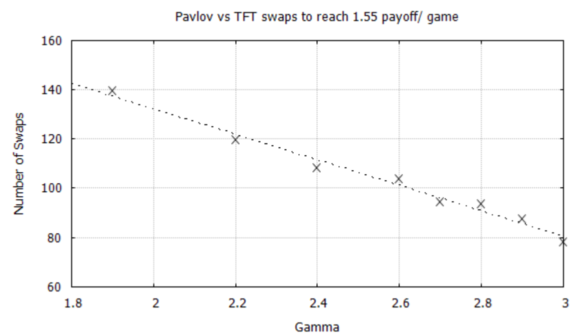

Fig. 6: STRT against scale-free exponent γ for nodes swapping Pavlov and Tit-for-Tat strategies.

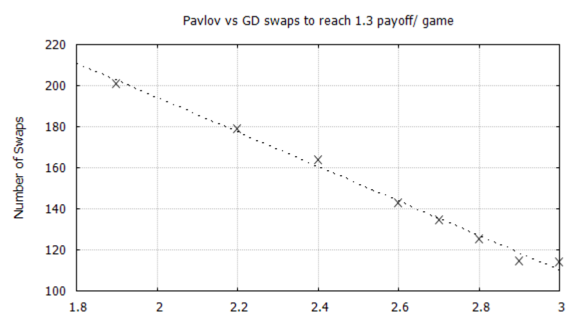

Fig. 7: STRT against scale-free exponent γ for nodes swapping Pavlov and General Defector strategies.

comparing figures 11 and 12 reveal ($\gamma = 2.8$ and $\gamma = 2.6$ reveal similar trends). The correlation however, becomes comparatively weaker when comparatively lower public good thresholds are used, or when mixed strategies (such as Pavlov and General Cooperator) instead of pure strategies are swapped, as comparing figure 11 with figure 14 indicates.

These trends could be explained by the fact that in scale-free networks with dissassortativity (that is, negative assortativity values), star-structures are often present, which allow strategies at the peripheral nodes to 'reach' the hubs relatively quickly, when swapped. So, for example, a lot of nodes with degree equal to

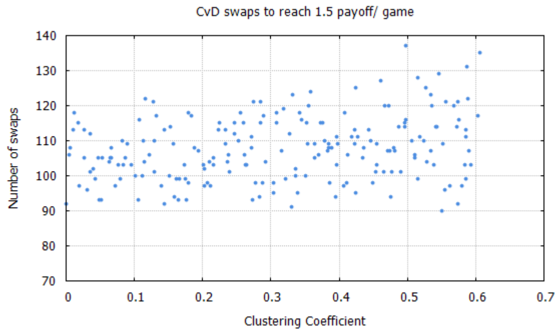

Fig. 8: STRT against network clustering coefficient $\bar{C}$ for nodes swapping Cooperator and Defector strategies. Here $\gamma = 2.8$.

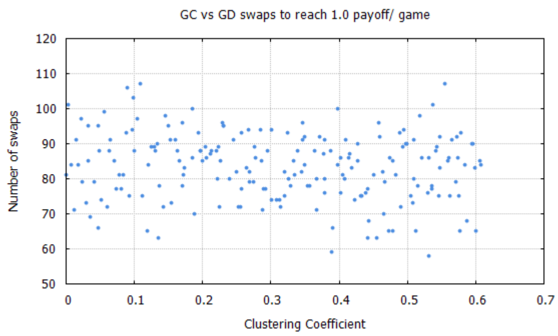

Fig. 9: STRT against network clustering coefficient $\bar{C}$ for nodes swapping General Cooperator and General Defector strategies. Here $\gamma = 2.8$.

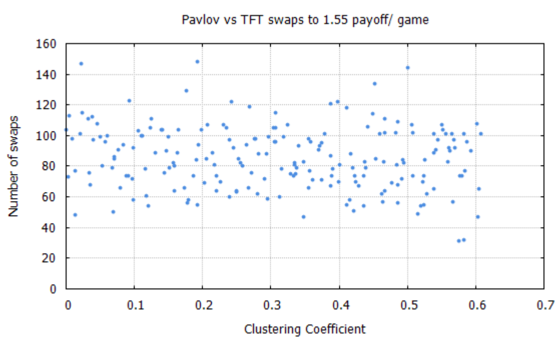

Fig. 10: STRT against network clustering coefficient $\bar{C}$ for nodes swapping Pavlov and Tit-for-Tat strategies. Here $\gamma = 2.8$.

