Development of a Virtual Platform for Rotary Inverted Pendulum Controlling

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ABSTRACT

Higher education sector is currently moving back towards onsite teaching/learning environment. And most of the universities are adapting some online teaching methods to enhance learning experience of students. Though hands-on experience is delivered through practical classes in physical mode, for control systems engineering undergraduates, it is important to understand the basics through interactive learning. As virtual platforms are well known for providing opportunity for interactive learning, this study was focused on developing a virtual platform, based on a commonly used Rotary Inverted Pendulum System. To benefit students from all the levels, Linear Time Invariant Model, Nonlinear Model and the actual model was considered for the study. LQR controller gains were derived based on the mathematical model, and further modifications were done to produce a Linear Time Invariant model closer to the prototype. Both linear and nonlinear models were observed after applying same LQR gains and the prototype was applied with gains derived for the physical system. The virtual platform visually presents the behavior of the nonlinear model. This platform can be used as a tool for students at every level as it does not require to install any additional software, other than widely used Matlab. Thus, this would be an effective tool to be used along with onsite teaching / learning environment.

KEYWORDS: *Rotary Inverted Pendulum, Virtual Platform, LQR, High Fluidity Simulations*

1 INTRODUCTION

After the unexpected challenges caused by Covid-19 pandemic, the world has never returned to its usual self. People had to change some aspects of their life styles permanently, and it opened new areas of research in the field of Economics, as well as Education. With all the safety regulations coming in to the play, the higher education institutes were forced to change their usual way of teaching in no time. After more than two years of online education, universities are returning to its usual onsite teaching/learning environments. Most of the universities have discovered that the hybrid mode of education will deliver the maximum benefit to the students, and thus, they are adapting some techniques from online teaching inside the onsite environment. With recorded online lectures, students are at ease with the flexibility it offers for studying. Therefore, studies have proven that students favor online education, considering the guidance provided to learn autonomously (Cacheiro-Gonzalez et al., 2019). But there are some other aspects hindering the process of learning in an online environment. Lack of a stable internet connection, lack of interactions and inability to provide hands-on experience are some of them (Cao et al., 2020). Therefore, it is required to focus on an onsite teaching/learning environment and promote an online learning environment only when required. It is important for control engineering undergraduates to have a thorough understanding of the dynamic of systems. To facilitate that, it is important to guide them for self-learning while continuing their studies in onsite mode. A virtual platform form would be an ideal option to increase their engagement in interactive learning.

So far in control systems engineering Inverted Pendulum can be considered as the one of the most popular problems throughout the history. There are number of variations done to this problem and therefore, its applicability ranges from teaching basics to analyzing far more complex systems (Furuta,

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et al., 1991). The inverted pendulum is a nonlinear system, and it is basically categorized in to two main types; rotary inverted pendulum and linear inverted pendulum. Out of these two types, the linear inverted pendulum consists of number of additional nonlinearities, making it far more difficult to be modelled, when compare to its rotary version(Furuta et al., 1991) ,(Chandrasekara & Davari, 2004). Therefore, in this study, a rotary inverted pendulum was used for the virtual platform. In the 3rd world nation, there are universities which cannot afford to buy such Inverted Pendulum apparatus. In order to overcome this issue, many studies have been revolved around remote controlling of existing apparatus in real-time (Jung & Ahn, 2011; Kolencik & Zakova, 2009; Masár et al., 2004; Sukontanakarn & Parnichkun, 2009). Though this approach is very commendable in terms of resource sharing, still there are multiple number of issues. For a system to be controlled real-time, there must be an operator near the apparatus or complex autonomous system to smoothen the process, as there may be multiple number of people trying to control it. Also, for this type of arrangement, it is necessary to have a stable internet connection (good bandwidth and low latency). In terms of autonomous learning, the students can only use the apparatus under specific guidelines and that becomes just another practical to them.

