Mathematical Modeling of Weft yarn Tension in Pirn Winding

EASK Fernando, TSS Jayawardana

***Abstract*— Pirn winding is an operation of winding yarn from supply yarn package onto pirns which are used for shuttle weft insertion. Firm and coherent winding tension is required to maintain in the pirn winding process to withstand the high forces produced by the deceleration of the shuttle at the end of each pick and to avoid permanent strains in yarns which will appears as fabric defects in weaving. The package size and its diameter vary due to subsequent winding of pirns. Hence the balloon effect varies and caused to change the thread tension at the winding point of the pirn. The thread tension variation is considerably significant from the first pirn wound from a package to the last pirn wound from the same if no tension controlling mechanism is devised. Placing dead weights on the disc tension controller which adds a tension to take off tension may help to compensate the yarn tension variation to some extent to combat with this problem. However, this is a stepwise manual compensation technique which needs the correct timing of compensation to avoid significant tension variations in pirn windings. The author attempted to theoretically analyze the tension variation in the yarn path of the pirn winding machine and theoretically model the tension variation with and without deadweight placement on the tensioners. Author also verified the accuracy and the validity of the model developed through the experimental results obtained at different locations along the yarn path..**

***Index Term*— Constant tension, pirn-winder, tension model**

I. INTRODUCTION

Winding can be considered as a special operation to produce a particular type of package; but in broader sense almost every textile operation involves winding of one sort of another. Whatever the type of winding, it should be done under constant tension. The variations in tension of a package may produce distortion of the intended yarn path when used for subsequent textile operation and consequent deformation or irregularity of the woven or knitted fabrics.

The pirn winding is a process of winding yarns from a large package to pirns which are used to weft insertion. Weft yarn package should fit into the shuttle loom and then send it across the loom shed in order to insert the weft at right angles to the warp threads.

There are two conflicting requirements in winding of yarn onto pirns when it is used to be at a loom. A firm and coherent package is necessary to withstand the high forces produced by the deceleration of the shuttle at the end of each pick which may be more than two hundred times of the acceleration due to gravity. The production of such packages is assisted by the use of high winding tension. However, excessive tension causes permanent strain in visco-elastic yarns and consequent defect may be appeared on the fabric. These defects are particularly observable when the strain variations are cyclic along the yarn.

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. Though shuttle looms are widely used due to low production, in case of modern fabrics such as smart textiles and technical textiles where woven fabrics are produced with electric conductive material incorporating various electronic devices, it is important to have a continuation of the weft yarn for a certain length in the fabric to obtain electric conductivity for certain distance. Only shuttle looms could provide above mentioned requirements as the weft is inserted from a pirn with a length of about 1000m to 2000m depending on the yarn count. Therefore it is vital to mathematically analyse tension variation of weft yarn during pirn winding so that good quality of fabric can be produced where number of weft breakages should zero during weaving in case of technical fabrics.

The wining tension at the pirn is the take off tension of the yarn package plus the tension added in the yarn path of the pirn winder due to friction. Unwinding tension is heavily depending on the package diameter, its shape and the winding pattern. The tension variation in unwinding is highly conditioned by the balloon effect and it varies with the yarn package diameter. Diameter variation of yarn package due to consumption of yarn causes to create tension variation in the pirn.

So the winding tension variation in pirn is considerably high from the first pirn wound from a package to the last pirn wound from the same if no tension controlling mechanism is devised. Experimentally it was found that the tension variation from full cone package to near empty cone package is approximately 10cN which is nearly one third of the winding tension. Placing dead weights on the disc tension controller which adds a tension to take off tension may help to partially compensate the yarn tension variation. However, this is a stepwise manual compensation technique and its accuracy depends on the step size (weight of each dead weight) and the correct timing of placing the dead weight. Tension variation is nonlinear with the time even the machine operates at a constant speed, and hence timing of placement cannot be done at equal time intervals. This leaves a challenge for devising a closed loop tension regulating device.