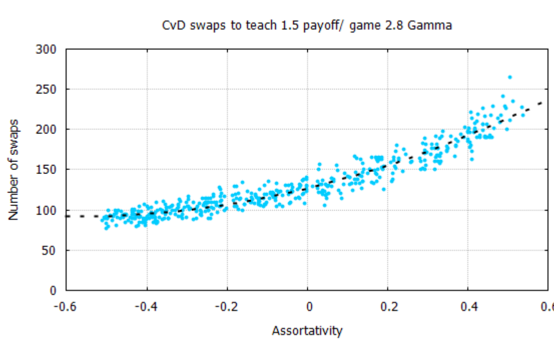

Fig. 11: STRT against assortativity r for nodes swapping Cooperator and Defector strategies. Here $\gamma = 2.8$. Public good threshold = 1.5

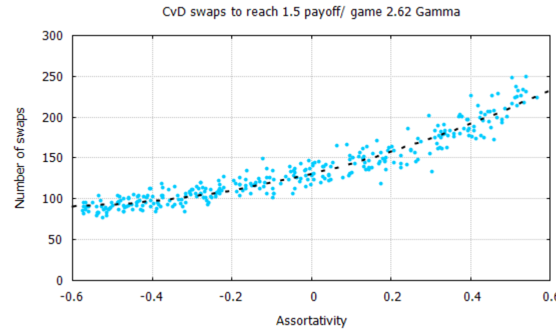

Fig. 12: STRT against assortativity r for nodes swapping Cooperator and Defector strategies. Here $\gamma = 2.6$. Public good threshold = 1.5

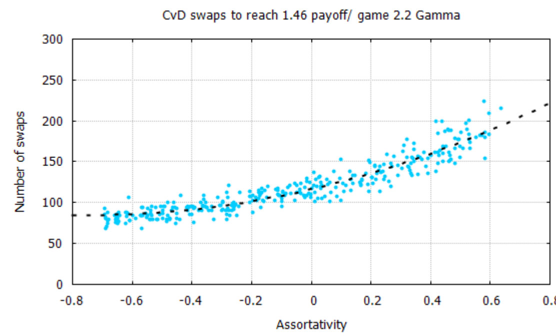

Fig. 13: STRT against assortativity $\bar{C}$ for nodes swapping Cooperator and Defector strategies. Each datapoint represents an average of 20 simulation runs on the same network with $\gamma = 2.8$. Public good threshold = 1.5 per game

one could reach a node with degree equal to a hundred, say, in a single swap. On the other hand, when assortativity is higher, this by definition means the degree difference between adjacent nodes is lower, therefore, when swapped, strategies from the peripheral nodes must 'travel' longer (go through more swaps) before they reach the hubs. Since, in any pair of strategies, where one strategy is more desirable at the hubs for the public good of the network, and initially all strategies are randomly assigned, disassortativity overall facilitates quicker movement of the desirable strategy towards the hubs, and thus, in disassortative networks the public

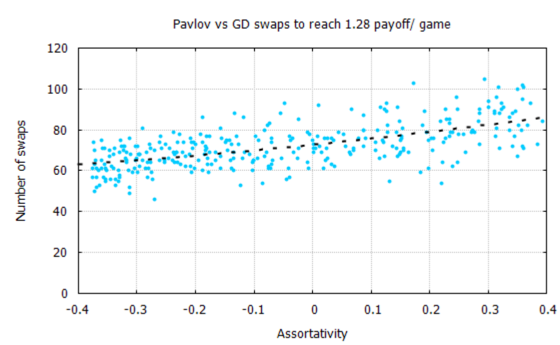

Fig. 14: STRT against assortativity r for nodes swapping Pavlov and General Cooperator strategies. Here $\gamma = 2.8$. Public good threshold = 1.2