To avoid the issues associated with internet stability, many studies have focused on creating a simulation environment, using linearized versions of inverted pendulum systems. With the obtained state space model, comparisons have been done with the real-time controlled systems to assess the performance (Lima et al., 2006; F. Pan et al., 2010; Y. H. Pan et al., 2014). As these systems are considerably deviated from the physical system performance, nonlinear models have been derived and controlled using various platforms. Some studies have focused on presenting data in terms of graphs with the absence of a physical system (Guo, 2012; Rawat et al., 2018), while some studies have taken a further step to link real time controlling of the actual apparatus to the simulation platform (Demirtas et al., 2013; Yuan & Zhang, 2013). The main drawback of these systems is the absence of visual aids to help the learner. Though these platforms are suitable for students with an understanding of the basics, for the beginners, lack of visualization may hinder the process of the learning. Use of Simscape multibody model has been suggested by many studies, and most of them have used the models for the purpose of robust controlling and to compare the performance of different controllers (Alkamachi, 2020),(Ghayoor, 2020). Comparison of performance of linear and nonlinear models, based on an actual system has been done in some studies, but vast difference between each model were presented as the actual system is a linear inverted pendulum with number of non-linearities (Kumarihami et al., 2021). More advance approach could be observed in research done by Ganganath and the team, as they have succeeded in creating a Simulink multibody model, which is closely resembling the actual system for remote teaching (Ganganath, 2022). A similar approach can be observed in another study, where a digital twin has been created for rotary motion platforms (Traver et al., n.d.). Although these systems are suitable for studying the system performance, exposure given to the students to interact with linear and nonlinear models are minimum.

In this study, main focus will be on creating a virtual platform, where the students can observe an existing system in its linear, nonlinear and real-time model. The platform will use Matlab software as the base, because almost all of the control engineering students have a thorough understanding on using Matlab software. This study has focused on creating a platform to be used without internet, allowing student to use and modify the models as they please. The procedure followed in achieving the task will be discussed under methodology.

2 METHODOLGY

2.1 System Modelling

Dynamic system model of the Rotary Inverted Pendulum unit is depicted in the Figure 1. The pendulum to be balanced is of length l_p and inertia J_p (about its center of mass). The pendulum forms an angle $\propto$ with the Z_0 axis, which is positively increasing in Counter Clockwise (CCW) direction. When the pendulum is at upright equilibrium position, the angle formed is zero. The pendulum is connected to the rotary arm through a pivot joint. As indicated in the Figure 1, the length of the rotary arm is l_r and inertia is J_r (about center of mass of rotary arm). θ is the angle formed between the rotary arm and the X_0 axis, which is indicating a positive increment in CCW direction.

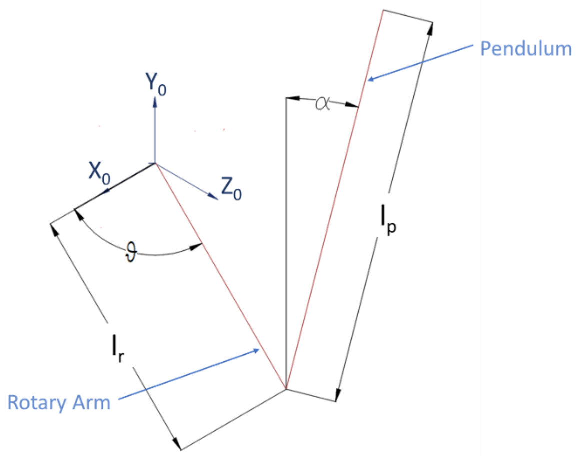

Figure 1. Conventions for Rotary Inverted Pendulum (Inc., 2011)

Lagrange method, which is often used in working with more complex systems like multiple jointed robot manipulators, is used to illustrate the motions of the pendulum and the rotary arm. The Euler-Lagrange equation as in equation 1, was used in the system dynamics.

$$\frac{\partial^2 L}{\partial t \partial q_i} - \frac{\partial L}{\partial q_i} = Q_i \qquad (1)$$

In rotary inverted pendulum system $\theta(t)$ and $\alpha(t)$ are the variables. Therefore, Euler-Lagrange equations will be as follows.