Sri Lanka is a country where the higher production of woven fabrics is done on conventional shuttle looms. Analysis of root causes for tension variation and a method to compensate the tension variation accurately are of paramount importance to eliminate the variation of winding tension and hence to eradicate the subsequent fabric defects caused.

In this paper, authors attempt to analyse the tension variation in the yarn path due to friction in theoretical perspective and identify the significance of tension variation due to change of yarn package diameter practically. Further, a closed loop tension control device was developed to emulate the timely placement of dead weight and practically evaluate the significance of optimally timed deadweight placement.

Section II briefly describes about the attempts made by the researches in the history to address the pirn tension variation. Methodology is illustrated in section III while section IV is dedicated for the results. Discussion and conclusion of the research work described in this paper could be found in section V.

II. LITERATURE REVIEW

The value of the winding tension in the yarn depends initially on the take off tension from the supply package and then on any subsequent increase of tension due to friction taking place over tensioning devices and yarn guides. An intensive research has been carried on modelling of take off tension in unwinding of yarn from a package. The take off tension depends on the speed of the yarn and also on a large number of other facts such as unwinding tension, tension with which the yarn was wound on the package, coefficient of friction of unwinding yarn against the surface of the package when yarn touches it, shape of the package and the geometry of winding the yarn on package, coefficient of friction of the unwinding yarn against the cap that may be used on the edge of the package tube to protect the yarn from damages and the speed of the yarn. [2], [3]. It has also been observed that the takeoff tension of the yarn during high speed over end unwinding from a helically wound stationary package is highly non-linear. [8]

A comprehensive investigation of dynamics of over-end winding has been reported by Padfield [8] and subsequently Kothari et. al. [2] carried out extensive numerical calculations based on Padfield's analysis. Goswami [9] continued analysis of non-linear dynamics of unwinding tension, however no exact model was developed and even in case of simplified mathematical model, parameters are situational. Dynamics of yarn balloon was investigated over a long period, and recently a complex model was developed under number of uncertainties in the process.

At higher speed yarn unwinding, a balloon is formed and as a result of that the contact of yarn with the package is lost. Due to the formation of balloon, friction effect between the yarn and package is no more significant, but a new factor of the air drag of the balloon is to be considered. This effect is called the balloon effect and it depends on the speed of the yarn and shape of the balloon. The shape of the balloon depends on the geometry of the yarn path on the package, the characteristics of the package (package diameter, circularity, conical angle), modulus of elasticity of the yarn and the position of the first guide Ormerod [4], Lord and Mohemed [5]. When the yarn speed is increased the yarn tension due to balloon effect will also be increased. JD Clark et. al. also attempted to model the tension of yarn in unwinding from a package [11]. TMJA Cooray and Sandun Fernando [6] mathematically modelled the over-end yarn withdrawal and proposed a device to minimize the tension variation during unwinding. EE Mignshov [7] gave an insight to the yarn dynamics in unwinding process from package to package.

Although the volume of literature on the balloon theory, few publications were found with numerical results based on general equations of motion. The reader can refer to Kothari and Leaf [2], Kothari and Leaf [3]. Controlling attempts of unwinding tension has drawn the attention of many researchers as its variation effect causes even to breakage of the yarn. Various control strategies were adapted to automatic control of take off tension of the yarn in the unwinding process. Adaptive control strategy, which can be able to adjust the model automatically, was used to reject and eliminate the disturbances. Active Disturbance Rejection Control (ADRC) concept proposed by Hou [10] is one of such strategies used for yarn tension control. In this method, disturbances were estimated using an extended state observer and compensated during each sampling period. The controller was designed to be intrinsically robust against plant parameter variations. Because of the robustness of the controller and adequate capability of disturbance rejection, the proposed ADRC method is particularly suitable for yarn tension regulation applications.

In the study of literature on tension yarn controlling many works has been in automation on unwinding and regulation of tension. In some studies, Micro Electro-Mechanical Systems (MEMS) were used to monitor and maintain more or less uniform yarn tension throughout the unwinding process. In terms of cost-effectiveness, is has not gained the popularity but still used in several large scale applications.