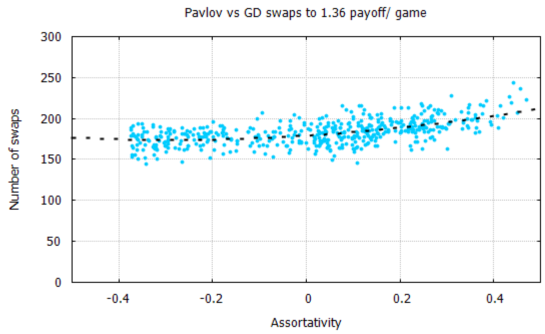

Fig. 15: STRT against assortativity $\bar{C}$ for nodes swapping Pavlov and General Cooperator strategies. Each datapoint represents an average of 20 simulation runs on the same network with $\gamma = 2.8$. Public good threshold = 1.5 per game

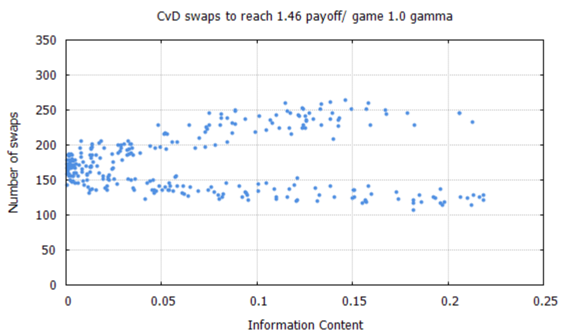

Fig. 16: STRT against mutual information for nodes swapping Cooperator and Defector strategies. Here $\gamma = 1.0$.

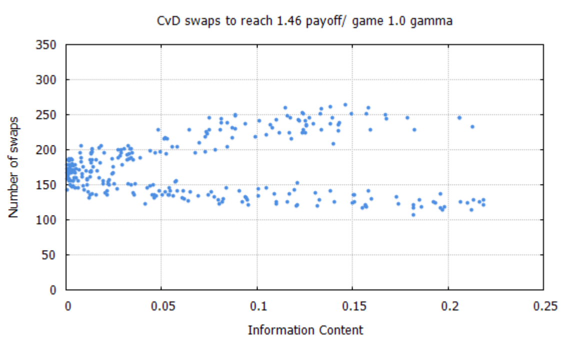

Fig. 17: STRT against mutual information for nodes swapping Cooperator and Defector strategies. Here $\gamma = 1.0$.

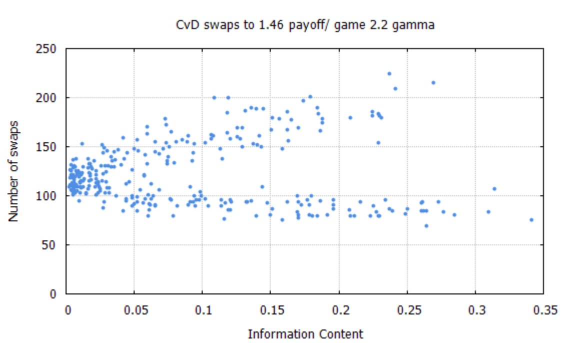

Fig. 18: STRT against mutual information for nodes swapping Cooperator and Defector strategies. Here $\gamma = 2.2$.

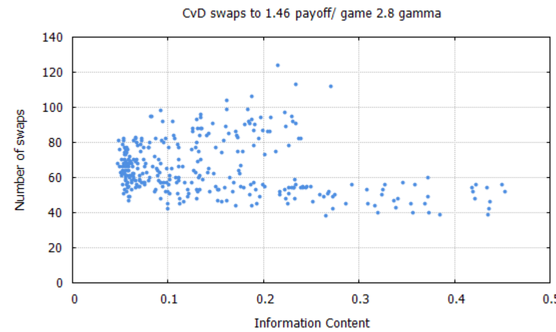

Fig. 19: STRT against mutual information for nodes swapping Cooperator and Defector strategies. Here $\gamma = 2.8$.

good threshold can be achieved with fewer swaps.