$$\frac{\partial^2 L}{\partial t \partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = Q_1 \qquad (2)$$

$$\frac{\partial^2 L}{\partial t \partial \dot{\alpha}} - \frac{\partial L}{\partial \alpha} = Q_2 \qquad (3)$$

As Lagrangian is the difference between the kinetic energy (T) and potential energy (V) of a system, it can be depicted as follows.

$$L = T - V \qquad (4)$$

When equating the non-conservative forces acting upon the system with respect to generalized coordinates to generalized forces, following equations can be derived for the rotary arm and the pendulum respectively. The values of the parameters used in following equations with respect to prototype are given in Table 1.

$$\left(m_p {l_r}^2 + \frac{1}{4} m_p l_p^2 - \frac{1}{4} m_p l_p^2 cos(\alpha^2) + J_r\right)\ddot{\theta} - \left(\frac{1}{2} m_p l_p l_r \cos(\alpha)\right)\ddot{\alpha} + \left(\frac{1}{2} m_p l_p^2 \sin(\alpha)\cos(\alpha)\right)\dot{\theta}\dot{\alpha} + \left(\frac{1}{2} m_p l_p l_r \sin(\alpha)\right)\dot{\alpha}^2 = \tau - B_r\dot{\theta} \qquad (5)$$

$$-\left(\frac{1}{2} m_p l_p l_r \cos(\alpha)\right)\ddot{\theta} + \left(J_p + \frac{1}{4} m_p l_p^2\right)\ddot{\alpha} - \left(\frac{1}{4} m_p l_p^2 \sin(\alpha)\cos(\alpha)\right)\dot{\theta}^2 - \frac{1}{2} m_p l_p g \sin(\alpha) = -B_p\dot{\alpha} \qquad (6)$$

At the rotary arm base, the applied torque (τ) can be described using the equation 7.

$$\tau = \frac{\eta_g K_g \eta_m k_t (V_m - K_g k_m \dot{\theta})}{R_m} \qquad (7)$$

2.2 Linear Time Invariant (LTI) Model

To obtain the state space equations of the model, the equations 5 and 6 were rearranged in to a matrix form and linearized. The terms “x” and “u” indicate the state vector of the system and control inputs respectively.

$$\dot{x} = Ax + Bu \tag{8}$$

$$y = Cx + Du \tag{9}$$

$$x = \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} \tag{10}$$

$$u = [\tau] \tag{11}$$

Thus, the matrices A, B, C, and D will be as follows.

$$A = \frac{1}{J_T}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{4}{m_p}^2{l_p}^2 l_r g & -\left(J_p + \frac{1}{4}m_p l_p^2\right)B_r & -\frac{1}{2}m_p l_p l_r B_p \\ 0 & \frac{1}{2}m_p l_p g\left(J_r + m_p l_r^2\right) & -\frac{1}{2}m_p l_p l_r B_r & -\left(J_r + m_p l_r^2\right)B_p \end{bmatrix} \tag{12}$$

$$B = \frac{1}{J_T}\begin{bmatrix} 0 \\ 0 \\ J_p + \frac{1}{4}m_p l_p^2 \\ \frac{1}{2}m_p l_p l_r \end{bmatrix} \tag{13}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{14}$$

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{15}$$

$$J_T = (m_p {l_r}^2 + J_r)(J_p + \tfrac{1}{4}m_p l_p^2) - \tfrac{1}{2}m_p l_p^2 l_r^2 \tag{16}$$

As the Quanser Rotary Inverted Pendulum was used as a reference in deriving the state space model, system parameters were also extracted from the actual system. Thus, the variables in the above equations 10, 11, 12 and 13 must be replaced with the values given in the Table 1.