In the literature of pirn winding, optimum winding conditions were discussed by Catlow and Walls [1]. The stated optimum winding condition could be met if the winding tension was constant and some attempts had been made to achieve this with the use of spring compensators and variable speed device.

The tension variation of pirn winding was tried to regulate by placing dead weights on the disc tension controller which adds a tension to take off tension. Several experiments are found in literature to investigate the significance of tension variation due to ballooning effect only by exact compensation of tension variation due to that effect [12]. An experimental study was illustrated by the author to investigate the exact impact of stepwise variation of incremental tension variation in the yarn path of pirn winder with automatic stepwise variation of dead weight effect by applying a stepwise varying electromagnet force on dead weights [13]. Still a detail analysis of exact model of incremental yarn tension variation along the yarn path of the pirn winder remain unsolved in literature and author attempts to address this gap through this paper. The results are also substantiated with experimental results.

III. METHODOLOGY

Winding tension of the yarn at the pirns depends initially on the take off tension from the supply package and subsequently increases the tension due to yarn guides and tension discs. A model was developed to analyse the yarn tension increment in detail along the yarn path.

A. Pirn winding machine and yarn path

Figure 1 shows the high speed pirn winding machine for which the yarn path was analysed and developed the tension model.

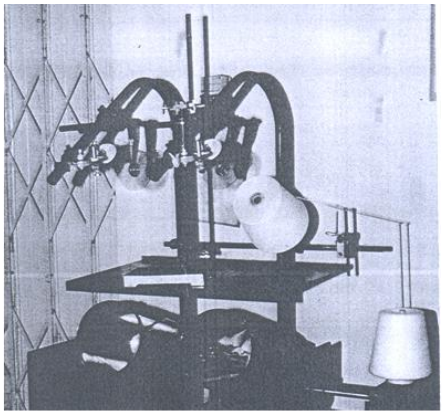

Fig. 1. High-speed pirn winding machine

The automatic pirn winding machine was designed to wind cotton staple and other kinds of yarn from stationary packages to pirns to be used in looms. It has the maximum spindle peed of 12000 rpm and capable of winding 10 inch long weft pirns.

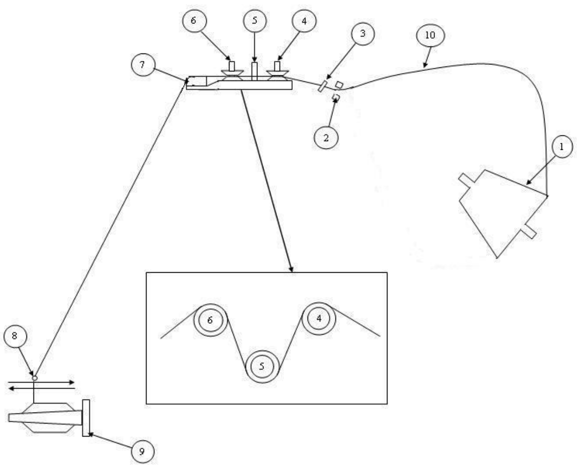

Fig. 2. Yarn path of pirn winding machine

Figure 2 concisely gives the yarn path of the machine illustrates the formation of the balloon. 1 through 10 denote supply yarn package, ceramic eye and thread guide, pigtail thread guide, first disc tension device, fixed post thread guide, second disc tension device, ceramic guide, reciprocating thread guide, pirn and yarn balloon respectively.

B. Mathematical model for tension variation

The friction influence which can be occurred in the yarn path due to different contact points such as yarn guide eyes, guide posts, tension devices have been added to the take off tension of the yarn package to constitute final winding tension. Take off tension measured after ceramic eye and thread guide is denoted by T_1 while it was increased to T_2 once it passes through pigtail thread guide. Due to two numbers of disc tension devices and fixed post thread guide yarn tension becomes T_3 and subsequently becomes T_4 after it passes through the ceramic guide. The yarn tension in different areas on the yarn path is denoted in figure 3. Since the tension control device is implemented with disc tension devices, tension variation across the disc tension devices is modelled and for that a detailed tension variation of the yarn across devices is given in figure 4.