D. *STRT against Information Content*

We next focus on information content (or mutual information) of networks, as defined before, against the STRT. Our results are shown in figures 17, 18 and 19 , and all trends show an interesting bifurcation. That is, in each figure, one class of networks shows an increasing trend, while another class shows a decreasing trend, even though both classes belong to scale-free networks with the same scale-free exponent. This bifurcation happens regardless of the value of the scale-free exponent, even though we may notice, curiously, that when $\gamma = 1$, this bifurcation is symmetric, and for all other values of γ, this bifurcation is asymmetric.

This phenomena could be explained by considering equation 4 and figure 3, and noting that there are two ways in which the $e_{j,k}$ distribution could deviate from the quantity $q_j * q_k$: upward on average, or downward on average. Since the mutual information essentially measures the variation of $e_{j,k}$ distribution from the q_k distribution, for any given q_k distribution, there will be two $e_{j,k}$ distributions which will result in the same mutual information: one that varies 'upwards' (on average) and another which varies 'downward' (on average). We postulate that the two classes of networks present in the figures 17, 18 and 19 correspond to these two types of $e_{j,k}$ distribution for a given q_k, with the 'upward variation' corresponding to the increasing trend and the 'downward variation' corresponding to the decreasing trend.

It is interesting to note, therefore, that while an increase in the scale-free exponent helps the network reach its public utility threshold quicker, an increase in assortativity has the opposite effect: it results in the network reaching its public utility threshold more slowly. Clustering coefficient seems to have no impact on how quickly a network reaches its public utility threshold, while an increase in topological information content helps a class of scale-free networks and hinders another class of scale-free networks in reaching the public utility threshold. It would be enlightening, as part of future research, to analyse what topological features distinguish these two classes of networks.

V. Conclusions

In networked game theory, pairs of strategies could be compared in terms of their topological placement, and this usually results in the broad conclusion that for optimum public good, one strategy needs to occupy the hubs and another strategy needs to occupy the peripheral nodes. This becomes even more important in scenarios where a certain fixed proportion of players play each strategy, and while these players can be swapped topologically, the proportion can not be changed. A real world example is a defensive system which is predefined topologically and when under attack, all of its positions need to be occupied at all times, and the number of defenders and their resources are fixed, but certain defenders or their instruments are most effective at certain positions or locations. The leadership of the defence can therefore only try to rearrange the allocation of people to positions by swapping the defenders locally.

It has already been identified in literature, when strategies are considered pairwise in networks, which strategies need to occupy the hubs for optimal public good. For example, it has been shown that when cooperation plays defection, cooperators need to be at the hubs for optimal public good. In this paper, we considered the related question of reaching a certain public good threshold, which could correspond to a certain level of performance / break-even profit / level of effectiveness within a system. We analysed how the topology of the social network affects how quickly this threshold can be reached by swapping players who play fixed strategies along topological links, from an initial random assignment. Theoretically, this is also equivalent to the question of players remaining in fixed locations and swapping strategies with their neighbours, though as it is hard to think why players might want to swap rather than adopt strategies, it is more realistic to consider players (for example, defenders) who are fixated in their strategies (for example, instruments or skill sets), but willing to swap positions with their neighbours in the organisational (for example, defensive) topology.

We considered four topological properties: scale-free exponent (since most real world social networks are scale-free), clustering coefficient, assortativity and topological information content. We found that clustering coefficient does not affect the number of swaps it takes to achieve a certain utility threshold. Scale-free exponent shows a decreasing trend, so that the higher it is, the quicker the threshold can be reached from a random initial assignment. Assortativity on the other hand shows an increasing trend, so that the higher the assortativity, the longer it takes to reach a public utility threshold from a random initial assignment. We found that in terms of mutual information, there are two classes of scale-free networks, where in one class higher information content makes it quicker to reach the utility threshold, whereas in another class the opposite is true.

Our results can be used to design social systems where a certain network payoff threshold can be quickly reached by swapping players among topological locations, regardless of the initial assignment. For example, defence experts can design defensive fortifications and topologies in ways that reduce the need to swap defenders who have fixed skills / strategies, before the defensive structure reaches a certain performance threshold. This is particularly interesting since a global outcome (increased effectiveness of defense) is achieved by undertaking local actions (swapping defenders among neighbouring defensive positions). It is our future research direction to identify and experiment with more practical contexts where such results will be useful.

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