Table 1. Parameters of the Rotary Inverted Pendulum System

Symbol	Description	Value	Unit
$\boldsymbol{m_p}$	Mass of the pendulum	0.127	kg
$\boldsymbol{l_p}$	Total Length of the pendulum	0.337	m
$\boldsymbol{J_p}$	Pendulum moment of inertia about center of mass	0.0012	$kg.m^2$
$\boldsymbol{B_p}$	Pendulum viscous damping coefficient as at pivot axis	0.0024	N.m.s/rad
$\boldsymbol{l_r}$	Rotary arm length	0.0619	m
$\boldsymbol{J_r}$	Rotary arm moment of inertia about its center of mas	0.000998	$kg.m^2$
$\boldsymbol{B_r}$	Rotary arm viscous damping coefficient as at pivot axis	0.0024	N.m.s/rad

With the parameters from the actual system, the LTI model was built in Simulink environment. The state space model was modified with the addition of a Kalman filter and an AWGN (Add White Gaussian Noise). The complete Simulink model for the LTI system is shown in Figure 2. In addition, a signal generator was utilized for feeding different types of signals to the system.

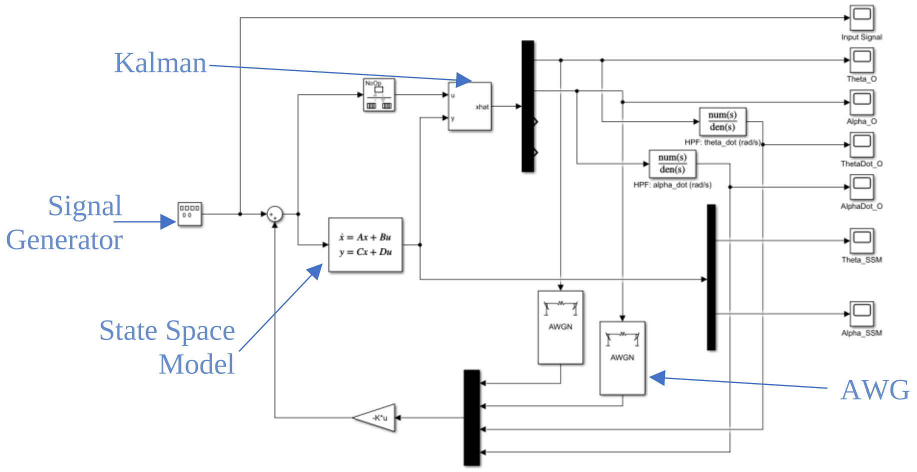

Figure 2. Linear Time Invariant Model developed in Simulink Environment

2.3 Nonlinear Model

The Quanser Rotary Pendulum system was used as a reference to build a CAD model using a 3D modelling software. The 3D model was consisting of all the main components and the movements between them. The rendered 3D model is depicted in Figure 3.

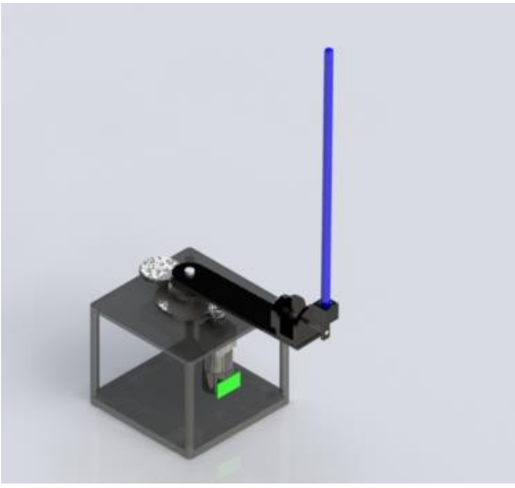

Figure 3. Rendered Image of the CAD Model

The 3D model was then imported in to Simscape Multibody environment, and the extracted model consist of two revolute joints, which is clearly depicted in Figure 4. The block diagram was then modified to be controlled by the LQR gains, and a signal generator was applied to observe the system behavior under different input signals. The complete block diagram in Simulink environment is shown Figure 5.