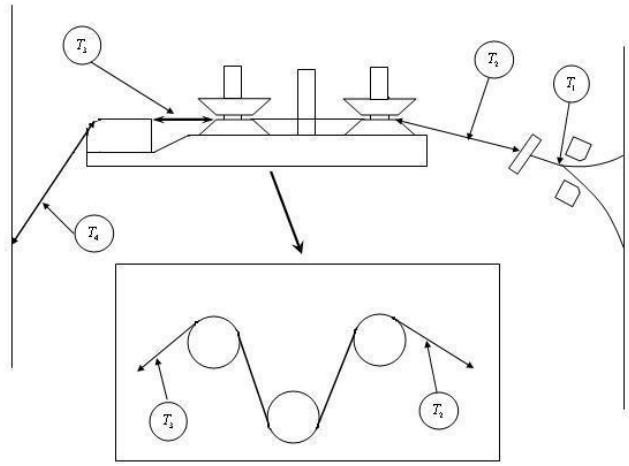

Fig. 3. Tension in different areas of the yarn path

The take off tension T_1 was theoretically modelled by Gordeev and Volkof [14] in 1984 and the theoritiical unwinding tension is given by the following formula.

$$T_1 = 2T\left(1 + k\,sin^2\beta_0.\frac{H^2}{r_0^2}\right)v^2\,10^{-8} \quad (1)$$

Where T is yarn count in Tex, k is the coefficient depending on the condition of winding, β_0 is the angle of incline of the yarn curl to the spindle axis, H is the height of the balloon in cm, r_0 is the radius at the point of unwinding in cm and v is the unwinding speed in cm/sec.

The take off tension T_1 from the yarn package is elevated due to the friction between the yarn and ceramic eye (denoted by no. 2 in Figure 2). The yarn tension after ceramic eye T_2 can de given as

$$T_2 = T_1 e^{\mu_1 \theta_1} \quad (2)$$

where θ_1 is the angle of touching inside the ceramic eye and μ_1 is the friction coefficient between the yarn and ceramic eye.

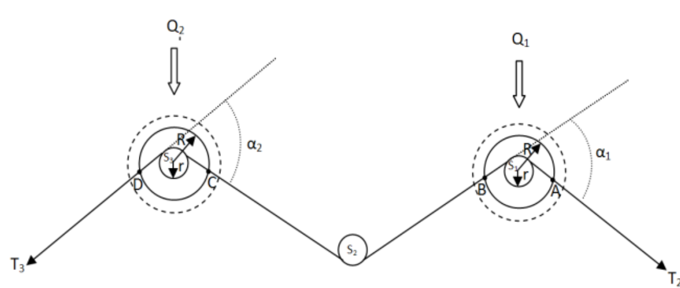

Fig. 4. Detailed tension variation across two disc tension devices and fixed post thread guide

In figure 4, centres of tension disk 1, fixed post thread guide and tension disc 2 are denoted by S_1, S_2 and S_3 respectively. R is the radius of contact surface of the washer and r is the radius of the spindle in which the washer is placed.

Then the thread passes through the first tension controller. In addition to sliding the yarn around the spindle of the first tension controller, it is subjected to axial force between disks. Therefore the tension added in the friction discs is due to a two fold mechanism. Hence yarn tension at the first contact point of the first disc tension controller T_A can be stated as

$T_A = T_2 + 2p_1\mu_2$ (3)

where μ_2 is the coefficient of friction of yarn against the washer and p_1 is the pressure exerted by the washer at the first tension device. It can be quantitatively expressed as [15]

$p_1 = 0.5Q_1a_1$ (4)

where Q_1 is the axial pressure force of the first disc tension device due to dead weights and

$a_1 = \frac{R}{\sqrt[2]{(R^2-r^2)}\sin\left(\frac{\alpha_1}{2}\right)-r\cos\left(\frac{\alpha_1}{2}\right)+R}$ (5)

α_1 is the angle of yarn bend around the spindle at the first tension device.