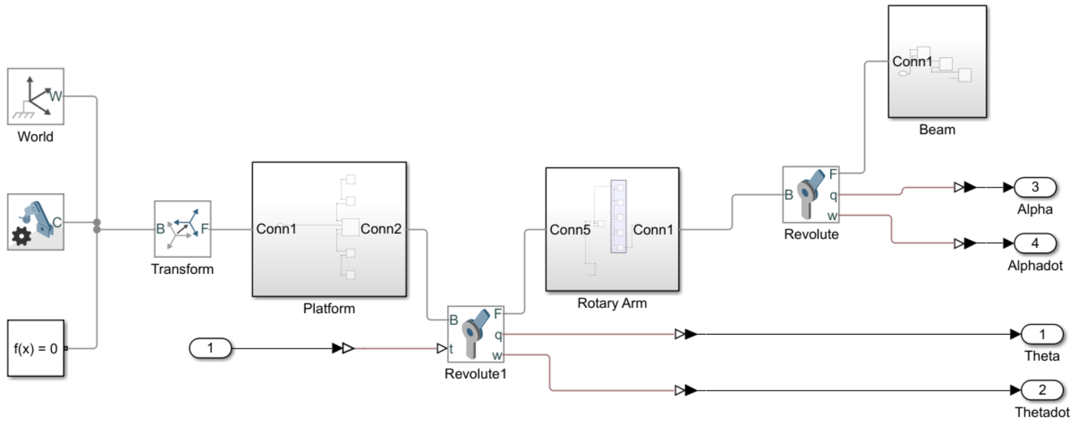

Figure 4. Simscape Multibody Diagram for the Rotary Inverted Pendulum System

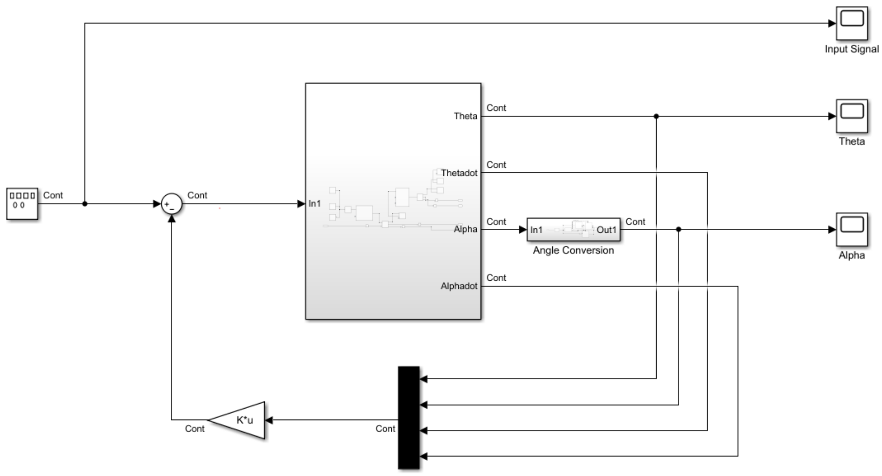

Figure 5. Non-linear Model Controlled using LQR gains

Shown below in Figure 6 is the appearance of the rotary inverted pendulum in Mechanics Explorer environment.

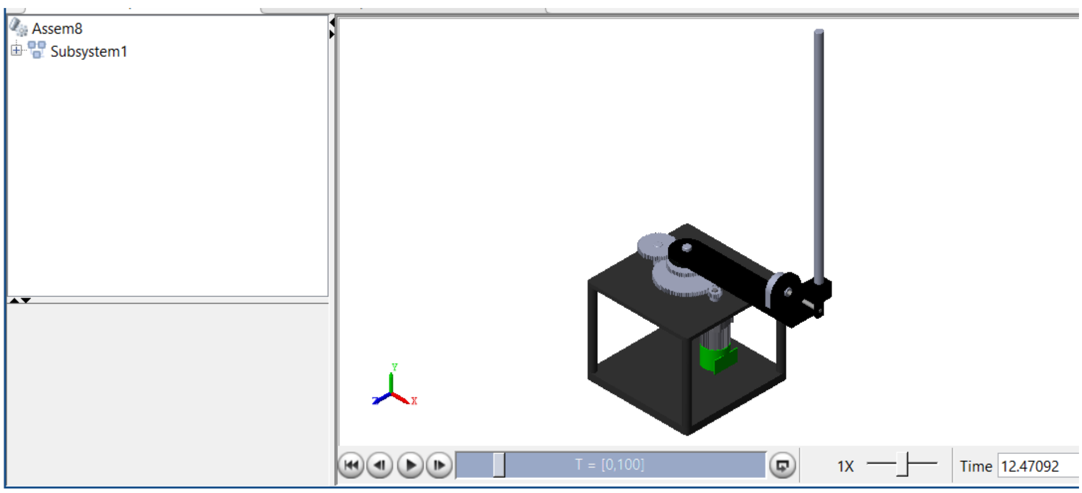

Figure 6. Visual Output of the Behaviour of Nonlinear Model

2.4 Prototype

The prototype shown in Figure 7 was actuated with same input signals, and was controlled with fine-tuned controller gains, considering the safety of the equipment.

Figure 7. Quanser Rotary Inverted Pendulum System

3 RESULTS & DISCUSSION

The LQR gains obtained from the mathematical model was used in controlling the LTI model as well as the nonlinear model. To observe the behavior of each system, signals shown in Figure 8, were fed to the both models using signal generators. In Case 1, the signal generator was producing a signal of amplitude 0 and in Case 2, the amplitude of the signal produced was ± 0.35 at 10 s intervals.

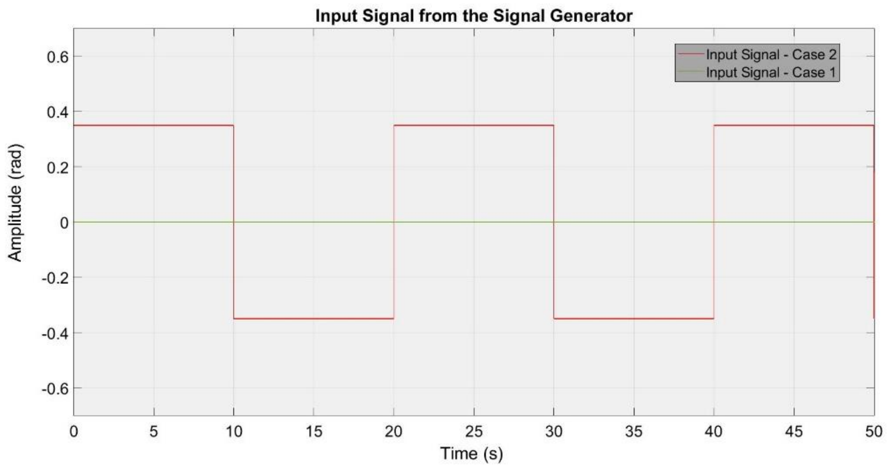

Figure 8. Input Signals from the Signal Generator Block

To compare the performance of both LTI system and nonlinear system under same conditions, the system was operated with same LQR gains, and considered to be operated with same initial conditions, which are shown in the Table 2.

Table 2. Initial Conditions of the Four States

State	$\boldsymbol{\theta}$	$\boldsymbol{\alpha}$	$\dot{\boldsymbol{\theta}}$	$\dot{\boldsymbol{\alpha}}$
Initial Condition	0.0242	-0.0152	0	0

The behavior of both systems when Case 1 input signal (as shown in Figure 8) is provided and its ability to stabilize was observed after applying the obtained LQR gains. It was observed that both systems were stable in terms of rotary arm angle, after 2 s. In terms of the stability of the pendulum, the nonlinear system was quicker to reach the stability but with more oscillations observed from the rotary arm, which is clearly indicated in Figure 9 and Figure 10. The overshoot was higher when considering the pendulum angle, which is clearly indicated in Figure 9. Behavior of the systems are almost similar expect for the overshoot and oscillatory movement at the end of the nonlinear model.