Due to yarn slippage wrapping around the spindle of the first disc tension device, yarn tension is further elevated and the tension of the yarn at the last contact point of the spindle is given by

$T_{S1} = T_A e^{\mu\alpha_1}$ (6)

where μ is the coefficient of friction between the spindle of the first disc tension device and the yarn. It can be proved that the lap angle α_1 is equal to the angle of yarn bend around the spindle at the first tension device with simple geometry.

Due to the second segment of the trapped yarn between two washers of the first disc yarn tension device causes to increase the yarn tension further and yarn tension at point B can be expressed as

$T_B = T_{S1} + 2p_1\mu_2$ (7)

As the yarn slides around the fixed post thread guide with a lap angle of β it also contributed to increase the yarn tension and the yarn tension after the fixed post thread guideT_{S2} can be stated as

$T_{S2} = T_B e^{\beta\mu_3}$ (8)

where μ_3 is the friction coefficient between the fixed post thread guide and the yarn.

Assuming the coefficient of friction of yarn against the washer (μ_2) in two disc tension devices are identical and the friction coefficient of yarn against the spindle of the first disc tension device and second disc tension device are equal, in a similar manner, yarn tension after second disc tension device T_3 can be expressed as follows.

$T_3 = (T_{S2} + 2p_2\mu_2)e^{\mu\alpha_2} + 2p_2\mu_2$ (9)

where α_2 is the lap angle of the yarn around the spindle of the second disc tension device and p_2 is the pressure exerted by the washer at the second tension device and it is equal to as in [15]

$p_2 = 0.5Q_2a_2$ (10)

where Q_2 is the axial pressure force of the first disc tension device due to dead weights and

$a_2 = \frac{R}{\sqrt[2]{(R^2-r^2)}\sin\left(\frac{\alpha_2}{2}\right)-r\cos\left(\frac{\alpha_2}{2}\right)+R}$ (11)

The final tension after two disc tension devices and the fixed post thread guide can be simplified to obtain the following equation.

$$T_3 = (((T_2 + 2p_1\mu_2)e^{\mu_2\alpha_1} + 2p_1\mu_2)e^{\beta\mu_3} + 2p_2\mu_2)e^{\mu\alpha_2} + 2p_2\mu_2$$

$$T_3 = T_2e^{\mu(\alpha_1+\alpha_2)+\mu_3\beta} + 2p_1\mu_2e^{\mu\alpha_2+\mu_3\beta}(1+e^{\mu\alpha_1}) + 2p_2\mu_2(1+e^{\mu\alpha_2}) \quad (12)$$

The yarn slips over ceramic guide and thereby increases the yarn tension due to the friction between the ceramic guide and the yarn. If the angle of wrap around the ceramic guide is θ_2 then the yarn tension after the ceramic guide T_4 can be given by

$T_4 = T_3e^{\mu_1\theta_2}$ (13)

where μ_1 is the coefficient of friction between the yarn and ceramic guide.

C. Experimental procedure

The actual unwinding tension T_1 and actual winding tension could not be measured either due to insertion of tension meter interrupt the actual yarn path and hence change in existing tension or difficulty in inserting the probe of tension meter due to space limitation. For instance, in case of inserting tension meter probe before the ceramic eye may deform the balloon and change the diameter, which will cause to change the unwinding tension. Due to such practical limitations, yarn tension was measured between ceramic eye & thread guide as well as between the ceramic guide and reciprocating thread guide with a digital tension meter. By varying the dead weights, same readings were repeated.

The specifications of the tension measuring equipment can be stated as follows.

Make: ZIVY
Model: EL-TEN
Range: 0-400cN
Resolution: 0.1cN

In order to check the validity of the yarn tension model over varying package diameter, the experiment was repeated with yarn packages with different diameters. During this experiment, yarn type, thickness of the yarn (Tex count), yarn winding speed and winding pattern of the package were keep constant.

IV. Results

The unwinding tension of the supply yarn package and subsequent tension increment due to friction was calculated based on the theory. Thereby the tension between the ceramic eye & thread guide (T_2) was theoretically calculated. Further, using the developed mathematical model, yarn tension between the ceramic guide and reciprocating thread guide (T_4) was theoretically calculated. Since Matlab is a good scientific package that yields precise numerical calculations, a Matlab code was used for the theoretical calculation. Values of few parameters were experimentally obtained while the remaining parameters were obtained from the technical manual of the pirn winding machine. For theoretical calculations, the following set of parameters given in table I was used.

Table I
Parameters and its estimated or specified values used for theoretical calculations

Symbol	Description	Value
T	Tex count of the yarn	20
k	coefficient clirundly	0.05
β_0	angle of incline of the yarn curl to the spindle axis	120
H	height of the balloon in cm	22.1
r_0	radius at the point of unwinding in cm	3.74, 4.06, 7.14, 9.12, 12.12, 15.06
v	unwinding speed in cm/sec	916.6
θ_1	angle of touching inside the ceramic eye (in ^{0})	60
θ_2	angle of wrap around the ceramic guide (in ^{0})	70
μ_1	friction coefficient between the yarn and ceramic eye or ceramic guide	0.15
R	radius of contact surface of the washer in cm	3.34
r	radius of the spindle in cm	0.81
μ_2	coefficient of friction of yarn against the washer	0.2
μ_3	coefficient of friction between the fixed post thread guide and the yarn	0.3
β	lap angle around fixed post thread guide (in ^{0})	90
μ	coefficient of friction between the spindle of the disc tension device and the yarn	0.3
α_1	lap angle of the yarn around the spindle of the first disc tension device in degrees	45
α_2	lap angle of the yarn around the spindle of the second disc tension device in degrees	45
Q_1	weight of the washer in the first disc tension device in cN	4.12, 8.64, 11.63, 15.64, 23.71
Q_2	weight of the washer in the second disc tension device in cN	4.12, 8.52, 11.63, 16.01, 23.65

The table II summarizes the theoretical yarn tension between the ceramic eye & thread guide (T_4) as well as that between the ceramic guide and reciprocating thread guide (T_4) for yarn packages with different diameters.

Table II
Theoretical yarn tension after ceramic eye and before reciprocating ceramic guide for different package diameters and different dead weight

Average package diameter (in cm)	Dead Weights (in cN) & Theoretical Yarn Tension T (in CN)									
	Q_1= 4.12 Q_2= 4.12		Q_1= 8.64 Q_2= 8.52		Q_1= 11.63 Q_2= 11.63		Q_1= 15.64 Q_2= 16.01		Q_1= 23.71 Q_2= 23.65	
	T_2	T_4	T_2	T_4	T_2	T_4	T_2	T_4	T_2	T_4
3.74	0.445	7.18	0.445	12.26	0.445	16.53	0.445	22.35	0.445	33.52
4.06	0.437	7.15	0.437	12.25	0.437	16.52	0.437	22.35	0.437	33.52
7.14	0.407	7.06	0.407	12.24	0.407	16.51	0.407	22.34	0.407	33.51
9.12	0.402	7.04	0.402	12.24	0.402	16.51	0.402	22.33	0.402	33.51
12.12	0.398	7.03	0.398	12.24	0.398	16.51	0.398	22.33	0.398	33.51
15.06	0.397	7.03	0.397	12.24	0.397	16.51	0.397	22.33	0.397	33.50
18.17	0.396	7.02	0.396	12.24	0.396	16.51	0.396	22.33	0.396	33.50

Table III
Experimental yarn tension after ceramic eye and before reciprocating ceramic guide for different package diameters and different dead weight