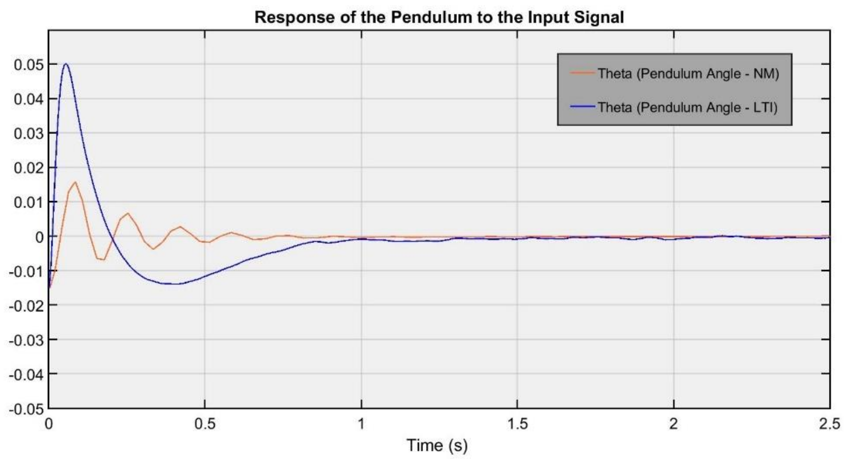

Figure 9. Response of the Pendulum for the Input Signal Case 1 (LTI and Nonlinear Models)

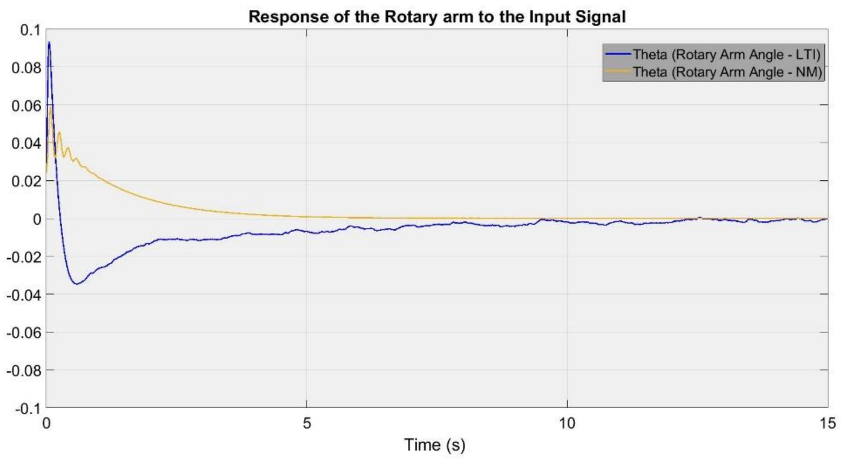

Figure 10. Response of the Rotary Arm for the Input Signal Case 1 (LTI and Nonlinear Models)

When the Case 2 signal (as shown in Figure 8), depicted in Figure 8 was fed in to the both systems, the pendulum was stabilized in less than 3 s, but the overshoot and oscillations are visibly higher in the nonlinear model. The behavior of the pendulum of both systems is plotted in Figure 11. The variation of the rotary arm angle under same conditions is plotted in Figure 12.

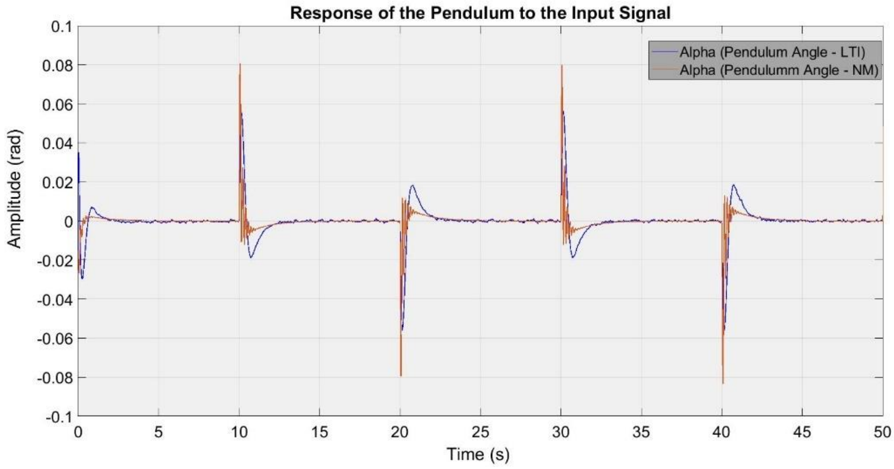

Figure 11. Behavior of the Pendulum for the Signal Case 2 (LTI and Nonlinear Models)