Average package diameter (in cm)	Dead Weights (in cN) & Experimental Yarn Tension T (in cN)									
	Q_1= 4.12 Q_2= 4.12		Q_1= 8.64 Q_2= 8.52		Q_1= 11.63 Q_2= 11.63		Q_1= 15.64 Q_2= 16.01		Q_1= 23.71 Q_2= 23.65	
	T_2	T_4	T_2	T_4	T_2	T_4	T_2	T_4	T_2	T_4
3.74	0.4	7.9	0.4	13.3	0.4	17.8	0.4	25.4	0.4	34.8
4.06	0.4	7.2	0.4	13.1	0.4	17.2	0.4	24.1	0.4	34.1
7.14	0.4	7.3	0.4	13.2	0.4	17.7	0.4	23.5	0.4	33.8
9.12	0.4	7.1	0.4	12.2	0.4	16.8	0.4	22.3	0.4	33.2
12.12	0.3	7.0	0.3	11.8	0.3	16.1	0.3	22.0	0.3	31.4
15.06	0.3	6.8	0.3	10.0	0.3	15.7	0.3	21.4	0.3	30.4
18.17	0.3	6.7	0.3	10.7	0.3	14.4	0.3	20.6	0.3	29.5

Based on the theoretical model developed for the tension increment along the yarn path, theoretical increment in yarn tension was calculated and tabulated in Table IV. For easy comparison, the corresponding experimental values were also tabulated in the adjacent column of Table IV.

Table IV
Theoretical and experiment increment in yarn tension between the reciprocating ceramic guide and ceramic eye for different package diameters and different dead weight

Average package diameter (in cm)	Theoretical & Experimental Increment in Yarn Tension T_T and T_P (T_4-T_2 in CN)									
	Q_1= 4.12 Q_2= 4.12		Q_1= 8.64 Q_2= 8.52		Q_1= 11.63 Q_2= 11.63		Q_1= 15.64 Q_2= 16.01		Q_1= 23.71 Q_2= 23.65	
	T_T	T_P	T_T	T_P	T_T	T_P	T_T	T_P	T_T	T_P
3.74	6.735	7.5	11.815	12.9	16.085	17.4	21.905	25	33.075	34.4
4.06	6.713	6.8	11.813	12.7	16.083	16.8	21.913	23.7	33.083	33.7
7.14	6.653	6.9	11.833	12.8	16.103	17.3	21.933	23.1	33.103	33.4
9.12	6.638	6.7	11.838	11.8	16.108	16.4	21.928	21.9	33.108	32.8
12.12	6.632	6.7	11.842	11.5	16.112	15.8	21.932	21.7	33.112	31.1
15.06	6.633	6.5	11.843	9.7	16.113	15.4	21.933	21.1	33.103	30.1
18.17	6.624	6.4	11.844	10.4	16.114	14.1	21.934	20.3	33.104	29.2

Figure 5 illustrates the experimental results and theoretical value obtained using the developed model for the increment in yarn tension vs. different yarn package diameters.

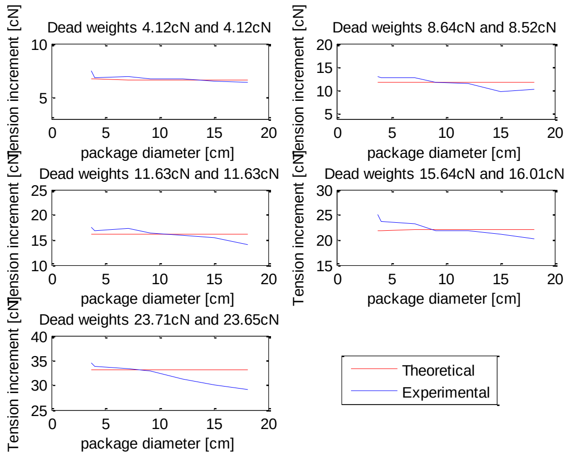

Fig. 5. Theoretical values and experimental results of yarn tension increment along the yarn path

V. Discussion

Fig.5 and Table 4, it is observed that theoretical tension increment along the yarn path is almost constant for a given dead weights placed on two disc tension devices. However, with the increment of dead weights placed on two disc tension devices, the increment in yarn tension is also elevated not proportionately. From Fig.5 it is evident that there is a close concurrence between the theoretical and experimental values of the increment in the yarn tension along the yarn path of the pirn winder. Thereby, the validity of the model developed is confirmed. However, experimental values have a slight negative trend.

In the yarn unwinding process from the supply package, yarn is subjected to longitudinal as well as lateral vibration which causes to develop dynamic tension. As the supply package diameter reduces, the balloon diameter of the unwinding yarn increases and due to which additional dynamic tension becomes slightly significant. This is evident that a downward trend in experimental yarn tension shown in the graphs of Fig. 5.

From machine manufacturer's point of view, the increment of yarn tension along the yarn path can be manipulated by varying the parameters r, R, α_1, α_2 and β. However, α_1, α_2 can be varied from 0^0 to 180^0 and r is subjected to the constraint of $r < R$ [15].

$$\frac{\partial a_i}{\partial r} = \frac{R\frac{r}{\sqrt{R^2-r}}\sin\frac{\alpha_i}{2} + \cos\frac{\alpha_i}{2}}{\left[\sqrt[2]{(R^2-r^2)}\sin\left(\frac{\alpha_i}{2}\right) - r\cos\left(\frac{\alpha_i}{2}\right) + R\right]^2}$$

Consequently, with the increase of r the value of a_i is increased and hence p_i and the increment in yarn tension.

$$\frac{\partial a_i}{\partial \alpha_i} = \frac{-0.5R\sqrt{R^2-r^2}\cos\frac{\alpha_i}{2} + \mathrm{r}\sin\frac{\alpha_i}{2}}{\left[\sqrt[2]{(R^2-r^2)}\sin\left(\frac{\alpha_i}{2}\right) - r\cos\left(\frac{\alpha_i}{2}\right) + R\right]^2} < 0$$

As the α_i increases, a_i is decreased and hence p_i and the increment in yarn tension along the yarn path. Therefore increment in α_1, α_2 caused to decrease the yarn tension whereas increase in r causes to increase the yarn tension. By varying the level of finish and by selecting appropriate materials with different coefficient of friction for the elements along the yarn path, the increment in yarn tension can be further varied by the manufacturer of the machine.

The scope of the developed model is limited to high peed pirn winding machine with tension controllers having dead weights. However, it can be portable to other tension controllers based on friction with slight modifications. In verifying the mathematical model, we have confronted with certain practical limitations such as fluctuation of tension within a cycle of weft winding and to overcome this we have obtained the average tension. Further, dynamic tension in weft winding has not been considered due to higher complexity in its nature and assumed that it has no significant influence to the final tension.

VI. Conclusions

A mathematical model for the yarn tension increment along the yarn path due to friction was developed. Hence the significance of tension variation and responsible technical parameters were identified in quantitative aspect. Further, experimentally yarn tension of different segment of the yarn path was obtained with a digital tension meter using a high speed pirn winding machine (at 550m/min speed). The experiment was repeated for different yarn package diameters and different dead weights. Thereby the validity of the yarn tension model was verified for different package diameters over different dead weights. However, the validity of the model could not be verified for different yarn counts and different yarn types due to practical constraints.

By using the above mathematical model it is possible to develop a continuous tension control device capable of regulating the winding tension of the weft package so as to ensure the minimum weft breakages during weaving. Though a large volume or research work carried out on unwinding tension for various packages, mathematical models developed for the unwinding tension of the weft yarn have certain limitations. Since the final unwinding tension of the pirn during shuttle weaving depends on the pirn winding tension during weft winding. Therefore, further research needs to be carried out on weft unwinding process in quantitative aspect during weaving.

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