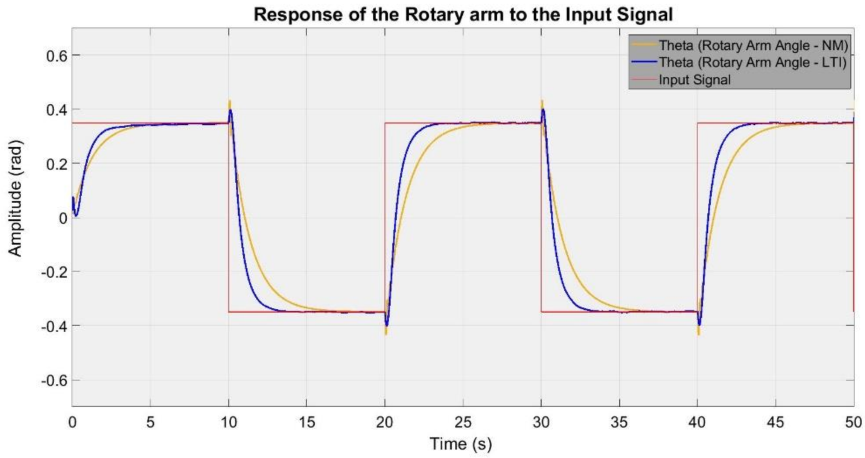

Figure 12. Behavior of the Rotary Arm for the Signal Case 2 (LTI and Nonlinear Models)

When observing the actual system response, for the Case 2 signal indicated in Figure 8, the system response was as in the Figure 13. For the safety of the actual system, the system was tested with more appropriate LQR gains, derived specifically for the actual system, as factors like damping, friction, disturbances…etc., which were not modelled plays a considerable role in the response of the prototype. The actual response in terms of pendulum angle and the rotary arm angle is depicted in Figure 13.

Figure 13. Behavior of the Rotary Arm of the Prototype

It was observed that though the Linear Time Invariant system was modified to attain a closer resemblance to the nonlinear model, there are still some deviations in smoothness when tested with same controller gains. The LQR controller gains applied for the three systems are as in the Table 3.

Table 3. LQR Gains Applied for the Models

Model	**LQR Gains (K)**			
LTI	-1.0000	12.8555	-1.4385	1.6943
Nonlinear	-1.0000	12.8555	-1.4385	1.6943
Prototype	-5.261	28.16	-2.758	3.219

4 CONLUSION

The use of state space model, nonlinear model and the actual apparatus in a virtual platform, to deliver knowledge on basics of modelling of dynamics system to the beginners in control systems engineering, has been presented in this paper. The availability of the state space model allows the students to improve their understanding on basics. With the addition of other parameters, a linear system closer to the nonlinear model of the Rotary Inverted Pendulum system has been derived, allowing the students to identify the factors contributing to the difference in linear and nonlinear systems. In addition, students who have the basic knowledge can use these systems as a reference to develop their own model from scratch. This platform can be used as a pre lab session for students, who are to perform advanced controlling experiments on the real system. With the visual representation of the system response, the system is more understandable than the graphs.

With already available software the virtual platform provides access to the students anytime, where they can perform different experiments by changing the parameters, which is not possible with the actual system. Matlab Simscape Multibody environment used by the virtual platform is common software, which is already available with the control systems engineering students. Therefore, this platform can be put into use even in the onsite teaching environment as it provides more exposure to the basics of control systems engineering than a simple lab experiment, where students are not allowed to perform experiments, which may threat the safety of the equipment and students. This virtual platform would be a great tool to guide students in various level of understanding towards autonomous learning. There is a considerable difference in the LQR gains applied for prototype from the other two models. This is mainly due to the unaccounted factors like friction, damping and disturbances. In future, the LTI and Nonlinear models can be further improved to resemble the protype closely, by modelling the factors mentioned above.

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